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A Generalized Model in the Performance Evaluation of Decision Making Sub-units

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Abstract

Data Envelopment Analysis (DEA) evaluates the efficiency of decision making units with multiple inputs and outputs. So far, a number of DEA models have been developed: The CCR model, the BCC model and the FDH model are well known as basic DEA models. In many instance, However, the decision making units can be separated into different sub-units. In this paper, we study a generalized model for this $DMUs$ by different sub-units.

Keywords : Data envelopment analysis; Decision making units; Sub-units; Efficiency; Generalized model

1 Introduction

Data Envelopment Analysis(DEA), originally proposed by Charnes, Cooper and Rodes (1978 and 1979)[1], has become one of the most widely used methods in management science. DEA measures the relative efficiency of comparable entities called Decision making units($DMUs$)essentially performing the same task using similar multiple inputs to produce similar multiple outputs. The purpose of DEA is to empirically estimate the so-called efficient frontier based on the set of available $DMUs$. A DMU is efficient if there is no other unit-existing or virtual that can either produce more outputs by consuming the same amount or less of inputs or produce the same amount or more of outputs by consuming less or the same amount of inputs as the DMU under consideration. The former approach is referred to as the output oriented and the latter as the input oriented DEA. DAE provides the user with information about the efficient and inefficient units, as well as the efficiency scores and reference sets for inefficient units. The result of the DEA analysis, especially the efficiency scores, had practical applications as performance indicators of $DMUs$.

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In many instances however, the decision making units can be separated into different sub-units. färe and grosskpof [3], for example, look at a multi-stage process where in intermediate product or output at one stage can be both final products and inputs to the later stages of production. Those authors are not explicitly interested in obtaining measures of efficiency at each stage, but rather are concerned with overall efficiency measurement. Another example is due to cook et al [4] and involves multi-component efficiency with shared inputs.
In this paper,we propose a generalized model for DEA when $DMUs$ has sub-units, which can treat basic DEA models for this $DMUs$, specifically, the CCR model, the BCC model and the FDH model in a unified way. In addition, we show theoretical properties on relationships among this model and those DEA models by sub-units, and this model makes it possible to calculate the efficiency of $DMUs$ incorporating various preference structure of decision makers.
The following sections of the paper provide a sub-units efficiency measurement.

2 Basic DEA models for sub-units

Assume that we have n $DMUs$, and a DMU_p consists of b sub-units. called $DMSU$.Each DMU_j transforms resources, or inputs into products, or outputs in particular, $DMU_j, 2 \leq j \leq b-1$, produces k_j different types of outputs and consumes I_j types of external inputs and I'_j types of internal inputs (i.e. a part of inputs coming from outside the whole DMU and the other part coming from inside the DMU).The internal input of DMS_jU is output produced by the last $DMSU_j$. The first $DMSU_1$ consumes the input vector $\overline{X_1}$ and produces the output vector Y_1 and the last $DMSU_b$ consumes the internal input vector X_b and the external input vector X_b produces the output vector Y_b. All the $DMSUs$ considered have the same types of outputs and internal and external inputs. Especially, $DMSU_j, 2 \leq j \leq b$, consumes I_j types of external inputs X_j and I'_j types of internal inputs $\overline{X_j} = Y_{j-1}$. Also $DMSU_j, 2 \leq j \leq b$, produces k_j types of outputs Y_j. See the fig. 1.
For notational purpose,let $y_j^{(p)}, j = 1, \ldots, b$, denote the output vectors produced by jth sub-DMU of DMU_p in which

$$Y_j^{(p)} = (y_{j1}^{(p)}, \ldots, y_{j,k_j}^{(p)}),$$

Also,let $X_j^{(p)}$ and $\overline{X_j}^{(p)}, j = 2, \ldots, b$, denote I_j and I'_j-dimensional vectors of external and internal inputs to jth sub-DMU of DMU_p, respectively, in which

$$X_j^{(p)} = (x_{j1}^{(p)}, \ldots, x_{j,I_j}^{(p)}),$$

$$\overline{x}_j^{(p)} = (\overline{x}_{j1}^{(p)}, \ldots, \overline{x}_{j,I'_j}^{(p)}) = (y_{j-1,1}^{(p)}, \ldots, y_{j-1,I'_j}^{(p)}),$$

