Robust H-infinity Takagi-Sugeno Fuzzy Controller Design for a Bilateral Tele-operation System via LMIs

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ABSTRACT:
This paper presents a new approach to a robust fuzzy controller design for the bilateral teleportation system with varying time delays using linear matrix inequalities. Communication channels are considered with different forwarding and returning time delays. The time delays of communication channels are assumed to be unknown and randomly time varying, but the upper bounds of the delay interval and the derivative of the delay are assumed to be known. In order to design the controllers, first, an impedance controller is designed for the master system to achieve desired impedance behavior for the master. Then, nonlinear Euler-Lagrange equation of motion of the slave system is linearized in the neighborhood of some operating points. In the sequel, an open-loop scheme is considered for the slave system. The linear model of the slave system has two imaginary/unstable poles. The slave system is stabilized by a PD-controller to be used in the open-loop scheme. To design the slave controller, the tele-operator block diagram is rearranged such that the tele-operator block diagram converts to a standard representation of a feedback control system which helps us to design a robust *H-infinity* controller for the slave system. The local linear models of the system are combined to form a Takagi-Sugeno fuzzy model of the whole tele-operation system. A Lyapunov-Krasovskii function is defined to analyze the closed-loop system's stability and derive a sufficient delay-dependent stability criterion. An *H-infinity* performance index is defined and the design criteria for the slave controller are expressed as a set of LMIs, which can be tested readily using standard numerical software.

KEYWORDS: Bilateral force feedback control, Robust control, Takagi-Sugeno fuzzy system, LMI.

1. INTRODUCTION

Stability analysis and control design problems of time delay systems have drawn an increasing attention during recent decades [1]-[2]. Since 2000, Takagi-Sugeno (T-S) fuzzy model approach has been extended to deal with analysis and control problems of nonlinear systems with time delay [3]. As an application, teleoperation is one of the most interesting areas of such systems [4]-[8]. A teleoperation system consists of a dual robot system in which a remote slave robot tracks motion of a master robot, which is, in turn, commanded by a human operator. Information is transmitted between master and slave via a communication media. The internet is of the most common communication channels used in this field. Time delay, induced by the internet, is time varying and it is well-known in the literature that time delay is a major source of instability and performance debasement [1].

In teleoperation, a human operator conducts a task in a remote environment via master and slave manipulators. Providing contact force information to the human operator can improve task performance. When this is done, the contact force is said to be reflected to the human operator and the teleoperator is said to be controlled bilaterally [4]. The field has wide applications in areas such as operations in hazardous environments, undersea exploration, robotic surgery, drilling, etc [9]. The reader is referred to [8]-[9] for detailed surveys of the various schemes developed for the problem of bilateral teleoperation.

A number of different control schemes have been proposed in the literature for teleoperation systems to provide a reliable and satisfactory control system [9]. Furthermore, over the last years, the number of applications of fuzzy logic to mobile robot control has increased significantly [10], mainly due to its capabilities to cope with imprecise information and the flexibility of nonlinear control laws.
However, this paper presents a new approach to deal with not only the nonlinear nature, but also the stability analysis and control design problem of the

teleoperation system. In other words, the approach allows further taking into account the nonlinearities of the teleoperation system. in addition, it utilizes a new Lyapunov-Krasovskii functional (LKF) which causes to derive a new less conservative stability analysis criterion, and also provides a robust *H-infinity* control system design for slave system. The criteria for stability and control design are presented as a set of matrix inequalities.

In this paper, the environment is assumed to be passive. In this scenario, and in the presence of delay, the nonlinear slave teleoperation system, which is modeled by Euler-Lagrange's equations of motion [11], is linearized in the neighborhood of some operating points. An impedance controller is considered for the master system. In the sequence of the design problem solution, the control system block diagram is rearranged such that the slave control system design is converted to design a robust controller for an equivalent standard feedback control system. Moreover, the resulting linearized models are used to construct a T-S fuzzy model of the teleoperation system. The obtained T-S fuzzy model is fed to a robust stability analysis problem. Furthermore, an LKF is defined and its derivative is used to construct the stability criteria based on a set of LMIs. In the sequel, an *H-infinity* index is defined and a set of LMIs is derived for slave controller design problem.

The paper is organized as follows. Section 2 presents the basic teleoperator configuration. The design problem is explained with details and formulated in a standard representation of control system. Section 3 provides the main results; the obtained sufficient delay-dependent conditions for stability and controllers design. Section 4 shows numerical simulations. Finally, in Section 5, a brief discussion about our results is presented.

