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Effects of Chemical Reaction, Heat and Mass Transfer and Radiation on MHD Flow along a Vertical Porous Wall in the Present of Induced Magnetic Field

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Abstract

An analysis is carried out analytically to study steady heat and mass transfer by mixed convection flow of a viscous, incompressible, electrically-conducting and radiating fluid which is an optically thin gray gas, along a vertical porous plate under the action of transverse magnetic field taking into account the induced magnetic field with viscous dissipation of energy. The presence of a homogeneous chemical reaction of first order is also taken into account. The porous plate is subjected to a constant suction velocity as well as a uniform free stream velocity. The boundary layer equations have been transformed into dimensionless coupled non-linear ordinary differential equations by using appropriate transformations. The similarity solutions of the transformed dimensionless equations for the flow field, induced magnetic field, current density and heat and mass transfer characteristics are obtained by series solution technique and their numerical results are presented in the form of graphs. It is found that the velocity is reduced considerably with a rise in the conduction-radiation parameter (R) whereas the temperature is found to be markedly boosted with an increase in the conduction-radiation or the chemical reaction parameter (K). An increase in the magnetic body parameter (M) or chemical reaction rate (K) is found to escalate the induced magnetic field whereas an increase in the conduction-radiation parameter (R) or the magnetic Prandtl number (Pm) is shown to exert the opposite effect. Similarly, the current density (J) and the shear stress (τ) are both considerably increased with an increase in the magnetic Prandtl number. Possible applications of

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the present study include laminar magneto-aerodynamics, materials processing and MHD propulsion thermo-fluid dynamics.

Keywords : Thermal Radiation; Wall Suction; Induced Magnetic field; Current Density; Mixed Convection; Mass diffusion; Magnetic Prandtl number.

Table 1
Nomenclature

H_0	$Uniformmagneticfield$
H_x	$Inducedmagneticfieldalongx-direction$
$\overline{C}$	$Speciesconcentration(kg.m^{-3})$
C_P	$Specificheatatconstantpressure(J.kg^{-1}.K)$
$\bar{C}_\infty$	$Speciesconcentrationinthefreestream(kg.m^{-3})$
$\bar{C}_w$	$Speciesconcentrationatthesurface(kg.m^{-3})$
D	$Chemicalmoleculardiffusivity(m^2.s^{-1})$
Ec	$Eckertnumber/dissipativeheat$
g	$Accelerationduetogravity(m.s^{-2})$
Gr	$ThermalGrashofnumber$
Gm	$MassGrashofnumber$
K	$Chemicalreactionparameter$
M	$Hartmannnumber/Magneticparameter$
$\overline{m}$	$Absorptioncoefficient$
$\overline{p}$	$Pressure(Pa)$
Pm	$MagneticPrandtlnumber$
Pr	$Prandtlnumber$
$\overline{n}$	$Stefan-Boltzmannconstant$
Sc	$Schmidtnumber$
$\overline{T}$	$Temperature(K)$
$\bar{T}_w$	$Fluidtemperatureatthesurface(K)$
$\bar{T}_\infty$	$Fluidtemperatureinthefreestream(K)$
u	$Velocitycomponentinx-direction(m.s^{-1})$
U_0	$Dimensionlessfreestreamvelocity(m.s^{-1})$
v_0	$Suctionvelocity(m.s^{-1})$
J	$Currentdensity$
q_r	$Radiativeheatflux$

Table 2
Greek symbols

β	$Coefficientofvolumeexpansionforheattransfer(K^{-1})$
$\overline{\beta}$	$Coefficientofvolumeexpansionformasstransfer(K^{-1})$
μ_0	$Magneticdiffusivity$
θ	$Dimensionlessfluidtemperature, \theta = \frac{(\bar{T}-\bar{T}_\infty)}{(\bar{T}_w-\bar{T}_\infty)}$
κ	$Thermalconductivity(W.m^{-1}.K^{-1})$
ν	$Kinematicviscosity(m^2.s^{-1})$
ρ	$Density(kg.m^{-3})$
σ	$Electricalconductivity$
τ	$Shearingstress(N.m^{-2})$
ϕ	$Dimensionlessspeciesconcentration$

Subscripts

- w Conditions on the wall.
- ∞ Free stream conditions.

1 Introduction

The experimental and theoretical studies of magneto-hydrodynamic flows for an electrically conducting fluid past a porous vertical surface are important from a technological point of view, because they have many engineering applications such as plasma studies, petroleum industries, MHD power generators, cooling of nuclear reactors, boundary layer control in aerodynamics, and crystal growth. In view of these applications, several investigators have made model studies on the effect of magnetic field on free convection flows. Along with the effects of magnetic field, wall transpiration effect being an effective method of controlling the boundary layer has been considered by Kafoussias et al. [11]. Raptis and Soundalgekar [14] investigated steady laminar free convection flow of an electrically conducting fluid along a porous hot vertical plate in the presence of a heat source/sink. The study of heat and mass transfer on free convective flow of a viscous incompressible fluid past an infinite vertical porous plate in the presence of a transverse sinusoidal suction velocity and a constant free stream velocity was presented by Ahmed [2].