Hence, a measure of aggregate performance $e_p^{(a)}$ can be represented by

$$e_p^{(a)} = \frac{\mu^{(1)T} y_1^{(p)} + \mu^{(2)T} y_2^{(p)} + \cdots + \mu^{(b)T} y_b^{(p)}}{v^{(1)T} x_1^{(p)} + v^{(2)T} x_2^{(p)} + \cdots + v^{(b)T} x_b^{(p)} + \overline{v}^{(1)T} y_1^{(p)} + \cdots + \overline{v}^{(b-1)T} y_{b-1}^{(p)}}$$

and performance for each sub-units of DMUp can be represented by

$$e_p^{(1)} = \frac{\mu^{(1)T} y_1^{(p)}}{v^{(1)T} x_1^{(p)}}$$

$$e_p^{(i)} = \frac{\mu^{(i)T} y_i^{(p)}}{v^{(i)T} x_i^{(p)} + \overline{v}^{(i-1)T} y_{i-1}^{(p)}}, \quad i = 2, \cdots, b.$$

Theorem 2.1. *The aggregate efficiency $e_p^{(a)}$ is a convex combination of $DMSU's$ efficiency.*

Proof: The proof is in [2].

Theorem 2.2. *DMUp is efficiency iff all of $DMUSp$ are efficiency.*

Proof: The proof is straightforward.

Then we have the following mathematical programming problem:

$$\begin{aligned} \max \quad & e_p^{(a)} \\ s.t. \quad & e_j^{(a)} \leq 1, \quad j = 1, \ldots, n \\ & e_j^{(i)} \leq 1, \quad j = 1, \ldots, b, \;\; j = 1, \ldots, n \\ & \mu^{(i)} \in \overline{\Omega}_1, \quad i = 1, \ldots, b \\ & (v^{(i)}, \overline{v}^{(i)}) \in \overline{\Omega}_2, \quad i = 1, \ldots, b \end{aligned} \tag{2.1}$$

The sets $\overline{\Omega}_1$ and $\overline{\Omega}_2$ are assurance regions defined by any restrictions imposed on multipliers [4]. The model (2.1) can be expressed in the following form

$$\begin{aligned} Max \quad & \sum_{i=1}^{b} \mu^{(i)T} y_i^{(p)} \\ S.t. \quad & \sum_{j=1}^{b} v^{(i)T} x_i^{(p)} + \sum_{j=1}^{b-1} \overline{v}^{(i)T} y_i^{(p)} = 1, \\ & \sum_{j=1}^{b} \mu^{(i)T} y_i^{(j)} - \sum_{j=1}^{b} v^{(i)T} x_i^{(j)} - \sum_{i=1}^{b-1} \overline{v}^{(i)T} y_i^{(j)} \leq 0, j = 1, \ldots, n \\ & \mu^{(i)T} y_i^{(j)} - v^{(i)T} x_i^{(j)} - \overline{v}^{(i-1)T} y_{i-1}^{(j)} \leq 0, \quad i = 2, \cdots, b, j = 1, \cdots, n \\ & \mu^{(i)} \in \Omega_1 \quad i = 1, \cdots, b \\ & (v^{(i)}, \overline{v}^{(i)}) \in \Omega_2 \quad i = 1, \cdots, b \end{aligned} \tag{2.2}$$

