



Defuzzification Method for Solving Fuzzy Linear Systems

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Abstract

In this article, it is attempted to develop a method for solving an arbitrary general fuzzy linear system by using the weighted metric. Using the distance between two fuzzy numbers, the best and nearest approximation fuzzy symmetric vector can be obtained. Defuzzification can replace the $n \times n$ fuzzy linear system with two $n \times n$ crisp linear systems. Consequently, to obtain the best approximate solution for $n \times n$ fuzzy linear system the researcher must solve two $n \times n$ crisp linear systems. First considering the existing and uniqueness of fuzzy solution to $n \times n$ fuzzy linear system is done. Then numerical examples are presented to illustrate the proposed model.

Keywords: Fuzzy number; Defuzzification; Fuzzy linear system; Weighted distance.

1 Introduction

More and more, modeling techniques, control problems and operation research algorithms have been designed to fuzzy data since the concept of fuzzy number and arithmetic operations with these numbers was introduced and investigated first by Zadeh. Whereas system of simultaneous linear equations plays major roles in various areas, it is immensely important to establish mathematical models and numerical procedures for fuzzy linear systems and solve them. A general model for solving an arbitrary $n \times n$ fuzzy linear system whose coefficients matrix are crisp and the right hand side column is an arbitrary fuzzy number vector was first proposed by Friedman et al. [8]. They also studied duality fuzzy linear systems in [9]. Moreover, they used the embedding method given in [5] and replaced the original fuzzy linear system with $2n \times 2n$ crisp linear system. The numerical solution of fuzzy linear system is studied in [1, 2, 3]. The solution of $m \times n$ ($m \leq n$) original fuzzy linear system by $2m \times 2n$ crisp system is done in [4]. Many authors obtained the least

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square symmetric solution of $m \times n$ fuzzy general linear systems in which $m \leq n$.

In this study, the researcher gives a new method for solving an $n \times n$ fuzzy linear system, whose coefficients matrix is crisp and the right hand side column is an arbitrary fuzzy number vector, by using the weighted metric. First of all this study uses the concept of the symmetric triangular fuzzy number and introduces an approach to defuzzify a general linear system. Hence, defuzzification [13, 14, 10, 15] can make a mapping from the set of all fuzzy numbers to positive half plane (\mathbb{R}^2), and this enables us to replace $n \times n$ fuzzy system with two $n \times n$ crisp linear systems and to solve an $n \times n$ fuzzy linear system in $n \times n$ space. It is clear that solving $n \times n$ large linear system is better than solving $2n \times 2n$ large linear system.

Since perturbation analysis is very important in numerical methods, the authors in [16] have presented the perturbation analysis for a class of fuzzy linear system which could be solved by an embedding method. Now, according to the presented method in this study, the researcher can investigate perturbation analysis in two $n \times n$ crisp linear systems instead of one $2n \times 2n$ linear system as the authors of [16] have done.

This paper is organized as follows. In Section 2, the researcher recalls some fundamental results on fuzzy numbers. In Section 3, this article obtains the nearest symmetric triangular fuzzy vector. In this Section, some theorems and remarks are proposed and illustrated. It is explained by two examples in Section 4. The paper ends with conclusions in Section 5.

2 Preliminaries

The basic definition of a fuzzy number given in [7, 11, 12] is as follows:

Definition 2.1. A fuzzy number is a mapping $u : \mathbb{R} \rightarrow [0, 1]$ with the following properties:

1. u is an upper semi-continuous function on \mathbb{R} ,
2. $u(x) = 0$ outside of some interval $[a_1, b_2] \subset \mathbb{R}$,
3. There are real numbers a_2, b_1 such as $a_1 \leq a_2 \leq b_1 \leq b_2$ and
 - 3.1 $u(x)$ is a monotonic increasing function on $[a_1, a_2]$,
 - 3.2 $u(x)$ is a monotonic decreasing function on $[b_1, b_2]$,
 - 3.3 $u(x) = 1$ for all x in $[a_2, b_1]$.