2. PROBLEM STATEMENT

Fig. 1 shows the block diagram representation of a bilateral teleoperation system. In Fig. 1, P_M denotes the overall master side dynamics, including the master robot and its controller. K_s and P_s represent the slave side controller and the robot dynamics, respectively. Furthermore, the forwarding and returning communication channels time delays are labeled as d_1 and d_2, respectively. f_h and f_s are the forces applied by the operator on the master and the force applied on the slave, respectively. Meanwhile, f_s^d shows the slave force signal sent to the master through the communication channel. In this scheme, the angular velocity information of the master is transmitted to the slave through the communication channel and the force information of the slave is reverted back to the master through the channel. The signals from and to the channel are related as follows:

$$\theta_m^d(t) = \theta_m(t - d_1) \quad (1)$$

$$f_s^d(t) = f_s(t - d_2) \quad (2)$$

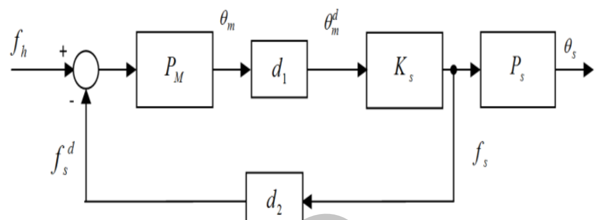

Fig. 1. The teleoperation control system configuration.

A computer network such as the internet can be employed as a communication channel. One of the main factors affecting teleoperation is the transmission time delay between the master and the slave.

Suppose the local dynamics of the master is expressed as a 1-degree-of-freedom (DOF) mass-damper system for an arbitrary operating point as follows:

$$m_m \dot{\theta}_m + b_m \theta_m = u_m + f_h \quad (3)$$

where θ and u are angular velocity and torque. m and b are mass and viscous coefficients, subscript *'m'* denotes the master, and finally, f_h is the force applied by the operator to the master.

Neglecting friction or other disturbances, as represented in [11], the Euler-Lagrange's equation of motion for an n-link slave robot is given as:

$$M_s(q_s)\ddot{q}_s + C_s(q_s, \dot{q}_s)\dot{q}_s + g_s(q_s) = \tau_s - f_e \quad (4)$$

where the subscript s indicates the slave variables, q_s is the $n \times 1$ vector of joint displacement, $\dot{q}_s$ is the $n \times 1$ vector of joint velocity, τ_s is the $n \times 1$ vector of applied torque, $M_s(q_s)$ is the $n \times n$ symmetric positive definite manipulator inertia matrix, $C_s(q_s, \dot{q}_s)$ is the $n \times n$ matrix of Centripetal and Coriolis torques and $g_s(q_s)$ is the $n \times 1$ gradient of the gravitational potential energy. f_e is the environmental force acting on the slave robot when it contacts the environment.

As a case study, consider the nonlinear dynamics of a 1-link robot [11] as following:

$$F(\theta, \dot{\theta}, \ddot{\theta}) = \frac{1}{12} m \ell^2 \ddot{\theta} + m(\ell - r)\ddot{\theta} + (mg \frac{\ell}{2} \cos(\theta))\theta = u \quad (5)$$

Linearizing the slave nonlinear dynamics using Taylor's series in the neighborhood of some equilibrium operating Q -points give us local linear models.

$$\left.\frac{\partial F}{\partial \theta}\right|_Q \cdot \theta + \left.\frac{\partial F}{\partial \dot{\theta}}\right|_Q \cdot \dot{\theta} + \left.\frac{\partial F}{\partial \ddot{\theta}}\right|_Q \cdot \ddot{\theta} = u \quad (6)$$

$$[mg\frac{\ell}{2}(\cos(\theta) - \theta\sin(\theta))]\theta + 0 \cdot \dot{\theta} + [\frac{1}{12}m\ell^2 + m(\ell - r)]\ddot{\theta} = u \quad (7)$$

The local model (6) can be shown as following:

$$\alpha\ddot{\theta} + \beta\theta = u \quad (8)$$

where:

$$\alpha \triangleq [\frac{1}{12}m\ell^2 + m(\ell - r)]$$

$$\beta \triangleq [mg\frac{\ell}{2}(\cos(\theta) - \theta\sin(\theta))]$$

α and β are calculated in the operating Q -points.

The open-loop transfer function of the local model of the slave system $T_{ols}(s)$ is:

$$T_{ols}(s) = \frac{\Theta(s)}{U(s)} = \frac{1}{\alpha s^2 + \beta} \quad (9)$$

Eq. (9) indicates the open-loop dynamics of the local slave model has one unstable or two imaginary poles.

2.1. Master controller design

The impedance control is one of the widely used approaches for teleoperation applications [8]. An impedance controller relates the applied force and the velocity of a manipulator tip. Assume that the desired impedance for the master is given by:

$$M\,\ddot{\theta}_m + B\,\dot{\theta}_m + K\,\theta_m = f_h - f_s^{\,d} \quad (10)$$

where M , B and K are the inertia, damping and stiffness of the desired impedance, respectively.

