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Sensitivity analysis for criteria values in decision making matrix of SAW method

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Abstract

All of organizations around the world, try to increase competitive ability regards to other similar companies. In this way, decision making processes are one of the most important activities for help them. The multiple criteria decision making methods create for help better decision making in multidimensional environment to monitor organizational resources and, generally, for ranking them and their departments.

One of the simplest and applicable methods in multiple criteria decision making methods is SAW method (simple additive weighting method).The general problem in MADM methods is lacking of complementary information for final decision making. In optimizations methods (for example linear programming) the sensitivity analysis are used for produce complementary information and this reason helps for popularity of these methods. Although MADM methods don't belong optimizations methods, but in this paper try to use sensitivity analysis approach for produce complementary information by determination of criteria values domain in decision making matrix.

Keywords : Multiple criteria decision making , Ranking Methods, SAW, Sensitivity analysis.

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1 Introduction

In world industrial revolution, especially, since World War II, most of the mathematician and management scientists pay attention to classical optimization methods with only one criterion.

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They change his paradigms to complex decision making, in recent decay and monitor carefully multi criteria decision making methods. These methods divided two main branches: MODM (multiple objective decision making) that use for designating activities and MADM (multiple attribute decision making) that use for find priority of alternatives and ranking them. As shows in figure no.1, total process of multiple criteria decision making begins with goal and criteria determination and then if it possible; decision making matrix must be created. Development of operations follows by some preprocesses functions, for example finding the utility of criteria, dimensionless activities and the most important stage means weighting function. Decision makers must be allocated some values to criteria as its importance that calls criteria weights. At the end of these processes the rank of each criterion must be finding.