Let \mathbb{R} be the set of all real numbers. The researcher assumes a fuzzy number u that can be expressed for all $x \in \mathbb{R}$ in the form

$$u(x) = \begin{cases} g(x) & \text{when } x \in [a, b), \\ 1 & \text{when } x \in [b, c], \\ h(x) & \text{when } x \in (c, d], \\ 0 & \text{otherwise.} \end{cases} \quad (2.1)$$

Where a, b, c, d are real numbers such as $a < b \leq c < d$, and g is a real valued function that is increasing and right continuous, and h is a real valued function that is decreasing and left continuous.

Definition 2.2. A fuzzy number u in parametric form is a pair $(\underline{u}, \overline{u})$ of functions $\underline{u}(r)$ and $\overline{u}(r)$ that $0 \leq r \leq 1$, which satisfies the following requirements:

1. $\underline{u}(r)$ is a bounded monotonic increasing left continuous function,
2. $\overline{u}(r)$ is a bounded monotonic decreasing left continuous function,
3. $\underline{u}(r) \leq \overline{u}(r), 0 \leq r \leq 1$.

Definition 2.3. The symmetric triangular fuzzy number $u = (x_0, \sigma)$, with defuzzifier x_0 and fuzziness σ is a fuzzy set where the membership function is as

$$u(x) = \begin{cases} \frac{1}{\sigma}(x - x_0 + \sigma) & x_0 - \sigma \leq x \leq x_0, \\ \frac{1}{\sigma}(x_0 - x + \sigma) & x_0 \leq x \leq x_0 + \sigma, \\ 0 & \text{otherwise} . \end{cases}$$

The parametric form of symmetric triangular fuzzy number is

$$\underline{u}(r) = x_0 - \sigma + \sigma r \quad , \quad \overline{u}(r) = x_0 + \sigma - \sigma r.$$

Definition 2.4. For fuzzy set u Support function is defined as follows:

$$\text{supp}(u) = \overline{\{x | u(x) > 0\}},$$

where $\overline{\{x | u(x) > 0\}}$ is closure of set $\{x | u(x) > 0\}$.

The addition and scalar multiplication of fuzzy numbers are defined by the extension principle and can be equivalently represented as follows. For arbitrary fuzzy numbers $u = (\underline{u}, \overline{u})$ and $v = (\underline{v}, \overline{v})$, this article defines addition $(u + v)$ and multiplication by scalar $k > 0$ as

$$(\underline{u} + \underline{v})(r) = \underline{u}(r) + \underline{v}(r) \quad , \quad (\overline{u} + \overline{v})(r) = \overline{u}(r) + \overline{v}(r), \quad (2.2)$$

$$(k\underline{u})(r) = k\underline{u}(r) \quad , \quad (k\overline{u})(r) = k\overline{u}(r). \quad (2.3)$$

To emphasis the collection of all fuzzy numbers with addition and multiplication as defined by (2.2) and (2.3) denoted by F , which is a convex cone.

Definition 2.5. [17]. For two arbitrary fuzzy numbers $u = (\underline{u}, \overline{u})$ and $v = (\underline{v}, \overline{v})$ this study calls

$$d_w(u, v) = \left(\int_0^1 f(r) d^2(u, v) dr \right)^{\frac{1}{2}}, \quad (2.4)$$

the weighted distance between fuzzy numbers u and v , where

$$d^2(u, v) = (\underline{u}(r) - \underline{v}(r))^2 + (\overline{u}(r) - \overline{v}(r))^2,$$

and the function $f(r)$ is nonnegative and increasing on $[0, 1]$ with $f(0) = 0$ and $\int_0^1 f(r) dr = \frac{1}{2}$.

The function $f(r)$ is also called weighting function. Both conditions $f(0) = 0$ and $\int_0^1 f(r) dr = \frac{1}{2}$, ensure that the distance defined by Eq. (2.4) is the extension of the ordinary distance in \mathfrak{R} .

Definition 2.6. The $n \times n$ linear system of equations

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = y_1, \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = y_2, \\ \vdots \qquad \qquad \qquad \vdots \qquad \qquad \vdots \\ a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n = y_n, \end{cases} \quad (2.5)$$

where the coefficient matrix $A = (a_{ij})$, $1 \leq i, j \leq n$ is a crisp $n \times n$ matrix and $y_i \in F$, $1 \leq i \leq n$ is called a fuzzy linear system (FLS).