Ahmed and Liu [3] analyzed the effects of mixed convection and mass transfer of a three-dimensional oscillatory flow of a viscous incompressible fluid past an infinite vertical porous plate in the presence of a transverse sinusoidal suction velocity oscillating with time and a constant free stream velocity. Chamkha and Issa [7] investigated the effects of heat generation/absorption and thermophoresis on hydromagnetic flow with heat and mass transfer over a flat surface. Chamkha [8] investigated the chemical reaction effects on heat and mass transfer laminar boundary layer flow in the presence of heat generation/absorption effects. Muthucumaraswamy and Kulaivel [12] presented an analytical solution to the problem of flow past an impulsively started infinite vertical plate in the presence of heat flux and variable mass diffusion, taking into account the presence of a homogeneous chemical reaction of first order.

The above studies have generally been confined to very small magnetic Reynolds numbers, allowing magnetic induction effects to be neglected. Such effects must be considered for relatively large values of the magnetic Reynolds number. Glauert [10] presented a seminal analysis for hydromagnetic flat plate boundary layers along a magnetized plate with uniform magnetic field in the stream direction at the plate. He obtained series expansion solutions (for both large and small values of the electrical conductivity parameter) for the velocity and magnetic fields, indicating that for a critical value of applied magnetic field, boundary-layer separation arises. Recently, Bg et al. [4] have obtained local non-similarity numerical solutions for the velocity, temperature and induced magnetic field distributions in forced convection hydromagnetic boundary layers, over an extensive range of magnetic Prandtl numbers and Hartmann numbers. Alom et al. [1] investigated the steady MHD heat and mass transfer by mixed convection flow from a moving vertical porous plate with induced magnetic, thermal diffusion, constant heat and mass fluxes.

On the other hand, at high temperature the effects of radiation is space technology, solar power technology, space vehicle re-entry, nuclear engineering applications and other industrial areas are very significant. In view of these applications, England and Emery [9] have studied the radiation effects of an optically thin gray gas bounded by a stationary plate. The hydromagnetic free convective flow of an optically thin gray gas in the presence of radiation has been investigated by Bestman and Adiepong [5], Naroua et al. [13], when the induced magnetic field is negligible. Raptis and Massalas [15] investigated the effects of radiation on the oscillatory flow of a gray gas, absorbing-emitting in the presence induced magnetic field. The hydrodynamic free convective flow of an optically thin gray gas in the presence of radiation, when the induced magnetic field is taken into account was studied by Raptis et al. [16]. Chamkha [6] considered the problem of steady, hydromagnetic boundary layer flow over an accelerating semi-infinite porous surface in the presence of thermal radiation, buoyancy and heat generation or absorption effects.

In spite of all these studies, the problem of steady MHD mixed convective heat and mass transfer of a chemically reacting fluid in the presence of induced magnetic field as well as conduction-radiation effect has received little attention. Hence, the main objective of the present investigation is to consider the effects of radiation and chemical reaction on steady mixed convective heat and mass transfer flow of an optically thin gray gas over an infinite vertical porous plate with constant suction taking into account the induced magnetic field, and viscous dissipation of energy. Such an attempt has been made in the present work owing to applications in magnetic materials processing.

2 Mathematical analysis

We consider steady MHD mixed convective heat and mass transfer flow of a Newtonian, electrically-conducting, viscous, incompressible and radiating fluid over a porous vertical infinite plate with viscous dissipation of energy and homogeneous chemical reaction of first order.

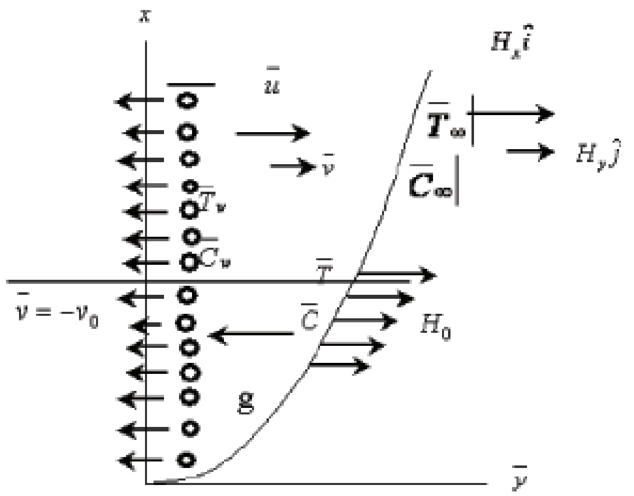

Fig. 1. Physical configuration and coordinate system

The following assumptions are implicit in our analysis:

- All of the fluid properties except the density in the buoyancy force term are constant;
- The Eckert number, Ec , is small, as appropriate for viscous incompressible regimes;
- The plate is subjected to a constant suction velocity;
- The plate is non-conducting and the applied magnetic field is of uniform strength H_0 and applied transversely to the direction of the main stream taking into account the induced magnetic field;
- There exists a homogeneous chemical reaction of first order with rate constant $\overline{K}$ between the diffusing species and the fluid;
- The magnetic Reynolds number of the flow is not taken to be small enough so that the induced magnetic field is not negligible;
- The concentration of the diffusing species in the binary mixture is assumed to be very small in comparison with the other chemical species, which are present, and hence, the Soret and Dufour effects are negligible;
- The equation of conservation of electric charge is $\nabla.J = 0$, where $J = (J_x, J_y, J_z)$. The direction of propagation is considered only along the y-axis and does not have any variation along the y-axis and so $\frac{\partial J_y}{\partial y} = 0$, which gives $J_y = constant$. Since the plate is electrically non-conducting, this constant is zero and hence $J_y = 0$ everywhere in the flow, following.

In the case of electrically conducting fluid, the flow and heat transfer are induced by an imposed magnetic field. An e.m.f. is produced in the fluid flowing across the transverse magnetic field. The resultant effect of current and magnetic fields produces a force, resisting the fluid motion. The fluid velocity also affects the magnetic field by producing an induced magnetic field which perturbs the original field. We introduce a coordinate system $(\overline{x}, \overline{y}, \overline{z})$ with the $\overline{x}$-axis vertically upwards along the plate, $\overline{y}$-axis normal to the plate into the fluid region and $\overline{z}$-axis along the width of the plate. Let the plate be long enough in the $\overline{x}$-direction for the flow to be parallel. Let $(\overline{x}, \overline{y}, 0)$ be the fluid velocity and $(H_x, H_y, 0)$ be the magnetic induction vector at a point $(\overline{x}, \overline{y}, \overline{z})$ in the fluid. Since the plate is infinite in length in the $\overline{x}$-direction, therefore, all of the physical quantities except possibly the pressure are assumed to be independent of $\overline{x}$. The wall is maintained at constant temperature $\bar{T}_w$ and concentration $\bar{C}_w$ higher than the ambient temperature $\bar{T}_\infty$ and concentration $\bar{C}_\infty$ respectively.

Within the frame of such assumptions and under the *Oberbeck-Boussinesq's* approximation and in consistency with boundary layer theory, the governing equations relevant to the problem are:

Conservation of Mass:

$\frac{\partial \overline{v}}{\partial \overline{y}} = 0$ which is satisfied with $\overline{v} = -v_0 =$ a constant.

Gauss's law of magnetism:

$\frac{\partial \overline{H}_y}{\partial \overline{y}} = 0$ which holds for $\overline{H_y} = -H_0$ = a constant=strength from applied magnetic field.

Conservation of Momentum:

$$-v_0 \frac{\partial \overline{u}}{\partial \overline{y}} = \nu \frac{\partial^2 \overline{u}}{\partial \overline{y}^2} + g\beta(\overline{T} - \overline{T}_\infty) + g\overline{\beta}(\overline{C} - \overline{C}_\infty) + \frac{\mu_0 H_0}{\rho} \frac{\partial \overline{H}_x}{\partial \overline{y}} \tag{2.1}$$

Conservation of Energy:

$$-v_0 \frac{\partial \overline{T}}{\partial \overline{y}} = \frac{1}{\rho C_P} \left[\kappa \frac{\partial^2 \overline{T}}{\partial \overline{y}^2} - \frac{\partial q_r}{\partial \overline{y}} \right] + \frac{\mu}{C_P} \left(\frac{\partial \overline{u}}{\partial \overline{y}} \right)^2 \tag{2.2}$$

Conservation of Magnetic Induction:

$$\frac{1}{\sigma \mu_0} \frac{\partial^2 \overline{H}_x}{\partial \overline{y}^2} + H_0 \frac{\partial \overline{u}}{\partial \overline{y}} + v_0 \frac{\partial \overline{H}_x}{\partial \overline{y}} 0 \tag{2.3}$$

Conservation of Species (Mass Diffusion):

$$-v_0 \frac{\partial \overline{C}}{\partial \overline{y}} = D \frac{\partial^2 \overline{C}}{\partial \overline{y}^2} - \overline{K} \left(\overline{C} - \overline{C}_\infty \right) \tag{2.4}$$

The boundary conditions are:

$$(\bar{y} = 0 :)(\bar{u} = 0), (\bar{T} = \bar{T}_w), (\bar{C} = \bar{C}_w), (\bar{H}_x = 0)$$

$$(\bar{y} \to \infty :)(\bar{u} \to U_0), (\bar{T} \to \bar{T}_\infty), (\bar{C} \to \bar{C}_\infty), (\bar{H}_x \to 0) \tag{2.5}$$