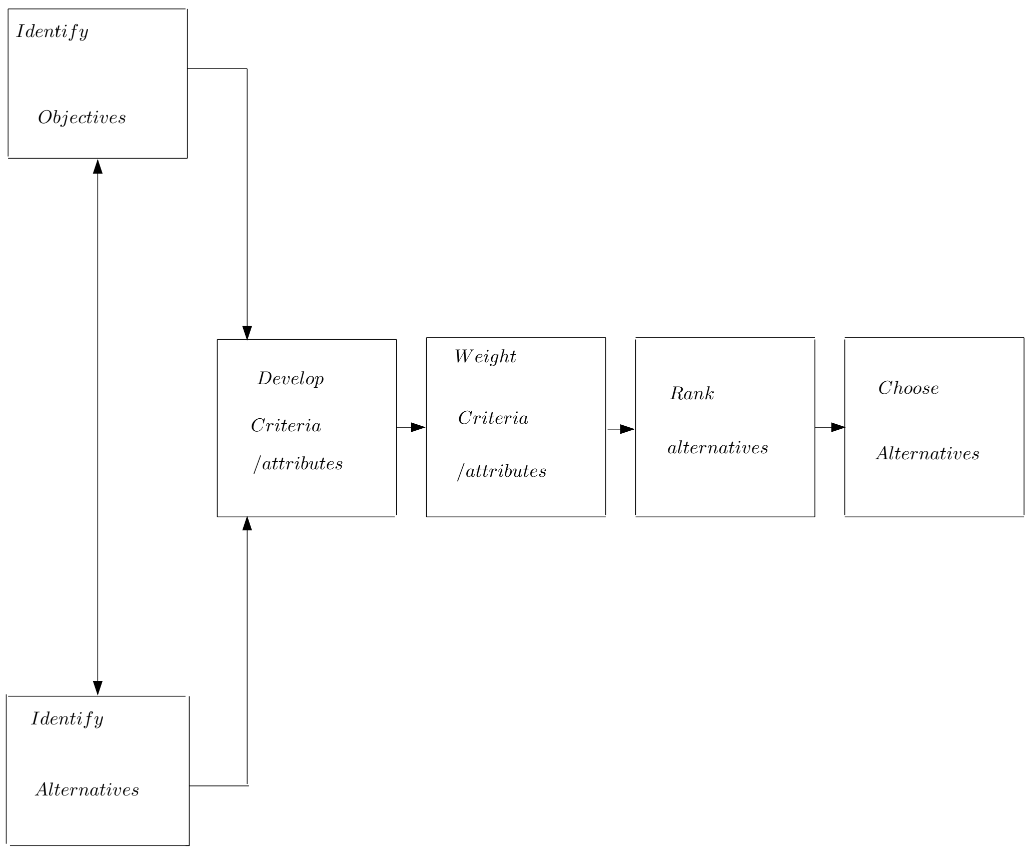

Fig. 1 Main process of multiple criteria decision making

Alternative ranking in decision making process for decision maker and alternatives is very important. Organizations are in efforts to establish a permanent relationship with set of suppliers, as in case of important and unpredicted events, able to replace suitable supplier. In industrial and competitive world, a decision maker try to attendance in the competitive market for resolve restrictions and select markets face that not exclusive.
On the other hand, organizations in order to maintain its competitive position and increasing competitive power, effort to reinforce their capabilities and values that important for customers. Any decision maker for select the best alternative from all possible alternatives set some criteria. Always Issues in multiple criteria decision making are selection base activities that select best alternative from m possible alternatives and to do this,

n criteria play their roles. The weight or importance of criteria has a very important effect on these processes. The SAW Technique is a common algorithm in multiple criteria decision making approach. This method with considering a set of criteria, calculate the value of each alternative and ultimately show the ranks of all possible alternatives. Since the change of data is often a problem in MADM, so sensitivity analysis after ranking can help us to effective adoption of a correct decision. In this article a new type of sensitivity analysis will be use in MADM problems. This type of sensitivity analysis use in the SAW method (one of the popular MADM methods) and study the relations obtained it subject to with changing the values of alternatives for each of criteria an effective decision making can be gained.
At the end of article, an example, presents for show the obtained relationship and method verifiability.

2 SAW technique

Compensatory models in MADM are very important in decision making because trade of between criteria is allowed. This method used to calculate each alternative values that product by criteria weights and at the end rank of them obtained. Suppose the set of alternatives are:

$$A = (a_1, a_2, \dots, a_n)$$

Set of criteria is:

$$C = (c_1, \dots, c_n)$$

And set of criteria weights are:

$$W = (w_1, w_2, \dots, w_n)$$

So, ranking of alternatives can be calculated as shown in table (1):

Table (1) the SAW method calculation matrix

Weights	w_1	w_2	w_j	$\cdots$	w_n	
alternatives/ Criteria	C_1	C_2	C_j	$\cdots$	C_n	Final Value
a_1	f_{11}	f_{12}	f_{13}	$\cdots$	f_{1n}	α_1
a_2	f_{21}	f_{22}	f_{23}	$\cdots$	f_{2n}	α_2
a_i	f_{i1}	f_{i2}	f_{i3}	$\cdots$	f_{in}	α_i
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
a_m	f_{m1}	f_{m2}	f_{m3}	$\cdots$	f_{mn}	α_m

Which:
w_j is scale less criteria weights
f_{ij} is scale less value of ith alternative for jth
α_i is final value for ith alternative that calculate as below:

$$\alpha_i = \sum_{j=1}^{n} w_j \times f_{ij} \tag{2.1}$$

3 Sensitivity analysis

The purpose of sensitivity analysis is determining the quantity of deviation of ith alternative value from jth criterion; subject to the arrangement of alternatives ranking still remain without changes. With considering the table (1), d_{ij} defined as, amount of deviation in f_{ij}, subject to the alternatives ranking still remain without change.
In this way:
d^+_{ij} is positive deviation of ith alternative from jth criterion
d^-_{ij} is negative deviation of ith alternative from jth criterion
d_{ij} is total deviation of ith alternative from jth criterion
α^+_i first dominant value into ith alternative
α^-_i first worse value into ith alternative

Since d_{ij} is total deviation of ith alternative from jth criterion, so it means distance between d^+_{ij} and d^-_{ij}, so. d_{ij} will be calculated as following formula:

$$d^-_{ij} < d_{ij} < d^+_{ij} \tag{3.2}$$

The d^+_{ij} through the formula no.(3.2) will be calculated as follow:

$$d^+_{ij} = (\alpha^+_i - \alpha_i) \times \sum_{j=1}^{n} w_j \tag{3.3}$$

Similarly, the d^-_{ij} through the formula no.(3.2) will be calculated as follow:

$$d^-_{ij} = (\alpha^-_i - \alpha_i) \times \sum_{j=1}^{n} w_j \tag{3.4}$$

Considering that the changes in value of alternative with lowest rank in negative direction should not have any affect on ranking, so, the only restriction in negative direction must be as follow:

$$f_{ij} + d^-_{ij} \geq 0 \tag{3.5}$$

and then:

$$-f_{ij} < d^-_{ij} \tag{3.6}$$

Therefore changes have allowed for value of alternative with lowest rank will be:

$$-f_{ij} \leq d_{ij} \leq (\alpha^+_i - \alpha_i) \times \sum_{j=1}^{n} w_j \tag{3.7}$$