Definition 2.7. A fuzzy number vector $(x_1, x_2, \dots, x_n)^t$ given by

$$x_j = (\underline{x}_j(r), \overline{x}_j(r)), \quad 1 \leq j \leq n, \quad 0 \leq r \leq 1,$$

is called a solution of (2.5) if

$$\begin{cases} \underline{\sum_{j=1}^n a_{ij}x_j} = \underline{\sum_{j=1}^n a_{ij}x_j} = \underline{y_i}, & i = 1, 2, \dots, n \\ \overline{\sum_{j=1}^n a_{ij}x_j} = \overline{\sum_{j=1}^n a_{ij}x_j} = \overline{y_i}, & i = 1, 2, \dots, n \end{cases} \quad (2.6)$$

Finally, This section concludes with a reviewing on the proposed method for solving fuzzy linear system in [8].

The authors in [8] wrote the linear system of Eq. (2.6) as follows:

$$SX = Y,$$

where the elements of $S = (s_{ij})$, $1 \leq i, j \leq 2n$, were as follows:

$$\begin{cases} a_{ij} \geq 0 & \Rightarrow & s_{ij} = a_{ij}, \quad s_{i,j+n} = a_{ij}, \\ a_{ij} < 0 & \Rightarrow & s_{i,j+n} = -a_{ij}, \quad s_{i+n,j} = -a_{ij}, \end{cases} \quad (2.7)$$

and any s_{ij} is not determined by Eq. (2.7) is zero, the unknowns and the right-hand side column were

$$X = (\underline{x}_1, \underline{x}_2, \dots, \underline{x}_n, -\overline{x}_1, -\overline{x}_2, \dots, -\overline{x}_n)^t,$$

$$Y = (\underline{y}_1, \underline{y}_2, \dots, \underline{y}_n, -\overline{y}_1, -\overline{y}_2, \dots, -\overline{y}_n)^t,$$

respectively. The structure of S implies that $s_{ij} \geq 0$, $1 \leq i, j \leq 2n$ and that

$$S = \begin{pmatrix} B & C \\ C & B \end{pmatrix}$$

where B contains the positive entries of A and C the absolute values of the entries of the negative entries of A and $A = B - C$.

Theorem 2.1. [8]. The matrix S is nonsingular if and only if the matrices $A = B - C$ and $B + C$ are both nonsingular.

Theorem 2.2. [8]. If S^{-1} exists it must have the same structure as S , i.e.

$$S^{-1} = \begin{pmatrix} D & E \\ E & D \end{pmatrix}$$

where $D = \frac{1}{2}[(B + C)^{-1} + (B - C)^{-1}]$ and $E = \frac{1}{2}[(B + C)^{-1} - (B - C)^{-1}]$.

It is known that if S is nonsingular then

$$X = S^{-1}Y,$$

but this solution may still not be an appropriate fuzzy vector, see the following example.

Example 2.1. Consider the 3×3 fuzzy system

$$\begin{cases} 4x_1 + 2x_2 - x_3 = (-27 + 7r, -7 - 13r), \\ 2x_1 + 7x_2 + 6x_3 = (1 + 15r, 40 - 24r), \\ -x_1 + 6x_2 + 10x_3 = (26 + 18r, 47 - 3r). \end{cases} \quad (2.8)$$

This fuzzy system will not be a fuzzy solution if the method of [8] is used.

In the next section, the researcher is going to investigate the fuzzy linear system that defines a fuzzy vector solution for fuzzy linear system.

3 The proposed model

In this section, the researcher wants to obtain the symmetric triangular fuzzy vector $x = (x_1, x_2, \dots, x_n)^t$ which is the nearest to the solution of the fuzzy linear system $Ax = y$. To carry out this purpose, this study minimizes functions as follows:

$$d_w(\sum_{i=1}^n a_{ij}x_j, y_i) = \left[\int_0^1 f(r) \left(\sum_{i=1}^n a_{ij}x_j(r) - \underline{y}_i(r) \right)^2 dr + \int_0^1 f(r) \left(\sum_{i=1}^n a_{ij}x_j(r) - \overline{y}_i(r) \right)^2 dr \right]^{\frac{1}{2}},$$

with respect to \underline{x}_{0j} and σ_j , since parametric form of components x is as follows:

$$\underline{x}_j = x_{0j} + \sigma_j(r - 1) \quad , \quad \overline{x}_j = x_{0j} + \sigma_j(1 - r).$$