The form of Ω_1 and Ω_2 depends on how $\overline{\Omega_1}$ and $\overline{\Omega_2}$ are structured.
Now the CCR model in present DMU's follows as:

$$
\begin{aligned}
&Min \quad \theta \\
&S.t. \quad \sum_{j=1}^{n} \lambda_j x_i^{(j)} + \sum_{j=1}^{n} \lambda_{ij} x_i^{(j)} \leq \theta x_i^{(j)}, \quad i = 1, \cdots, b \\
&\sum_{j=1}^{n} \lambda_j y_i^{(j)} + \sum_{j=1}^{n} \lambda_{ij} y_i^{(j)} \leq \theta y_i^{(p)}, \quad i = 1, \cdots, b-1 \\
&\sum_{j=1}^{n} \lambda_j y_i^{(j)} - \sum_{j=1}^{n} \lambda_{ij} y_i^{(j)} \geq y_i^{(p)}, \quad i = 1, \cdots, b \\
&\lambda_j \geq 0, \quad j = 1, \cdots, n \\
&\lambda_{ij} \geq 0, \quad i = 1, \cdots, b \quad j = 1, \cdots, n.
\end{aligned}
\tag{2.3}
$$

And the BCC model in present DMU's follows as:

$$
\begin{aligned}
&Min \quad \theta \\
&S.t. \quad \sum_{j=1}^{n} \lambda_j x_i^{(j)} + \sum_{j=1}^{n} \lambda_{ij} x_i^{(j)} \leq \theta x_i^{(j)}, \quad i = 1, \cdots, b \\
&\sum_{j=1}^{n} \lambda_j y_i^{(j)} + \sum_{j=1}^{n} \lambda_{ij} y_i^{(j)} \leq \theta y_i^{(p)}, \quad i = 1, \cdots, b-1 \\
&\sum_{j=1}^{n} \lambda_j y_i^{(j)} - \sum_{j=1}^{n} \lambda_{ij} y_i^{(j)} \geq y_i^{(p)}, \quad i = 1, \cdots, b \\
&\sum_{j=1}^{n} \lambda_j + \sum_{i=1}^{b} \sum_{j=1}^{n} \lambda_{ij} = 1, \\
&\lambda_j \geq 0, \quad j = 1, \cdots, n \\
&\lambda_{ij} \geq 0, \quad i = 1, \cdots, b, \quad j = 1, \cdots, n.
\end{aligned}
\tag{2.4}
$$

The multiplier form of the BCC model in present DMSU follows as:

$$
\begin{aligned}
Max \quad & \sum_{i=1}^{b} \mu^{(i)T} y_i^{(p)} + u_0 \\
S.t. \quad & \sum_{i=1}^{b} v^{(i)T} x_i^{(p)} + \sum_{i=1}^{b-1} \overline{v}^{(i)T} y_i^{(p)} = 1, \\
& \sum_{i=1}^{b} \mu^{(i)T} y_i^{(j)} - \sum_{i=1}^{b} v^{(i)T} x_i^{(j)} - \sum_{i=1}^{b-1} \overline{v}^{(i)T} y_i^{(j)} + u_0 \leq 0, j = 1, \ldots, n \\
& \mu^{(i)T} y_i^{(j)} - v^{(i)T} x_i^{(j)} - \overline{v}^{(i-1)T} y_{i-1}^{(j)} + u_0 \leq 0, \quad i = 2, \cdots, b, j = 1, \cdots, n \\
& \mu^{(i)} \in \Omega_1 \quad i = 1, \cdots, b \\
& (v^{(i)}, \overline{v}^{(i)}) \in \Omega_2 \quad i = 1, \cdots, b
\end{aligned}
\tag{2.5}
$$