Noise problems make the measurement of the angular acceleration very difficult. Therefore, the master side control law is derived such that the acceleration $\ddot{\theta}_m$ is not measured directly [12]. So, by combining (9) and (4), the control law of the master is obtained as follows:

$$u_m = \left(b_m - \frac{m_m}{M}B\right)v_m + \left(\frac{m_m}{M} - 1\right)f_h - \frac{m_m}{M}\left(f_s^{\,d} + Kx_m\right) \quad (11)$$

2.2. Slave controller design formulation

We consider an open-loop controller scheme for the slave manipulator to track the position of the master. The open-loop slave dynamics (8) has one unstable or two poles on the imaginary axis and the system will be faced with the stability problem. To overcome the instability problem, at first, a PD controller is used for each local slave model to stabilize it. Consider a PD-controller as following:

$$C_{PDi\,s}(s) = k_{pi} + k_{di}s \quad (12)$$

where k_{pi} is proportional gain and k_{di} is derivative constant. For each local model, the PD-controller is designed such that the overall slave side transfer function gives a desired dynamics. In other words, suppose the controller $C_{PDi\,s}(s)$ is imposed to the local slave model (8), the closed-loop transfer function of the slave system will be:

$$P_{si}(s) = \frac{C_{PDi\,s}(s)T_{ols}(s)}{1 + C_{PDi\,s}(s)T_{ols}(s)} = \frac{(k_{di}/\alpha)s + (k_{pi}/\alpha)}{s^2 + (k_{di}/\alpha)s + (k_{pi} + \beta)/\alpha} \quad (13)$$

Assume that the desired characteristic polynomial of the slave dynamics is given by:

$$M_s\ddot{\theta}_s + B_s\dot{\theta}_s + K_s\theta_s = 0 \quad (14)$$

Therefore, the PD-controller constants are obtained as following:

$$k_{di} = B_s/M_s$$
$$k_{pi} = (K_s - \beta)/M_s \quad (15)$$

Furthermore, one of the most difficulties to use an open-loop control is to reject the disturbance imposed on the plant [13]. In order to reject disturbances, a (sub)optimal disturbance canceling technique was developed [13]. This action does not enter to the stage of the control system design stage, directly. Therefore, for time being, to have more simplicity, the disturbance cancelling-loop has not been shown in Fig. 1. Then, we rearrange the control system configuration components of Fig. 1 such that the slave side controller design is converted to be a controller of a standard representation of feedback control system design as shown in Fig. 2.

In Fig. 2, w shows the exogenous input (human operator force) and z denotes the controlled output which is the slave side velocity and/or displacement. Also, y and u are the equivalent measured output (delayed master side velocity command) and control inputs (slave robot force), respectively.

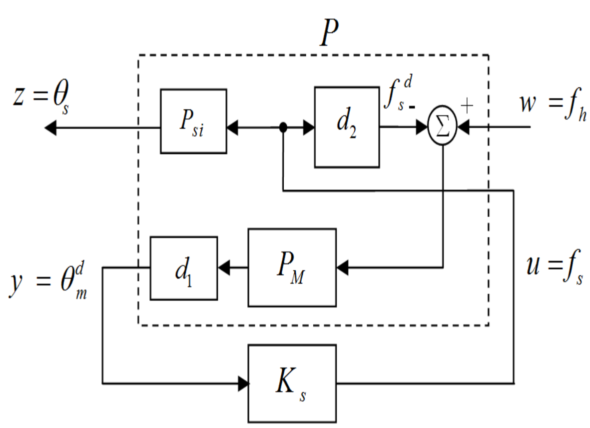

Fig. 2. The equivalent teleoperation control system configuration in a standard representation.

Now, the stat space representation of the equivalent plant P is derived. The slave plant state space representation is:

$$\begin{cases} \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & -\dfrac{K_s}{M_s} \\ 1 & -\dfrac{B_s}{M_s} \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} \dfrac{k_{pi}}{M_s} \\ \dfrac{k_{di}}{M_s} \end{bmatrix} u(t) \\ z_P(t) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \end{cases} \tag{16}$$

where $x_2 = \theta_s$ and its derivative gives the angular velocity. The master side overall dynamics state space representation is shown in Eq. (16).

$$\begin{cases} \begin{bmatrix} \dot{x}_3(t) \\ \dot{x}_4(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\dfrac{K}{M} & -\dfrac{B}{M} \end{bmatrix} \begin{bmatrix} x_3(t) \\ x_4(t) \end{bmatrix} + \begin{bmatrix} 0 \\ \dfrac{1}{M} \end{bmatrix} w(t-d_1(t)) + \begin{bmatrix} 0 \\ -\dfrac{1}{M} \end{bmatrix} u(t-d_1(t)-d_2(t)) \\ y_P(t) = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} x_3(t) \\ x_4(t) \end{bmatrix} \end{cases} \tag{17}$$

where $[x_3 \quad x_4]^T = [\theta_m \quad \dot{\theta}_m]^T$. Then, the overall state space representation of the local equivalent plant P will be:

$$\begin{cases} \dot{x}_P(t) = A_P x_P(t) + B_1 w(t-d_1(t)) + B_2 u(t) + B_3 u(t-d_1(t)-d_2(t)) \\ z_P(t) = C_{P1} x_P(t) \\ y_P(t) = C_{P2} x_P(t) \end{cases} \tag{18}$$