In order to minimize $d_w(\sum_{i=1}^n a_{ij}x_j, y_i)$, it suffices to minimize function

$$d_i(x_{01}, \dots, x_{0n}, \sigma_1, \dots, \sigma_n) = d_w^2(\sum_{i=1}^n a_{ij}x_j, y_i) \quad , \quad \forall i = 1, 2, \dots, n.$$

Therefore equality minimizes functions as follows:

$$\begin{aligned} d_i(x_{01}, \dots, x_{0n}, \sigma_1, \dots, \sigma_n) &= \int_0^1 f(r) \left(\sum_{a_{ij}>0} a_{ij}\underline{x}_j(r) + \sum_{a_{ij}<0} a_{ij}\overline{x}_j(r) - \underline{y}_i(r) \right)^2 dr \\ &+ \int_0^1 f(r) \left(\sum_{a_{ij}>0} a_{ij}\overline{x}_j(r) + \sum_{a_{ij}<0} a_{ij}\underline{x}_j(r) - \overline{y}_i(r) \right)^2 dr, \end{aligned}$$

with to x_{0j} and σ_j for each $1 \leq i, j \leq n$.

In order to minimize $d_i(x_{01}, \dots, x_{0n}, \sigma_1, \dots, \sigma_n)$, the researcher considers

$$\frac{\partial d_i(x_{01}, \dots, x_{0n}, \sigma_1, \dots, \sigma_n)}{\partial x_{0j}} = 2a_{ij} \int_0^1 \left(2 \sum_{j=1}^n a_{ij} x_{0j} - \underline{y}_i(r) - \overline{y}_i(r) \right) f(r) dr,$$

$$\frac{\partial d_i(x_{01}, \dots, x_{0n}, \sigma_1, \dots, \sigma_n)}{\partial \sigma_j} = 2a_{ij} \int_0^1 \left(2 \sum_{j=1}^n a_{ij} \sigma_j (1-r) - (\overline{y}_i - \underline{y}_i(r))(1-r) \right) f(r) dr,$$

for $1 \leq j \leq n$.

Therefore, a stationary point $(x_{01}, \dots, x_{0n}, \sigma_1, \dots, \sigma_n)$ ought to be found for which

$$\begin{cases} \frac{\partial d_i(x_{01}, \dots, x_{0n}, \sigma_1, \dots, \sigma_n)}{\partial x_{0j}} = 0, \\ j = 1, 2, \dots, n, \\ \frac{\partial d_i(x_{01}, \dots, x_{0n}, \sigma_1, \dots, \sigma_n)}{\partial \sigma_j} = 0. \end{cases} \quad (3.9)$$

Therefore the researcher obtains two $n \times n$ normal equation systems as follows:

$$\int_0^1 \left(2 \sum_{j=1}^n a_{ij} x_{0j} - \underline{y}_i(r) - \overline{y}_i(r) \right) f(r) dr = 0,$$

$$\int_0^1 \left(2 \sum_{j=1}^n |a_{ij}| \sigma_j (1-r)^2 - (\overline{y}_i - \underline{y}_i(r))(1-r) \right) f(r) dr = 0,$$

there is

$$\sum_{j=1}^n a_{ij} x_{0j} = \int_0^1 (\underline{y}_i + \overline{y}_i) f(r) dr,$$

$$\sum_{j=1}^n |a_{ij}| \sigma_j = \frac{\int_0^1 (\overline{y}_i - \underline{y}_i)(1-r) f(r) dr}{2 \int_0^1 (1-r)^2 f(r) dr},$$

for $i = 1, 2, \dots, n$. The matrix form of these are as

$$Ax_0 = p, \quad B\sigma = q, \quad (3.10)$$

which $B = (b_{ij})$ contains the absolute value of elements of A i.e.

$$b_{ij} = |a_{ij}| \quad \text{for each } 1 \leq i, j \leq n, \quad (3.11)$$

and

$$p_i = \int_0^1 (\underline{y}_i + \overline{y}_i) f(r) dr, \quad q_i = \frac{\int_0^1 (\overline{y}_i - \underline{y}_i)(1-r) f(r) dr}{2 \int_0^1 (1-r)^2 f(r) dr}. \quad (3.12)$$

If it will be assumed that $f(r) = r$, there is

$$p_i = \int_0^1 (\underline{y}_i + \overline{y}_i) r dr, \quad q_i = 6 \int_0^1 (\overline{y}_i - \underline{y}_i)(1-r) r dr.$$

Then, to obtain the nearest symmetric triangular fuzzy vector solution, two $n \times n$ crisp linear systems should be solved. As a consequence for obtaining the nearest approximation symmetric triangular fuzzy vector solution of an $n \times n$ fuzzy linear system, it takes more time to calculate.