We now introduce the following non-dimensional quantities into the equations (2.1) to (2.4):

$$(y = \frac{v_0 \overline{y}}{\nu}), (u = \frac{\bar{u}}{U_0}), (U = \frac{\bar{U}_p}{v_0}), (\theta = \frac{(\bar{T} - \bar{T}_\infty)}{(\bar{T}_w - \bar{T}_\infty)}), (Sc = \frac{\nu}{D}),$$

$$(Gm = \frac{\nu g \bar{\beta}(\bar{C}_w - \bar{C}_\infty)}{U_0 v_0^2}), (Gr = \frac{\nu g \beta(\bar{T}_w - \bar{T}_\infty)}{U_0 v_0^2}), (P_m = \sigma \nu \mu_0),$$

$$(M = \sqrt{\frac{\mu_0}{\rho}} \frac{\bar{H}_x}{U_0}), (H = \sqrt{\frac{\mu_0}{\rho}} \frac{\bar{H}_0}{v_0}), (Pr = \frac{\mu C_P}{\kappa}), (K = \frac{\nu \bar{K}}{v_0^2}),$$

$$(Ec = \frac{U_0^2}{C_P(\bar{T}_w - \bar{T}_\infty)}), (R = \frac{64 \bar{m} \bar{n} \nu \bar{T}_\infty^3}{\rho C_P v_0^2}), (\phi = \frac{(\bar{C} - \bar{C}_\infty)}{(\bar{C}_w - \bar{C}_\infty)}) \tag{2.6}$$

For the case of an optically thin gray gas, the local radiant absorption is expressed as [11, 4, 12, 2, 3]:

$$\frac{\partial q_r}{\partial \overline{y}} = 4\overline{m}\overline{n} \left(\overline{T}_\infty^4 - \overline{T}^4 \right) \tag{2.7}$$

where $\overline{m}$ is the absorption coefficient and $\overline{n}$ is the Stefan-Boltzmann constant.

We assume that the temperature differences within the flow are sufficiently small such that $\overline{T}^4$ may be expressed as linear function of the temperature. This is accomplished by expanding $\overline{T}^4$ in Taylor series about $\overline{T}_\infty$ and neglecting higher-order terms, thus

$$\overline{T}^4 \cong 4\overline{T}_\infty^3 \overline{T} - 3\overline{T}_\infty^4 \tag{2.8}$$

Using the transformations (2.6) and with the help of (2.7)-(2.8), the non-dimensional forms of (2.1) to (2.4) are

$$\frac{d^2u}{dy^2} + \frac{du}{dy} + M\frac{dH}{dy} + Gr\theta + Gm\phi = 0 \tag{2.9}$$

$$\frac{d^2\theta}{dy^2} + Pr\frac{d\theta}{dy} + \frac{PrR}{4}\theta + PrEc\left(\frac{du}{dy}\right)^2 = 0 \tag{2.10}$$

$$\frac{d^2\phi}{dy^2} + Sc\frac{d\phi}{dy} - ScK\phi = 0 \tag{2.11}$$

$$\frac{d^2H}{dy^2} + MPm\frac{du}{dy} + Pm\frac{dH}{dy} = 0 \tag{2.12}$$

The corresponding boundary conditions are:

$$(y = 0) : (u = 0), (\theta = 1), (H = 0), (\phi = 1)$$

$$(y \to \infty) : (u \to 1), (\theta \to 0), (H \to 0), (\phi \to 0) \tag{2.13}$$

3 Method of solution

The perturbation theory leads to an expression for the desired solution in terms of a power series in some "small" parameter that quantifies the deviation from the exactly solvable problem. The leading term in this power series is the solution of the exactly solvable problem, while further terms describe the deviation in the solution, due to the deviation from the initial problem. The perturbation theory is applicable if the problem at hand can be formulated by adding a "small" term (Eckert number in this work) to the mathematical description of the exactly solvable problem. The solution of the equation (2.11) subject to the boundary condition (2.13) is

$$\phi = e^{-\eta y} \tag{3.14}$$

Now, in order to solve the equations (2.9), (2.10) and (2.12) under the boundary condition (2.13), we note that $Ec << 1$ for all incompressible fluids and it is assumed the solutions of the equations to be of the form

$$\Re(y) = \Re_0(y) + Ec\Re_1(y) + O(Ec^2), \tag{3.15}$$

where $\Re$ stands for u , θ or b_x. Substituting (3.15) into the equations (2.9), (2.10) and (2.12) and equating the coefficients of the same degree terms and neglecting terms of $O(Ec^2)$, the following differential equations are obtained:

$$u_0'' + u_0' = -Gr\theta_0 - Gm\phi - MH_0' \tag{3.16}$$

$$u_1'' + u_1' = -Gr\theta_1 - MH_1' \tag{3.17}$$

$$\theta_0'' + Pr\theta_0' - \frac{PrR}{4}\theta_0 = 0 \tag{3.18}$$

$$\theta_1'' + Pr\theta_1' - \frac{PrR}{4}\theta_1 = -Pr\left(u_0'\right)^2 \tag{3.19}$$

$$H_0'' + PmH_0' = -MPmu_0' \tag{3.20}$$

$$H_1'' + PmH_1' = -MPmu_1' \tag{3.21}$$

The boundary conditions (2.10) reduce to

$$(y=0) : (u_0 = 0), (u_1 = 0), (\theta_0 = 1), (\theta_1 = 0), (H_0 = 0), (H_1 = 0)$$

$$(y \to \infty) : (u_0 \to 1), (u_1 \to 0), (\theta_0 \to 0), (\theta_1 \to 0), (H_0 \to 0), (H_1 \to 0), \tag{3.22}$$

The solutions of equations (3.16) to (3.21) subject to the boundary conditions (3.22) are:

$$\theta_0(y) = e^{-\xi y} \tag{3.23}$$

$$u_0(y) = 1 + A_1 e^{-\xi y} + A_2 e^{-\eta y} + A_3 e^{-\lambda y} \tag{3.24}$$

$$H_0(y) = A_4 e^{-\xi y} + A_5 e^{-\eta y} + A_6 e^{-\lambda y} + A_7 e^{-Pmy} \tag{3.25}$$

$$\begin{aligned} \theta_1(y) &= D_1 e^{-2\xi y} + D_2 e^{-2\eta y} + D_3 e^{-2\lambda y} + D_4 e^{-(\xi+\eta)y} + D_5 e^{-(\lambda+\xi)y} \\ &+ D_6 e^{-(\lambda+\eta)y} + D_7 e^{-\xi y} \end{aligned} \tag{3.26}$$

$$\begin{aligned} u_1(y) &= F_1 e^{-2\xi y} + F_2 e^{-2\eta y} + F_3 e^{-2\lambda y} + F_4 e^{-(\xi+\eta)y} + F_5 e^{-(\lambda+\xi)y} \\ &+ F_6 e^{-(\lambda+\eta)y} + F_7 e^{-\xi y} + F_8 e^{-\lambda y} + F_9 e^{-y} \end{aligned} \tag{3.27}$$

$$\begin{aligned} H_1(y) &= E_1 e^{-2\xi y} + E_2 e^{-2\eta y} + E_3 e^{-2\lambda y} + E_4 e^{-(\xi+\eta)y} + E_5 e^{-(\lambda+\xi)y} \\ &+ E_6 e^{-(\lambda+\eta)y} + E_7 e^{-\xi y} + E_8 e^{-\lambda y} + E_9 e^{-y} \end{aligned} \tag{3.28}$$

4 Skin-Friction

The boundary layer produces a drag on the plate due to the viscous stresses which are developed at the wall. The viscous stress at the surface of the plate is given by

$$\tau = \left(\frac{\partial u}{\partial y}\right)_{y=0} = \tau_0 + Ec\tau_1, \tag{4.29}$$

where

$$u_0'(0) = -\xi A_1 - \eta A_2 - \lambda A_3$$

and

$$u_1'(0) = -2\xi F_1 - 2\eta F_2 - 2\lambda F_3 - (\xi+\eta)F_4 - (\lambda+\xi)F_5 - (\lambda+\eta)F_6 - \xi F_7 - \lambda F_8 - F_9$$

5 Current density

The non-dimensional current density at the plate is given by

$$J = \left(\frac{\partial H}{\partial y}\right)_{y=0} = J_0 + EcJ_1, \tag{5.30}$$

where

$$J_0 = H_0^{'}(0) = -\xi A_4 - \eta A_5 - \lambda A_6 - PmA_7$$

and

$$J_1 = H_1^{'}(0) = -2\xi E_1 - 2\eta E_2 - 2\lambda E_3 - (\xi+\eta)E_4 - (\lambda+\xi)E_5 - (\lambda+\eta)E_6 - \xi E_7 - \lambda E_8 - E_9$$

6 Results and discussion

In order to get a clear insight of the physical problem, numerical results are displayed with the help of graphical illustrations. A representative set of results is shown in Figs. 2-12. Computations were carried out for various values of the physical parameters such as the chemical reaction parameter K, Hartmann number M , magnetic Prandtl number Pr , and the radiation parameter R. In addition, the values of the induced magnetic field H, temperature field θ and the current density J are tabulated in Tables 1 and 2 for various values of the Eckert number Ec.