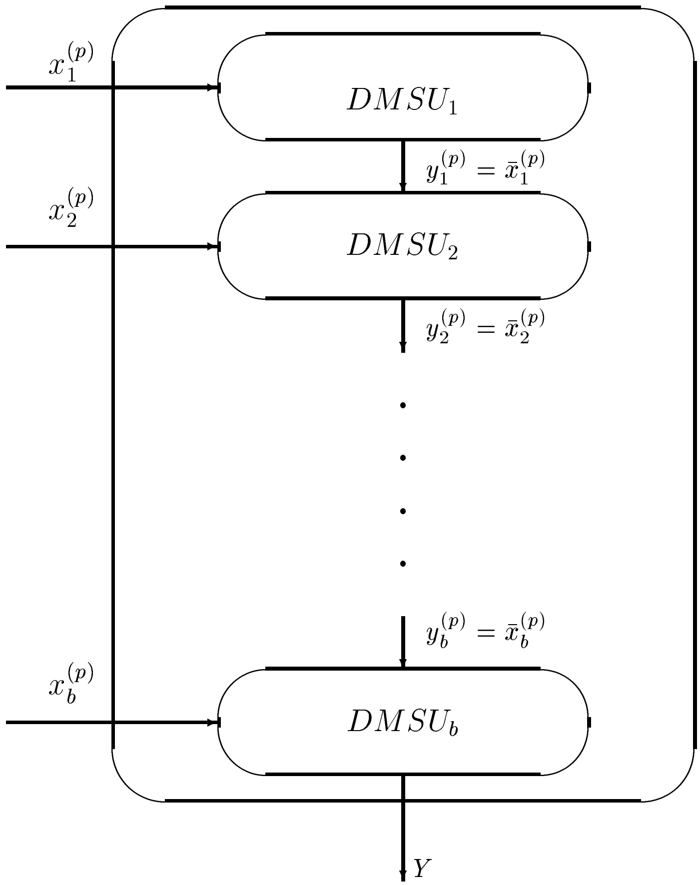

Fig. 1. The DMU_p by DMSUs

3 A generalized model in present sub-units

In this section, we formulate the generalized model in present sub-units, based on a domination structure and define a new efficiency in this model. Next we establish relationships between this generalized model and basic DEA models mentioned in section 2. Now, we formulate a generalized DEA model in present sub-units by employing the augmented Tchebyshev secularizing function [5]. This model, which can evaluate the efficiency in several basic models which are special cases for all DMUs, follows as:

$$
\begin{aligned}
&Max \quad \Delta \\
&S.t. \quad \Delta \leq \widetilde{d}_j + \alpha\Big(\sum_{i=1}^{b} \mu^{(i)T}(y_i^{(p)} - y_i^{(j)}) + \sum_{i=1}^{b} v^{(i)T}(-x_i^{(p)} + x_i^{(j)}) + \sum_{i=1}^{b-1} \overline{v}^{(i)}(-y_i^{(p)} + y_i^{(j)})\Big) \\
&\qquad \mu^{(i)T} y_i^{(j)} - v^{(i)T} x_i^{(j)} - \overline{v}^{(i-1)T} y_{i-1}^{(j)} \leq 0, \quad i = 2, \cdots, b, j = 1, \cdots, n \\
&\qquad \sum_{i=1}^{b} \mu^{(i)} + \sum_{i=1}^{b} v^{(i)} + \sum_{i=1}^{b-1} \overline{v}^{(i)} = 1 \\
&\qquad \mu^{(i)} \geq 0, \quad i = 1, \cdots, b \\
&\qquad v^{(i)} \geq 0, \quad i = 1, \cdots, b \\
&\qquad \overline{v}^{(i)} \geq 0, \quad i = 1, \cdots, b-1
\end{aligned}
\tag{3.6}
$$

where $\alpha > 0$ is appropriately given according to given problems, and $\widetilde{d}_j (j = 1, \cdots, n)$ is defined by following:

$$
\widetilde{d}_j = \max_{i=1,\cdots,bt=1,\cdots,b-1} \{\mu^{(i)}(y_i^{(p)} - y_i^{(j)}), v^{(i)}(-x_i^{(p)} + x_i^{(j)}) + \overline{v}^{(t)}(-y_t^{(p)} + y_t^{(j)})\} \tag{3.7}
$$

Note that when $j = p$ then $\Delta \leq 0$.