While changes in value of alternative with the highest rank in positive direction have not any affect on ranking, therefore, the changes have allowed the value of ith alternative for jth criterion with the highest rank is:

$$(\alpha^-_i - \alpha_i) \times \sum_{j=1}^{n} w_j \leq d_{ij} \tag{3.8}$$

4 Case study

Executive Managers in information technology projects, for implementation of management information systems, are trying to select the best contractor between four contractors.In this way, four people as experts, have to do this decision making.Criteria are including, time to complete the project (C_1), commissioning costs (C_2), Contractors records for projects implementation (C_3) and workforce capability (C_4). Table (2) is initial decision making matrix for the best contractor selection. The matrix includes integrated opinions of each four experts. Numbers within the matrix represent the amount of raw values and the first row represent of each criterion weights.

Table (2) initial decision making matrix

	C_1	C_2	C_3	C_4
Weights	0.4	0.8	0.6	0.2
a_1	7	8000	5	7
a_2	8	6500	3	5
a_3	10	6000	1	5
a_4	6	4000	9	10

First of all, the decision making matrix must be changes in scale less form, by probability scale less method. Table no. 3 shows the scale less decision making matrix. The row of weights should be done in same manner.

Table (3) scale less decision making matrix

	C_1	C_2	C_3	C_4
Weights	0.2	0.4	0.4	0.1
a_1	0.226	0.327	0.278	0.259
a_2	0.258	0.265	0.165	0.185
a_3	0.323	0.245	0.056	0.185
a_4	0.194	0.163	0.500	0.370

Total value for each alternative regards to formula no.(2.1) can be find and the results is as follow (Rounded to 3 digits):

$$\alpha_1 = 0.285, \quad \alpha_2 = 0.226, \quad \alpha_3 = 0.198, \quad \alpha_4 = 0.291$$

So, the ranks of all alternatives are:

$$\alpha_4 > \alpha_3 > \alpha_2 > \alpha_1$$

Since the company policy is permanent connection with information systems contractors and also for produce some extra operational, and also for obtain some extra operational and executive information about them, sensitivity analysis may be useful. Sensitivity analysis table is produce regards to above formula. Results have been shown in table no.(3); we can see the domain of each f_{ij}. For example the allowable deviation for alternative a_1 for criterion C_1 is about 0.03 in positive direction (it means that value can be increase) and 0.294 in negative direction (it means that value can be decrease) without any rank reversals in alternatives ranking.

Table (3) total deviations for each value of final decision making matrix

	C_1	C_2	C_3	C_4
a_1	$-0.294 < d_{11} < 0.030$	$-0.147 < d_{12} < 0.015$	$-0.196 < d_{13} < 0.020$	$-0588 < d_{14} < 0.060$
a_2	$0.0143 < d_{21} < 0.294$	$-0071 < d_{22} < 0.147$	$-0.095 < d_{23} < 0.196$	$-0.286 < d_{24} < 0.588$
a_3	$0.323 < d_{31} < 0.143$	$-0.245 < d_{32} < 0.071$	$-0.056 < d_{33} < 0.095$	$-0.185 < d_{34} < 0.286$
a_4	$-0.030 < d_{41}$	$-0.015 < d_{42}$	$0.020 < d_{43}$	$0.060 < d_{44}$

In this paper should be noted that sensitivity analysis briefly introduced for value of alternatives in SAW method. Sensitivity analysis for the weights and also, for other MADM methods will be discussed in next papers.

5 Conclusion

This article introduced sensitivity analysis in SAW method. In SAW method, alternatives ranks regards to criteria. This method is one of the individual, multiple criteria decision making methods but simply can be used for group decision making. Also, criteria weights can be finding with various methods. After obtaining alternatives rank, managers need to find the sensitivity of values and also, the domain of deviations in decision making matrix. This paper shows that by sensitivity analysis, decision makers can find extra information as decision supports, without any changes in alternatives ranking.
In this article, a new method for sensitivity analysis of numerical values in decision making matrix is presented, and also a case study done for model verification.

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