Figures 2-4 show the effects of both of the Hartmann and magnetic Prandtl numbers M and Pm on the velocity, temperature and induced magnetic field distributions, respectively. In general, application of a transverse magnetic field has the tendency to decrease the velocity due to the resistive Lorentz force. However, in the presence of the induced magnetic field, increasing the magnetic parameter M increases the induced magnetic field J causing the velocity of the fluid to increase yielding a net increase in the velocity profile. This behavior is depicted in the increases in both the fluid velocity and induced magnetic field profiles as M and increase in figures 2 and 4. In addition, it is also seen that, due to the presence of viscous dissipation, the increases in the velocity profiles as M and increase cause increases in the temperature profiles as is clearly depicted in Fig. 3.

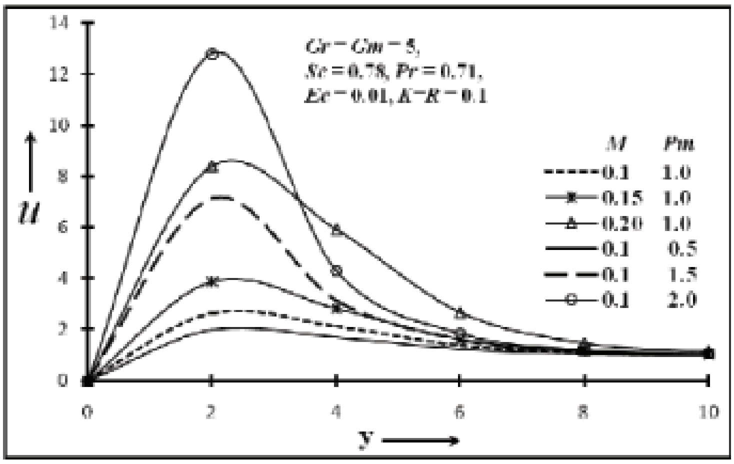

Fig. 2. Velocity field distribution with Hartmann and magnetic Prandtl number.

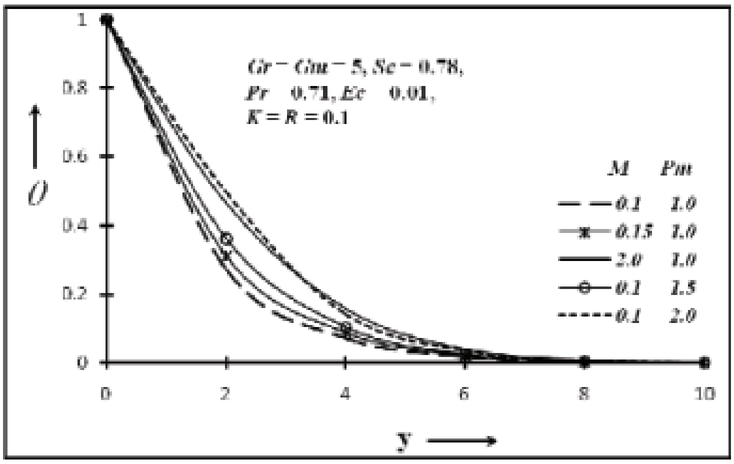

Fig. 3. Temperature field distribution with Hartmann and magnetic Prandtl number.

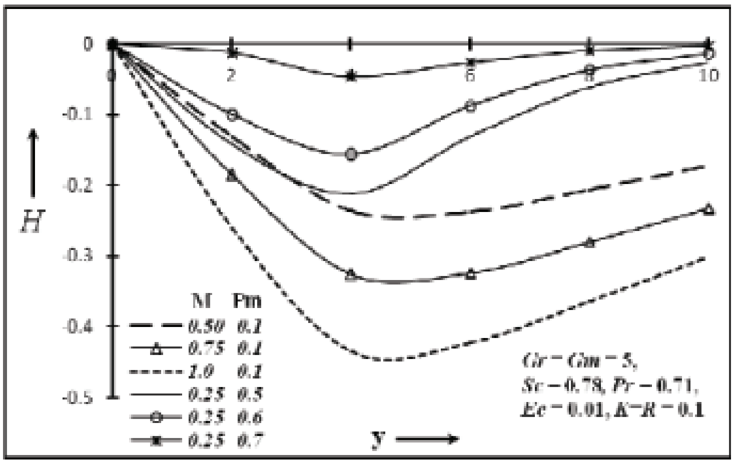

Fig. 4. Induced magnetic field distribution with Hartmann and magnetic Prandtl number.

Figures 5-7 present the effects of both of the chemical reaction and the radiation parameters K and R on the velocity, temperature and induced magnetic field profiles, respectively. It is seen that the velocity distribution decreases as the radiation parameter increases while it increases as the chemical reaction parameter increases. In addition, increasing either of the radiation parameter or the chemical reaction parameter causes the fluid temperature to increase. It is also predicted that the induced magnetic field distribution increases as the radiation parameter increases while it decreases as the chemical reaction parameter increases. The behaviors are clear from figures 5-7.