Definition 3.1. Let $x_0 = (x_{01}, x_{02}, \dots, x_{0n})^t$ and $\sigma = (\sigma_1, \sigma_2, \dots, \sigma_n)^t$ denote the unique solutions of two Eqs. (3.10). Vector $x = (x_1, x_2, \dots, x_n)^t$ which parametric form of x_i defined by

$$\underline{x}_i(r) = x_{0i} + |\sigma_i|(r - 1), \quad \overline{x}_i(r) = x_{0i} + |\sigma_i|(1 - r), \quad (3.13)$$

is called the nearest fuzzy solution of the fuzzy linear system (2.5).

If $\sigma_i \geq 0$, for $1 \leq i \leq n$, then

$$\underline{x}_i(r) = x_{0i} + \sigma_i(r - 1), \quad \overline{x}_i(r) = x_{0i} + \sigma_i(1 - r),$$

and x is called a strong nearest symmetric fuzzy solution. Otherwise x is a weak nearest symmetric fuzzy solution.

Theorem 3.1. The nearest symmetric fuzzy vector solution of fuzzy linear system (3.9) exists if and only if the solution of crisp system $B\sigma = q$ (3.10) is positive i.e.

$$\sigma_i \geq 0, \quad 1 \leq i \leq n.$$

Proof. By definition of fuzzy number the proof is evident.

Theorem 3.2. Let $D = (d_{ij})$ is inverse of B . The nearest solution X of (2.5) is a fuzzy vector for each arbitrary Y if only if D is nonnegative i.e.

$$d_{ij} \geq 0, \quad 1 \leq i \leq n, \quad 1 \leq j \leq n.$$

Proof. By definition of q_i in Eq. (3.12) the proof is evident.

Theorem 3.3. [5]. The inverse of a non-negative matrix A is non-negative if and only if A is a generalized permutation matrix.

According to Theorem (3.3), since B is non-negative then, D is non-negative if and only if D is generalized permutation matrices.

4 Examples

In this section, several types of fuzzy solution by the means of examples are discussed. Throughout this section, the researcher assumes that $f(x) = x$.

Example 4.1. Consider the 2×2 fuzzy linear system,

$$\begin{cases} x_1 - x_2 = (-7 + 2r, -3 - 2r), \\ x_1 + 3x_2 = (19 + 4r, 27 - 4r). \end{cases} \quad (4.14)$$

Two crisp linear systems should be solved as follows:

$$\begin{bmatrix} 1 & -1 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} x_{01} \\ x_{02} \end{bmatrix} = \begin{bmatrix} -5 \\ 23 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} 1 & 1 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix},$$

and the solution of Eq.(3.10) are

$$x_{01} = 2, \quad x_{02} = 7, \quad \sigma_1 = 1, \quad \sigma_2 = 1.$$

Therefore the fuzzy solution is

$$\underline{x}_1 = 2 + (r - 1), \quad \bar{x}_1 = 2 + (1 - r),$$

$$\underline{x}_2 = 7 + (r - 1), \quad \bar{x}_2 = 7 + (1 - r).$$

It is a strong nearest solution. The exact and obtained solutions by the proposed method are plotted and compared in Fig. 4.