Definition 3.1. **($\alpha-$efficiency)** *For a given positive number α, DMU_o is defined to be $\alpha-$efficiency if and only if the optimal value to the problem (3.6) is equal to zero. Otherwise, DMUp is said to be $\alpha-$ inefficiency.*

Theorem 3.1. *If $\Delta \neq 0$ the existence DMU where dominated DMUp.*

Proof: Let $\Delta \neq 0$, by contradiction suppose that there is not DMU where dominated DMUp.
On the other hand, for all j we have

$$
\begin{bmatrix} Y^{(j)} \\ -X^{(j)} \\ -\overline{X}^{(j)} \end{bmatrix} \gneq \begin{bmatrix} Y^{(p)} \\ -X^{(p)} \\ -\overline{X}^{(p)} \end{bmatrix}.
$$

We denote $Z_j = \begin{bmatrix} Y^{(j)} \\ -X^{(j)} \\ -\overline{X}^{(j)} \end{bmatrix}$. Therefore

$$Z^{(j)} \lneqq Z^{(p)} \quad (\forall j). \tag{3.8}$$

And from inequalities of the model (3.6) in present sub-units for all DMUs we have

$$\Delta \le \widetilde{d}_j + \alpha(\sum_{i=1}^{b} \mu^{(i)T}(y_i^{(p)} - y_i^{(j)}) + \sum_{i=1}^{b} v^{(i)T}(-x_i^{(p)} + x_i^{(j)}) + \sum_{i=1}^{b} \overline{v}^{(i)T}(y_i^{(p)} - y_i^{(j)}))$$

But, $\Delta < 0$, and if free variable, then necessary and sufficient condition for existence above inequality $(for \quad somej \neq p)$ is

$$\widetilde{d}_j + \alpha(\sum_{i=1}^{b} \mu^{(i)T}(y_i^{(p)} - y_i^{(j)}) + \sum_{i=1}^{b} v^{(i)T}(-x_i^{(p)} + x_i^{(j)}) + \sum_{i=1}^{b} \overline{v}^{(i)T}(y_i^{(p)} - y_i^{(j)})) < 0$$

We have

$$\widetilde{d}_j + \alpha(\mu, v, \overline{v}) \begin{bmatrix} Y_i^{(p)} - Y_i^{(j)} \\ -X_i^{(p)} + X_i^{(j)} \\ -\overline{X}_i^{(p)} + \overline{X}_i^{(j)} \end{bmatrix} < 0 \quad (for \quad somej \neq p).$$

That is we have the following

$$\widetilde{d}_j + \alpha(\mu, v, \overline{v})(Z_p^{(i)} - Z_j^{(i)}) < 0 \quad (for \quad somej \neq p).$$

Now by (3.8) and $\alpha > 0$ and $(\mu, v, \overline{v}) \ge 0$ we must have

$$\widetilde{d}_j < 0 \quad (for \quad somej \neq \rho).$$

And by definition $\widetilde{d}_j$ $(for \quad somej \neq p)$ we have:

$$\widetilde{d}_j = \max_{i=1,\cdots,bt=1,\cdots,b-1} \{\mu^{(i)}(y_i^{(p)} - y_i^{(j)}), v^{(i)}(-x_i^{(p)} + x_i^{(j)}) + \overline{v}^{(t)}(y_t^{(p)} - y_t^{(j)})\} < 0.$$

Hence by $(\mu, v, \overline{v}) \ge 0$, $Z_p^{(i)} - Z_j^{(i)} < 0$ (for some $j \neq p$). Where contradiction by (3.8). This contradiction asserts that there is not existence DMU where dominated $DMU\rho$, and the proof is complete.

4 Relationships between generalized model and BCC (CCR) model in present sub- units

In this section, we establish theoretical properties on relationships among efficiencies in the basic DEA model and generalized model in present sub-units.