where:

$$x_P(t) = [x_1(t) \quad x_2(t) \quad x_3(t) \quad x_4(t)]^T$$

$$A_P = \begin{bmatrix} 0 & -K_s/M_s & 0 & 0 \\ 1 & -B_s/M_s & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -K/M & -B/M \end{bmatrix}$$

$$B_1 = [0 \quad 0 \quad 0 \quad 1/M]^T$$

$$B_2 = [k_{pi}/M_s \quad k_{di}/M_s \quad 0 \quad 0]^T$$

$$B_3 = [0 \quad 0 \quad 0 \quad -1/M]^T$$

$$C_{P1} = [I_{2\times2} \quad 0_{2\times2}]$$

$$C_{P2} = [0 \quad 0 \quad 0 \quad 1]$$

A local dynamic output feedback controller $u(t) = K_s\, y(t)$ is considered for slave side control system with the following state space representation:

$$K_s : \begin{cases} \dot{x}_k(t) = A_k x_k(t) + B_k u_k(t), & u_k(t) \equiv y(t) \\ y_k(t) = C_k x_k(t) + D_k u_k(t), & y_k(t) \equiv u(t) \end{cases} \tag{19}$$

Define the tracking error e as:

$$e(t) = w(t-d_1(t)) - S_w z(t) \tag{20}$$

where S_w determines the output required to track. Now, the open loop augmented stat space system model is defined as:

$$\begin{cases} \dot{x}_a(t) = A_a x_a(t) + B_{a1} w(t-d_1(t)) + B_{a2} u(t) + B_{a3} u(t-d_1(t)-d_2(t)) \\ z_a(t) = C_{a1} x_a(t) \\ y_a(t) = C_{a2} x_a(t) \end{cases} \tag{21}$$

where the augmented state vector is defined as: $x_a(t)^T = [x_e \quad x_P]^T$ with:

$$x_e(t) = \int_0^t e(\tau) d\tau \quad , \quad A_a = \begin{bmatrix} 0 & -S_w C_{P1} \\ 0 & A_P \end{bmatrix}$$

$$B_{a1} = \begin{bmatrix} I \\ B_1 \end{bmatrix}, \qquad B_{a2} = \begin{bmatrix} 0 \\ B_2 \end{bmatrix}, \qquad B_{a3} = \begin{bmatrix} 0 \\ B_3 \end{bmatrix},$$

$C_{a1} = [C_e \quad C_{P1}]$ and $C_{a2} = [0 \quad C_{P2}]$.

With the dynamic output feedback controller K_s, the closed-loop system of the local augmented system (20) is described by the following state space equations:

$$\begin{cases} \dot{x}(t) = A x(t) + A_d x(t-d(t)) + E w(t-d_1(t)) \\ z(t) = C_1 x(t) \\ y(t) = C_2 x(t) \end{cases} \tag{22}$$

where:

$x(t)^T = [x_a(t) \quad x_k(t)]^T = [x_e \quad x_P \quad x_k]^T$,

$d(t) := d_1(t) + d_2(t)$, $A = \begin{bmatrix} A_a + B_{a2} D_k C_{a2} & B_{a2} C_k \\ B_k C_{a2} & A_k \end{bmatrix}$,

$A_d = \begin{bmatrix} B_3 D_k C_{P2} & 0 \\ 0 & B_3 C_k \end{bmatrix}$, $C_1 = \begin{bmatrix} C_{a1} & 0 \end{bmatrix}$,

$C_2 = \begin{bmatrix} C_{a2} & 0 \end{bmatrix}$, and $E = [B_{a1}^T \quad 0]^T$.

2.3. T-S Fuzzy model of the teleoperator system

Local linear models are used to construct the T-S fuzzy model of the system. So, we can choose arbitrary membership functions for the desired operating points in order to collect the different local linear models of the system in various operating points together to obtain a T-S fuzzy representation of the teleoperator system. A deep introduction to T-S fuzzy systems with state time delay is found in [3].

Consider the local linear models of the augmented equivalent plant P as Eq. (20):

$$\begin{cases} \dot{x}_a(t) = A_{ai} x_a(t) + B_{a1i} w(t-d_1(t)) + B_{a2i} u(t) + B_{a3i} u(t-d_1(t)-d_2(t)) \\ z_a(t) = C_{a1i} x_a(t) \\ y_a(t) = C_{a2i} x_a(t) \end{cases}$$

$$i = 1, 2, \ldots, r \tag{23}$$

where r is the number local linear models. The nonlinear master-salve teleoperator system with time delay can be approximated by a time-delay T-S fuzzy model of the following form:

Plant Rule i **:** IF θ_1 is μ_{i1} and $\cdots$ and θ_p is μ_{ip}, THEN:

$$\begin{cases}\dot{x}_a(t)=A_{ai}x_a(t)+B_{a1i}w(t-d_1(t))+B_{a2i}u(t)+B_{a3i}u(t-d_1(t)-d_2(t))\\ z_a(t)=C_{a1i}x_a(t)\\ y_a(t)=C_{a2i}x_a(t)\end{cases}$$

$$i=1,2,\ldots,r \tag{24}$$

where r is the number of IF-THEN rules and equal to local linear models; $\theta_j(x)$ and μ_{ij} $(i=1,\ldots,r,\ j=1,\ldots,r)$ are respectively the premise variables and the fuzzy sets; the forwarding and returning time delays $d_1(t)$ and $d_2(t)$ are unknown but are assumed that:

$$\begin{cases}0\le d_1(t)\le \bar{d}_1<\infty, & \dot{d}_1(t)\le\tau_1<\infty\\ 0\le d_2(t)\le \bar{d}_2<\infty, & \dot{d}_2(t)\le\tau_2<\infty\end{cases} \tag{25}$$

and $\bar{d}:=\bar{d}_1+\bar{d}_2$

By fuzzy blending, the overall fuzzy model of the equivalent plant P is inferred as follows:

$$\begin{cases}\dot{x}_a=\sum_{i=1}^{r}h_i(\theta)[A_{ai}x_a(t)+B_{a1i}w(t-d_1(t))\\ \qquad +B_{a2i}u(t)+B_{a3i}u(t-d_1(t)-d_2(t))]\\ z_a(t)=\sum_{i=1}^{r}h_i(\theta)[C_{a1i}x_a(t)],\\ y_a(t)=\sum_{i=1}^{r}h_i(\theta)[C_{a2i}x_a(t)],\end{cases}$$

$$i=1,2,\ldots,r \tag{26}$$

where $\theta=[\theta_1,\ldots,\theta_p]$ and $h_i(\theta)$ is the membership function corresponding to plant rule i such that $\sum_{i=1}^{r}h_i(\theta)=1$ with $h_i(\theta)\ge 0$.

In addition to the local PD-controller for stabilizing the slave dynamics, consider a dynamic output feedback controller for each local model; as follows:

Slave Controller Rule i **:** IF θ_1 is μ_{i1} and $\cdots$ and θ_p is μ_{ip}, THEN

$$K_s:\begin{cases}\dot{x}_k=A_{ki}x_k+B_{ki}u_k,\\ y_k(t)=C_{ki}x_k+D_{ki}u_k,\end{cases}\quad i=1,2,\ldots,r \tag{27}$$

$u_k(t)=y_a(t)$,

$y_k(t)=u(t)$

where slave controllers parameters A_{ki}, B_{ki}, C_{ki}, and D_{ki} are to be chosen.

The overall slave control law is thus inferred as:

$$\begin{cases}\dot{x}_k=\sum_{i=1}^{r}h_i(\theta)[A_{ki}x_k+B_{ki}u_k],\\ y_k(t)=\sum_{i=1}^{r}h_i(\theta)[C_{ki}x_k+D_{ki}u_k]\end{cases} \tag{28}$$

Combining (25) with (27), the closed loop fuzzy time-delay system Σ is written as:

$$\Sigma:\begin{cases}\dot{x}=\sum_{i=1}^{r}\sum_{j=1}^{r}h_i(\theta)h_j(\theta)[A_{ij}x+A_{dij}x(t-d(t))+E_{ij}w(t-d_1(t))],\\ z(t)=\sum_{i=1}^{r}h_i(\theta)[C_{1i}x], \qquad i=1,2,\ldots,r\end{cases} \tag{29}$$

where :

$x(t)^T=[x_a(t)^T \quad x_k(t)^T]^T=[x_e^T \quad x_P^T \quad x_k^T]^T$,

$d(t):=d_1(t)+d_2(t)$,

$$A_{ij}=\begin{bmatrix}A_{ai}+B_{a2i}D_{kj}C_{a2i} & B_{a2i}C_{kj}\\ B_{kj}C_{a2i} & A_{kj}\end{bmatrix},$$

$$A_{dij}=\begin{bmatrix}B_{a3i}D_{kj}C_{a2i} & 0\\ 0 & B_{a3i}C_{kj}\end{bmatrix},$$

$C_{1i}=\begin{bmatrix}C_{a1i} & 0\end{bmatrix}$ and $E_{ij}=[B_{a1i}^T \quad 0]^T$.

So, the control problem is to design a controller which robustly stabilizes the overall system Σ and causes the slave robot to track the master side commands.

3. MAIN RESULTS

3.1. Robust Stability Analysis

Now, we will present the stability condition for the general form of the proposed teleoperation control scheme.