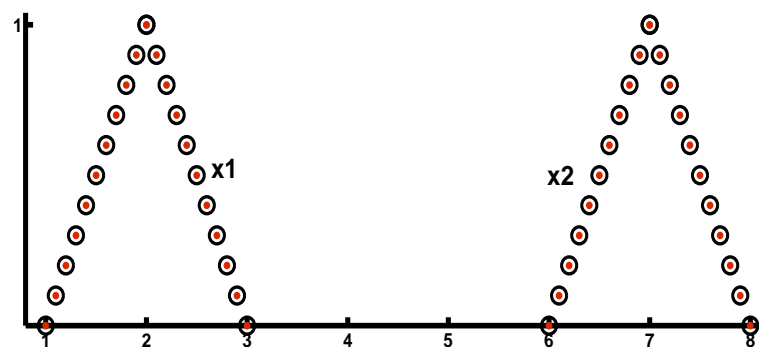


Fig. 4. The solid lines with (- . -) and (- o -) shows the exact and weighted solutions, respectively.

Example 4.2. [4]. Consider the 5×5 fuzzy linear system

$$\begin{cases} 6x_1 + x_2 + 3x_3 - x_4 + 6x_5 = (1 + r, 3 - r), \\ 5x_1 + 9x_2 + x_3 + 2x_4 + 3x_5 = (6 + r, 8 - r), \\ 2x_1 + 3x_2 + 9x_3 + 2x_4 + 3x_5 = (5 + r, 7 - r), \\ -x_1 + x_2 + 3x_3 + 8x_4 + 3x_5 = (3 + r, 5 - r), \\ x_1 + 2x_2 + 2x_3 + x_4 + 9x_5 = (2 + r, 4 - r). \end{cases} \quad (4.15)$$

The exact solutions are

$$x_1 = (\underline{x}_1(r), \bar{x}_1(r)) = (-0.04090368152 + 0.04621435588r, 0.0515250302 - 0.04621435586r),$$

$$\begin{aligned}x_2 &= (\underline{x}_2(r), \bar{x}_2(r)) = (0.6130455649 + 0.03883972467r, 0.6907250142 - 0.03883972474r), \\x_3 &= (\underline{x}_3(r), \bar{x}_3(r)) = (0.3194452931 + 0.04646017704r, 0.4123656473 - 0.04646017704r), \\x_4 &= (\underline{x}_4(r), \bar{x}_4(r)) = (0.1854134257 + 0.06710914455r, 0.3196317149 - 0.06710914454r), \\x_5 &= (\underline{x}_5(r), \bar{x}_5(r)) = (-0.001054606754 + 0.07956407729r, 0.1580735480 - 0.07956407732r).\end{aligned}$$

By using the proposed method, two crisp linear systems should be solved as follows:

$$\begin{bmatrix} 6 & 1 & 3 & -1 & 6 \\ 5 & 9 & 1 & 2 & 3 \\ 2 & 3 & 9 & 2 & 3 \\ -1 & 1 & 3 & 8 & 3 \\ 1 & 2 & 2 & 1 & 9 \end{bmatrix} \begin{bmatrix} x_{01} \\ x_{02} \\ x_{03} \\ x_{04} \\ x_{05} \end{bmatrix} = \begin{bmatrix} 2 \\ 7 \\ 6 \\ 4 \\ 3 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} 6 & 1 & 3 & 1 & 6 \\ 5 & 9 & 1 & 2 & 3 \\ 2 & 3 & 9 & 2 & 3 \\ 1 & 1 & 3 & 8 & 3 \\ 1 & 2 & 2 & 1 & 9 \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \sigma_5 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix},$$

and the solutions of Eq.(3.10) are

$$x_{01} = 0.0053, \quad x_{02} = 0.6519, \quad x_{03} = 0.3659, \quad x_{04} = 0.2525, \quad x_{05} = 0.0785,$$

$$\sigma_1 = 0.0462, \quad \sigma_2 = 0.0388, \quad \sigma_3 = 0.0465, \quad \sigma_4 = 0.0671, \quad \sigma_5 = 0.0796.$$

Therefore the fuzzy solutions are

$$\underline{x}_1 = 0.0053 + 0.0462(r - 1), \quad \bar{x}_1 = 0.0053 + 0.0462(1 - r),$$

$$\underline{x}_2 = 0.6519 + 0.0388(r - 1), \quad \bar{x}_2 = 0.6519 + 0.0388(1 - r),$$

$$\underline{x}_3 = 0.3659 + 0.0465(r - 1), \quad \bar{x}_3 = 0.3659 + 0.0465(1 - r),$$

$$\underline{x}_4 = 0.2525 + 0.0671(r - 1), \quad \bar{x}_4 = 0.2525 + 0.0671(1 - r),$$

$$\underline{x}_5 = 0.0785 + 0.0796(r - 1), \quad \bar{x}_5 = 0.0785 + 0.0796(1 - r),$$

The exact and obtained solutions by the proposed method are plotted and compared in Fig. 4.2.