Theorem 4.1. *$DMU\rho$ is BCC-efficiency in present sub-units if and only if DMUp is α-efficiency for some sufficiently large positive number α.*

Proof: Suppose that DMUp is α-efficient for some sufficiently large positive α. That is for all optimal solution we have:

$$0 = \Delta^* \leq \widetilde{d}_j + \alpha(\mu, v, \overline{v})(Z_p - Z_j).$$

The necessary and sufficient condition for some sufficiently large positive number α for this inequality follows as:

$$\begin{cases} Z_p - Z_j \geq 0 \\ \widetilde{d}_j \geq 0 \end{cases} \tag{4.9}$$

Then we have

$$(\mu^*, v^*, \overline{v}^*)(Z_p - Z_j) \geq 0.$$

Therefore

$$(\mu^*, v^*, \overline{v}^*)Z_p - (\mu^*, v^*, \overline{v}^*)Z_j) \geq 0. \tag{4.10}$$

Suppose that $(v, \overline{v}) \begin{bmatrix} X^p \\ \overline{X}^p \end{bmatrix} = \gamma$ then $(\frac{v}{\gamma}, \frac{\overline{v}}{\gamma}) \begin{bmatrix} X^p \\ \overline{X}^p \end{bmatrix} = 1.$

We denote $u_0^* = -(\mu^*, v^*, \overline{v}^*)Z_p$. Hence by (4.10) we have

$$(\mu^*, v^*, \overline{v}^*)Z_j + u_0^* \leq 0 \Rightarrow (\frac{\mu^*}{\gamma}, \frac{v^*}{\gamma}, \frac{\overline{v}^*}{\gamma})Z_j + \frac{u_0^*}{\gamma} \leq 0$$

Therefore$(\frac{\mu^*}{\gamma}, \frac{v^*}{\gamma}, \frac{\overline{v}^*}{\gamma})$ is a feasible solution for BCC model, in present sub-units where the value of objective function is one. Then DMUp is efficient in present DMU's.

Now by additional restriction $\sum_{i=1}^{b} \mu^{(i)T}(y_i^{(p)}) = \sum_{i=1}^{b} v^{(p)T}(x_i^{(j)}) + \sum_{i=1}^{b-1} \overline{v}^{(i)}(y_i^{(p)})$ we study the generalized model in present sub-units for all DMUs

$$\begin{aligned}
&Max \quad \Delta \\
&S.t. \quad \Delta \leq \widetilde{d}_j + \alpha(\sum_{i=1}^{b} \mu^{(i)T}(y_i^{(p)} - y_i^{(j)}) + \sum_{i=1}^{b} v^{(i)T}(-x_i^{(p)} + x_i^{(j)}) + \sum_{i=1}^{b-1} \overline{v}^{(i)}(-y_i^{(p)} + y_i^{(j)})) \\
&\sum_{i=1}^{b} \mu^{(i)}(y_i^{(p)}) = \sum_{i=1}^{b} v^{(i)T}(x_i^{(p)} + \sum_{i=1}^{b-1} \overline{v}^{(i)}(y_i^{(p)}) \\
&\mu^{(i)T} y_i^{(j)} - v^{(i)T} x_i^{(j)} - \overline{v}^{(i-1)T} y_{i-1}^{(j)} \leq 0, \\
&\sum_{i=1}^{b} \mu^{(i)} + \sum_{i=1}^{b} v^{(i)} + \sum_{i=1}^{b-1} \overline{v}^{(i)} = 1 \\
&\mu^{(i)} \geq 0, \quad i = 1, \cdots, b \\
&v^{(i)} \geq 0, \quad i = 1, \cdots, b \\
&\overline{v}^{(i)} \geq 0, \quad i = 1, \cdots, b-1,
\end{aligned} \tag{4.11}$$

Theorem 4.2. *DMUp is CCR-efficient if and only if DMUp is α-efficient for sufficient large positive α is present sub-units by (3.8) model.*

Proof: Suppose that DMUp is α-efficient for some sufficient large positive α. That is for all solution $(\hat{\Delta}, \hat{\mu}, v^*, \hat{\overline{v}})$ we have $\widehat{\Delta} = 0$

$$0 = \hat{\Delta} \leq \widetilde{d}_j + \alpha(\hat{\mu}, \hat{v}, \hat{\overline{v}})(Z^{(p)} - Z^{(j)})$$