Theorem 1: *System* Σ *in Eq. (28) with delays* $d_1(t)$ *and* $d_2(t)$ *satisfying Eq. (24) is asymptotically stable if there exist matrices* $P>0$, $Q_1\ge Q_2>0$, $Q_3\ge Q_4>0$, $M_1\ge M_2>0$, $M_3\ge M_4>0$, N_k, $k=1,\ldots,8$*, satisfying (29);*

$$\begin{bmatrix} \Pi_{11ij} & -N_1+N_2^T & -N_5+N_6^T & PA_{dij} & A_{ij}^T\Pi_{55} & N_1 & 0 & N_5 & 0 \\ * & \Pi_{22} & 0 & -N_3+N_4^T & 0 & N_2 & N_3 & 0 & 0 \\ * & * & \Pi_{33} & -N_7+N_8^T & 0 & 0 & 0 & N_6 & N_7 \\ * & * & * & \Pi_{44} & A_{dij}^T\Pi_{55} & 0 & N_4 & 0 & N_8 \\ * & * & * & * & -\Pi_{55} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -\bar{d}_1^{-1}M_1 & 0 & 0 & 0 \\ * & * & * & * & * & * & -\bar{d}_2^{-1}M_2 & 0 & 0 \\ * & * & * & * & * & * & * & -\bar{d}_2^{-1}M_3 & 0 \\ * & * & * & * & * & * & * & * & -\bar{d}_1^{-1}M_4 \end{bmatrix} < 0 \quad (29)$$

$for\ i, j = 1, 2, \ldots, r$

where:

$\Pi_{11ij} \triangleq A_{ij}^T P + PA_{ij} + Q_1 + Q_3 + N_1 + N_1^T + N_5 + N_5^T$,

$\Pi_{22} \triangleq -(1-\tau_1)(Q_1 - Q_2) - N_2 - N_2^T + N_5 + N_5^T$,

$\Pi_{33} \triangleq -(1-\tau_2)(Q_3 - Q_4) - N_6 - N_6^T + N_7 + N_7^T$,

$\Pi_{44} \triangleq -(1-\tau_1-\tau_2)(Q_2 + Q_4) - N_4 - N_4^T - N_8 - N_8^T$,

$\Pi_{55} \triangleq (\bar{d}_1 M_1 + \bar{d}_2 M_2 + \bar{d}_2 M_3 + \bar{d}_1 M_4)$.

$$V(t) = V_1 + V_2 + V_3 + V_4 + V_5 \quad (30)$$

$V_1 \triangleq x^T(t)Px(t)$,

$V_2 \triangleq \int_{t-d_1(t)}^{t} x^T(s)Q_1 x(s)ds + \int_{t-d_1(t)-d_2(t)}^{t-d_1(t)} x^T(s)Q_2 x(s)ds$,

$V_3 \triangleq \int_{-\bar{d}_1}^{0}\int_{\beta}^{0} \dot{x}^T(t+\alpha)M_1\dot{x}(t+\alpha)d\alpha d\beta + \int_{-\bar{d}_1-\bar{d}_2}^{-\bar{d}_1}\int_{\beta}^{0} \dot{x}^T(t+\alpha)M_2\dot{x}(t+\alpha)d\alpha d\beta$,

$V_4 \triangleq \int_{t-d_2(t)}^{t} x^T(s)Q_3 x(s)ds + \int_{t-d_1(t)-d_2(t)}^{t-d_2(t)} x^T(s)Q_4 x(s)ds$,

$V_5 \triangleq \int_{-\bar{d}_2}^{0}\int_{\beta}^{0} \dot{x}^T(t+\alpha)M_3\dot{x}(t+\alpha)d\alpha d\beta + \int_{-\bar{d}_1-\bar{d}_2}^{-\bar{d}_2}\int_{\beta}^{0} \dot{x}^T(t+\alpha)M_4\dot{x}(t+\alpha)d\alpha d\beta$.

where $P>0$, $Q_i>0$, $M_i>0$ are matrices to be determined. The time derivative of $V(t)$ along the solution, using Newton-Lebniz formula and Schur complement theorem give us the stability conditions. Because a similar rule to that of [14] is used, the proof of the theorem is omitted.

Remark 1. Using new and different LKF causes to derive new LMI conditions for stability criteria. When we use more information to derive the stability condition, it will be less conservative. There are two different delays in forwarding and returning channels. Since the properties of the delays are different due to transmission conditions, it is not acceptable to combine them together. Theorem 1 presents a delay dependent stability condition for system Σ which is a matrix inequality and can be readily checked using well known numerical software.

Remark 2. The stability condition of Eq. (29) is for the nominal system. However, the results can be readily extended to the system with uncertainties.

3.2. Slave side *H-infinity* controller synthesis

We introduce the following *H-infinity* performance index for system Σ, with zero initial conditions, as:

$$J = \int_0^\infty [z^T(t)S_w^T S_w z(t) - \gamma^2 w^T(t-d_1(t))w(t-d_1(t))]dt \quad (31)$$

Remark 3. Human operator is one of the important components of teleoperation system. S/he is considered as a passive subsystem in a teleoperation system which does not intentionally destabilize the system. Therefore, human operator avoids sudden changes in commands, imposed on the teleoperator system. This remark is in-line with the assumptions that usually imposes by the other researchers which use passivity theory for their designs [15]-[16]. To formulate this description in design procedure, assume that:

$$q_1 w(t) \le w(t-d_1(t)) \le q_2 w(t) \quad (32)$$

where q_1 and q_2 are real numbers. Then, one has:

$$\int_0^\infty [(z^T(t)S_w^T S_w z(t) - \gamma^2 q_2^2 w^T(t)w(t))]dt \le J \le \int_0^\infty [(z^T(t)S_w^T S_w z(t) - \gamma^2 q_1^2 w^T(t)w(t))]dt \quad (33)$$

Satisfying H_∞ performance index in Eq. (31) requires the right-hand side term of Eq. (33) to be negative.