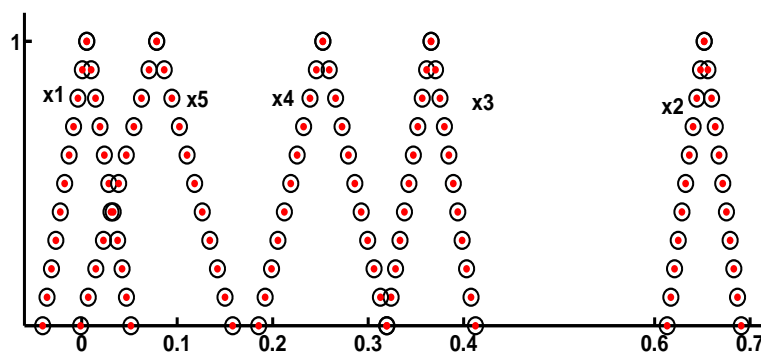


Fig. 4.2. The solid lines with (- . -) and (- o -) shows the exact and weighted solutions, respectively.

According to this fact that $\underline{x}_i \leq \bar{x}_i$, $i = 1, 2, \dots, 5$, are monotonic decreasing functions, then the fuzzy solutions $x_i = (\underline{x}_i, \bar{x}_i)$, $i = 1, 2, \dots, 5$, is a strong fuzzy solution.

Example 4.3. [8] Consider the 3×3 fuzzy linear system

$$\begin{cases} x_1 + x_2 - x_3 = (r, 2 - r), \\ x_1 - 2x_2 + x_3 = (2 + r, 3), \\ 2x_1 + x_2 + 3x_3 = (-2, -1 - r). \end{cases} \quad (4.16)$$

The matrix forms of Eqs. (3.10), are the same as follow:

$$\begin{bmatrix} 1 & 1 & -1 \\ 1 & -2 & 1 \\ 2 & 1 & 3 \end{bmatrix} \begin{bmatrix} x_{01} \\ x_{02} \\ x_{03} \end{bmatrix} = \begin{bmatrix} 1 \\ 2.83 \\ -1.83 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0.5 \\ 0.5 \end{bmatrix},$$

and the solution of Eq.(3.10) are

$$x_{01} = 1.2685, \quad x_{02} = -1.2931, \quad x_{03} = -1.0246, \quad \sigma_1 = 3.5, \quad \sigma_2 = -0.5, \quad \sigma_3 = -2.$$

Therefore the solution is

$$\underline{x}_1 = 1.2685 + 3.5(r - 1), \quad \bar{x}_1 = 1.2685 + 3.5(1 - r),$$

$$\underline{x}_2 = -1.2931 - 0.5(r - 1), \quad \bar{x}_2 = -1.2931 - 0.5(1 - r),$$

$$\underline{x}_3 = -1.0246 - 2(r - 1), \quad \bar{x}_3 = -1.0246 - 2(1 - r).$$

Since $\sigma_2, \sigma_3 < 0$, then x_2 and x_3 are not fuzzy numbers. Therefore fuzzy solution in this case is a weak weighted solution as follows:

$$x_1 = (1.2685 + 3.5(r - 1), \quad 1.2685 + 3.5(1 - r)),$$

$$x_2 = (-1.2931 + 0.5(r - 1), \quad -1.2931 + 0.5(1 - r)),$$

$$x_3 = (-1.0246 + 2(r - 1), \quad -1.0246 + 2(1 - r)).$$

5 Conclusion

The researcher proposed a model for solving fuzzy linear system, and the original system is replaced by two $n \times n$ crisp linear systems in this paper. Moreover an interesting approach to symmetric triangular approximation of solution of a fuzzy linear system is suggested. The proposed method leads to the triangular fuzzy number which is the best one with respect to a certain measure of weighted distance among fuzzy numbers, $d_w(u, v)$. This method is simple, natural and applicable to anywhere.

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