The necessary and sufficient condition for this formula is

$$\begin{cases} Z^{(p)} - Z^{(j)} \geq 0 \\ \widetilde{d}_j \geq 0 \forall j \end{cases} \tag{4.12}$$

But, we suppose that $\hat{v}x^{(p)} + \hat{\overline{v}}\overline{x}^{(p)} = \beta$ then

$$\frac{\hat{v}}{\beta}x^{(p)} + \frac{\widehat{\overline{v}}}{\beta}\overline{x}^{(p)} = 1. \tag{4.13}$$

Now $\hat{\mu}y^{(p)} = \hat{v}x^{(p)} + \hat{\overline{v}}\ \overline{x}^{(p)}$therefore $\frac{\hat{\mu}}{\beta}y^{(p)} = \frac{\hat{v}}{\beta}x^{(p)} + \frac{\hat{\overline{v}}}{\beta}\ \overline{x}^{(p)}$ and by (4.13) we have

$$\frac{\hat{\mu}}{\beta}y^{(p)} = 1 \tag{4.14}$$

and by (4.12) we have

$$(\hat{\mu}, \hat{v}, \hat{\overline{v}})(Z^{(p)} - Z^{(j)}) \geq 0$$

$$(\hat{\mu}, \hat{v}, \hat{\overline{v}})Z^{(p)} - (\hat{\mu}, \hat{v}, \hat{\overline{v}})Z^{(j)} \geq 0$$

Hence

$$-(\hat{\mu}, \hat{v}, \hat{\overline{v}})Z^{(j)} \geq 0.$$

Then

$$\frac{\hat{\mu}}{\beta}y^{(j)} - \frac{\hat{v}}{\beta}x^{(j)} - \frac{\hat{\overline{v}}}{\beta}\ \overline{x}^{(j)} \leq 0$$

and

$$\sum_{i=1}^{b} \frac{\hat{\mu}^{(i)T}}{\beta}y^{(j)} - \sum_{i=1}^{b} \frac{\hat{v}^{(i)T}}{\beta}x_i^{(j)} - \sum_{i=1}^{b-1} \frac{\hat{\overline{v}}^{(i)T}}{\beta}\ \overline{y}_i^{(j)} \leq 0, \quad j = 1, \cdots, n.$$

Therefore $(\frac{\hat{\mu}}{\beta}, \frac{\hat{v}}{\beta}, \frac{\hat{\overline{v}}}{\beta})$ is a feasible solution for CCR model in present sub-units and the value of objective function is$\frac{\hat{\mu}}{\beta}y^{(p)} = 1$. Then DMUp is efficient in present sub-units.

5 Conclusion

In this paper, we have suggested the GDEA model for performance evaluation based on parametric domination structure and defined $\alpha-$efficiency in the GDEA model. The method presented here can be used for the analysis of any real situation where a DMU is separated in to several different sub-units. Then we explain relationship between generalized model and BCC(CCR) model in present sub-units.

References

[1] A. Charnes, W. W. Cooper, E. Rhodes, Measuring the efficient of decision making unit, European Journal of Operation Research 2 (1978) 429-444.

[2] A. Amirteimoori, S. Kordrostami, DEA-like models for multi-component performance measurment, Applied Mathematics and Computer 163 (2005) 735-743.

[3] R. F. äre, S. Grosskopf, Productivity and intermediate products: a frontier approach, Economic Letters 50 (1) (1996)65-70.

[4] W. D. Cook, M. Hababuu, H. J. H. Tuenter, Multicomponent efficiency measurement and shared inputs in DEA: an application to sales and service performance in bank branches, Journal of Productivity Analysis 14 (2000) 209-224.

[5] R. G. Thompson, L. N. Langemeier, T. C. Lee, E. Lee, R. M. Thrull, The role of multiplier bounds in efficiency analysis with application to kansas Framiny, Journal of Econometrics 46 (1990) 91-108.

[6] Sawaragi Y., H. Nakayama and T. Tanino, Theory of multi objective optimization, Academic Press (1985).