Theorem 2: *The closed loop system Σ is robustly stable with tracking error bound γ for time delay satisfying Eq. (24), if there exist matrices $P>0$, $Q_1 \ge Q_2 > 0$, $Q_3 \ge Q_4 > 0$, $M_1 \ge M_2 > 0$, $M_3 \ge M_4 > 0$, N_k, $k = 1,\ldots,8$ and scalar q satisfying Eq. (34);*

$$\begin{bmatrix} \Pi_{11ij}+C_{1i}^T S_w^T S_w C_{1i} & -N_1+N_2^T & -N_5+N_6^T & PA_{dij} & PE_{ij}+A_{ij}^T\Pi_{66}E_{ij} & A_{ij}^T\Pi_{66} & N_1 & 0 & N_5 & 0 \\ * & \Pi_{22} & 0 & -N_3+N_4^T & 0 & 0 & N_2 & N_3 & 0 & 0 \\ * & * & \Pi_{33} & -N_7+N_8^T & 0 & 0 & 0 & 0 & N_6 & N_7 \\ * & * & * & \Pi_{44} & A_{dij}^T\Pi_{66}E_{ij} & A_{dij}^T\Pi_{66} & 0 & N_4 & 0 & N_8 \\ * & * & * & * & \Pi_{55ij}-\gamma^2 q^2 I & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -\Pi_{66} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & -\bar{d}_1^{-1}M_1 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & -\bar{d}_2^{-1}M_2 & 0 & 0 \\ * & * & * & * & * & * & * & * & -\bar{d}_2^{-1}M_3 & 0 \\ * & * & * & * & * & * & * & * & * & -\bar{d}_1^{-1}M_4 \end{bmatrix} < 0$$

for $i, j = 1, 2, \ldots, r$ (34)

where

$\Pi_{55ij} \triangleq E_{ij}^T \Pi_{66} E_{ij}$,

$\Pi_{66} \triangleq (\bar{d}_1 M_1 + \bar{d}_2 M_2 + \bar{d}_2 M_3 + \bar{d}_1 M_4)$

and the other notations are the same as in Theorem 1.

Proof: Following the procedure like the stability analysis and the assuming system Σ with zero initial conditions, it can be concluded that:

$$J = \int_0^\infty [z^T(t) S_w^T S_w z(t) - \gamma^2 w^T(t-d_1(t)) w(t-d_1(t)) + \dot{V}(\cdot)]\, dt \le \int_0^\infty [(z^T(t) S_w^T S_w z(t) - \gamma^2 q^2 w^T(t) w(t) + \dot{V}(\cdot))]\, dt \quad (35)$$

since $V(\cdot)\big|_{t=0} = 0$ and $V(\cdot)\big|_{t=\infty} \to 0$. Then, one obtains $J < 0$, if the LMI Eq. (33) holds over the entire uncertain domain. The LMI implies that $J < 0$ for any nonzero $w \in L_2$ and over the entire uncertain domain, i.e. the closed-loop system Σ is robustly stable for any time delay satisfying the constraints in Eq. (24), which concludes the proof.

4. SIMULATION RESULTS

The step response is the most common and generic dynamic test for controls. Hence, the tracking control performance is evaluated by applying a step force exerted by a human operator. Two different sets of parameters of desired behavior and manipulators were considered with equal varying time delay in forwarding and returning communication paths.

Example 1: The parameters were set to $M = 1\,kg$, $B = 1\,Ns/m$ and $K = 1\,N/m$ for the master robot. For the slave robot, the parameters were set to $\ell = 0.1\,m$, $r = 0.05\,m$, $m = 5\,kg$ and $g = 9.8\,N/kg$. We linearize the slave system about $\bar{\theta} = 30^\circ$ and $\bar{\theta} = 60^\circ$. The linearized models are obtained as: $0.245\ddot{\theta} + 1.47\theta = u$ and $0.245\ddot{\theta} - 0.98\theta = u$, respectively. The desired parameters for slave system were set to: $M_s = 1$, $B_s = 6$ and $K_s = 9$. The local PD-controllers were obtained as: $K_{PDs1} = 1.47s + 0.735$ and $K_{PDs2} = 1.47s + 3.185$ respectively. Also, H_∞ performance index was set to $\gamma = 0.01$.