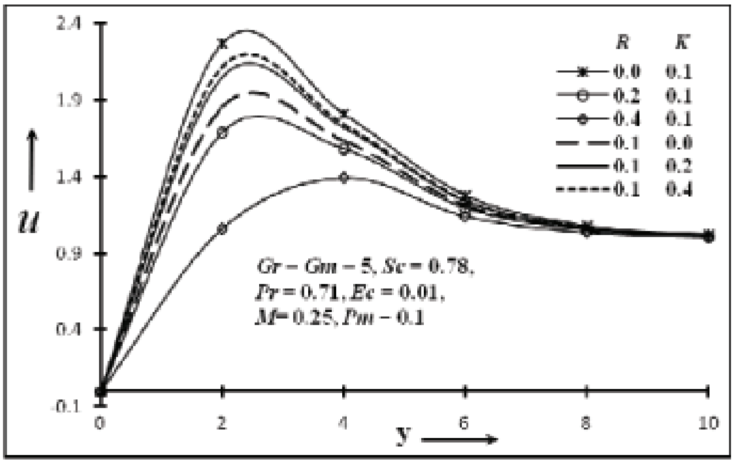

Fig. 5. Velocity field distribution with radiation and chemical reaction parameter.

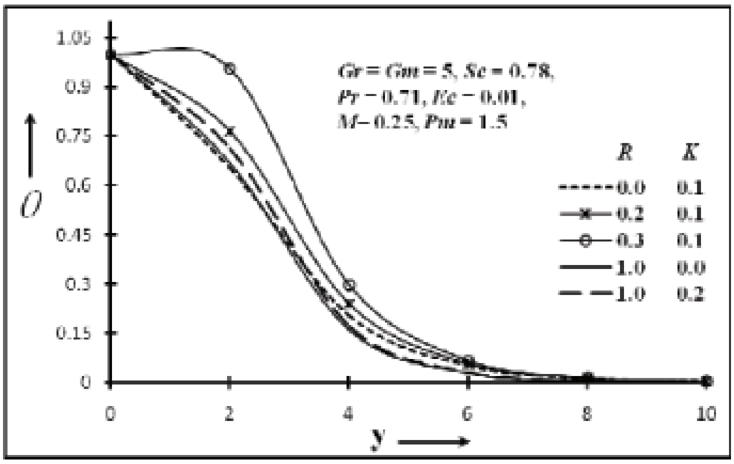

Fig. 6. Temperature field distribution with radiation and chemical reaction parameter.

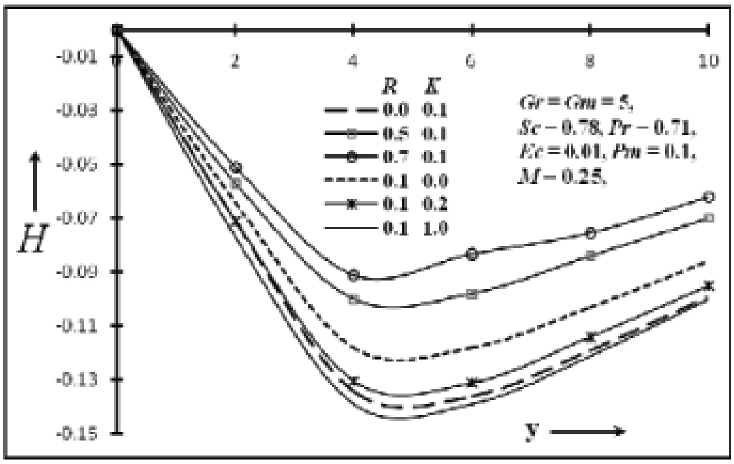

Fig. 7. Induced magnetic field distribution with Radiation and chemical reaction parameter.

Figures 8 and 9 present the current density distribution J versus the Hartmann number M for various values of the radiation parameter, chemical reaction parameter and the magnetic Prandtl number, respectively. It is observed that, in general, for small magnetic Prandtl numbers ($Pm < 1$), the current density increases as the Hartmann number increases reaching a maximum and then decreases slightly thereafter. However, for $Pm \geq 1$, the current density increases as the Hartmann number increases. Moreover, the current density increases as either of the chemical reaction parameter or the magnetic Prandtl number increases while it decreases as the radiation parameter increases.