Time delay constraints were considered as $\bar{d}_1 = \bar{d}_2 = 0.4$ sec and $\tau_1 = \tau_2 = 0.4$. The LMIs were solved by PENBMI Software [17]. The corresponding stable slave side controller was obtained with the following minimum realization:

A_k = [-2.439e+004 -864.9; -941.5 -155],

B_k = [-36.94; -46.32],

C_k = [84.37 3.202], D_k = -7.288e-017 (36)

To obtain a slave side controller with unit DC gain, a negative proportional gain was cascaded with the slave side controller equals to -4.8715. Fig. 3 shows the simulation results of H_∞ slave side controller. The figure shows excellent tracking performance. However, the transient behavior should be improved.

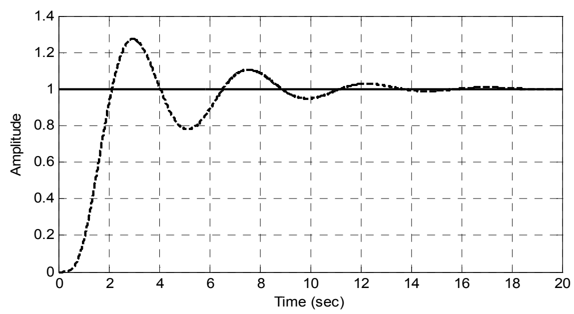

Fig. 3. Step response of H_∞ Controller for position tracking by x_s (--).

Example 2: In the second simulation, the parameters were set to $M = 1\,kg$, $B = 3\,Ns/m$ and $K = 2\,N/m$ for the master robot. For the slave robot, the parameters were set to $\ell = 0.2\,m$, $r = 0.1\,m$,

$m = 10\,kg$ and $g = 9.8\,N/kg$. We linearize the slave system about $\bar{\theta} = 30^\circ$ and $\bar{\theta} = 60^\circ$. The linearized models are obtained as: $1.03\ddot{\theta} + 4.45\theta = u$ and $1.03\ddot{\theta} - 7.81\theta = u$, respectively. The desired parameters for the slave system were set to: $M_s = 1$, $B_s = 4$ and $K_s = 4$. The local PD-controllers were obtained as: $K_{PDs1} = 4s - 0.45$ and $K_{PDs2} = 4s + 11.81$ respectively. Also, H_∞ performance index was set to $\gamma = 0.01$.

Time delay constraints were considered as $\bar{d}_1 = \bar{d}_2 = 0.4\,\text{sec}$ and $\tau_1 = \tau_2 = 0.4$. The LMIs were solved by PENBMI Software [17]. The corresponding stable slave side controller was obtained with the following minimum realization:

A_k = 1.0e+005*[-1.7841 -1.0910; -0.0189 -4.9885],

B_k = 1.0e+005*[1.9028; 0.6423],

C_k = 1.0e+003*[-3.7816 -5.5297],

D_k = 5.8218e+004

To obtain a slave side controller with unit DC gain, a negative proportional gain was cascaded with the slave side controller equals to -5.425. Fig. 4 shows the simulation results of H_∞ slave side controller. The figure shows excellent tracking performance. The transient behavior has been improved compared to the example 1 by selecting different desired impedance behavior.

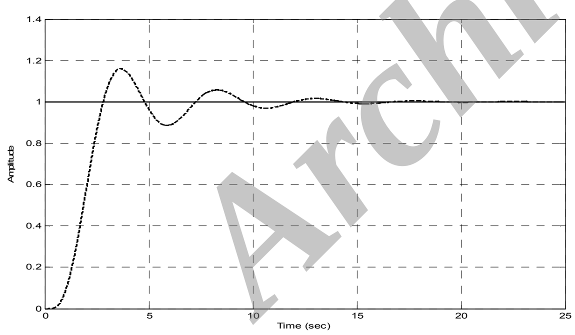

Fig. 4. Step response of H_∞ Controller for position tracking by x_s (--).

5. CONCLUSIONS

In this paper, a new formulation was presented for control system design of a bilateral teleoperator system. A local closed-loop configuration was considered for the master system and a local open-loop configuration was considered for the slave system. Open-loop scheme of the slave side allowed us to formulate the design problem in a standard representation of a control system in an H_∞ framework. To design the controllers, first, an impedance controller was designed for the master system to achieve desired impedance behavior for the master. Then, the nonlinear Euler- Lagrange's equation of motion of the slave system was linearized in the neighborhood of some operating points. The slave system dynamics was located in a series configuration relative to the other portions of the control system. Because of the oscillatory or unstable dynamics of the local linear models of the slave system, local PD-controllers were considered in addition to the main slave controller to stabilize the local slave system dynamics. The main slave side controller was designed based on a Takagi-Sugeno fuzzy model strategy. Using a Lyapunov-Krasovskii functional, new criteria were derived to analyze the close loop system stability. In the sequel, an *H-infinity* output tracking performance index was considered and the design criteria for the slave controller were expressed as a set of LMIs, which could be solved using standard numerical software.

(37)

6. ACKNOWLEDGEMENTS

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