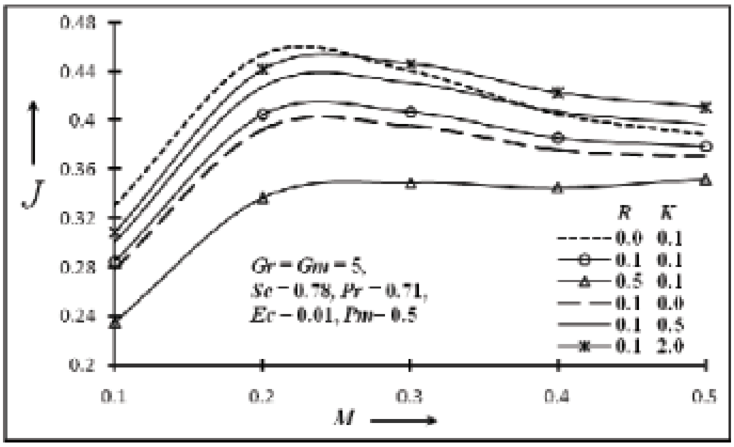

Fig. 8. Current density versus Hartmann number for various radiation and chemical reaction parameter.

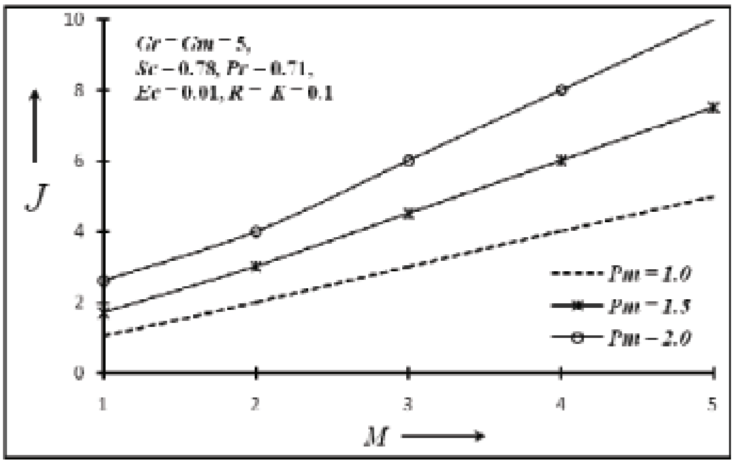

Fig. 9. Current density versus Hartmann number for various magnetic Prandtl number.

Figures 10-12 show the effects of the radiation parameter, magnetic Prandtl number and the chemical reaction parameter on the distribution of the shear stress versus the Hartmann number, respectively. In general, it is observed that increases in the Hartmann number cause increases in the shear stress values. In addition, the values of shear stress are predicted to increase as either of the radiation parameter or the magnetic Prandtl number increases while the values of the shear stress decrease as the chemical reaction parameter increases.

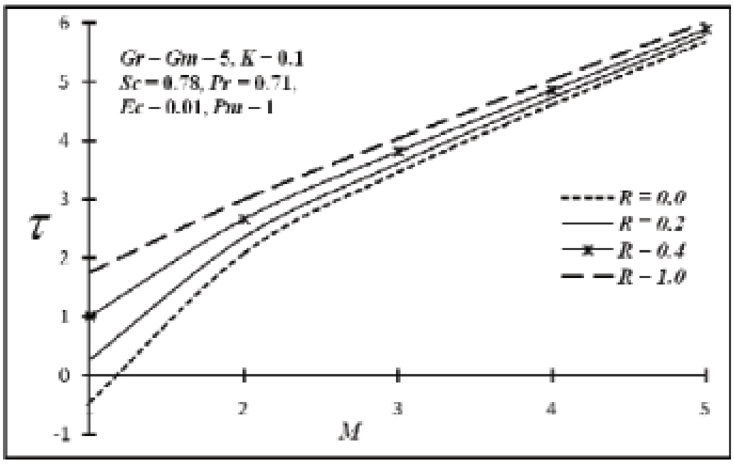

Fig. 10. Shear stress versus Hartmann number for various radiation parameters.

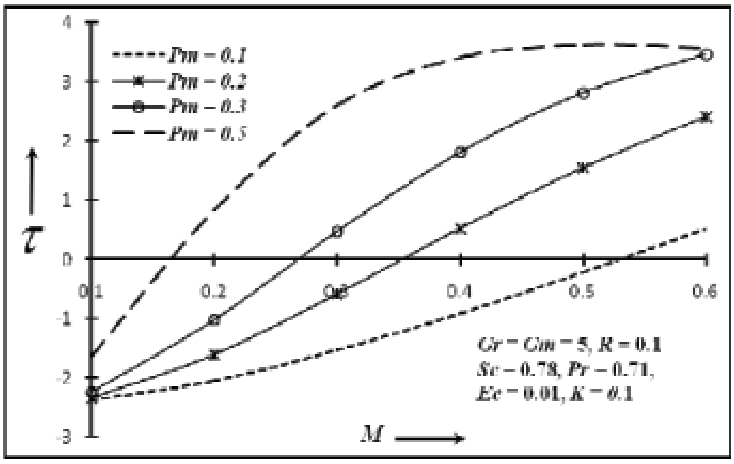

Fig. 11. Shear stress versus Hartmann number for various magnetic Prandtl numbers.

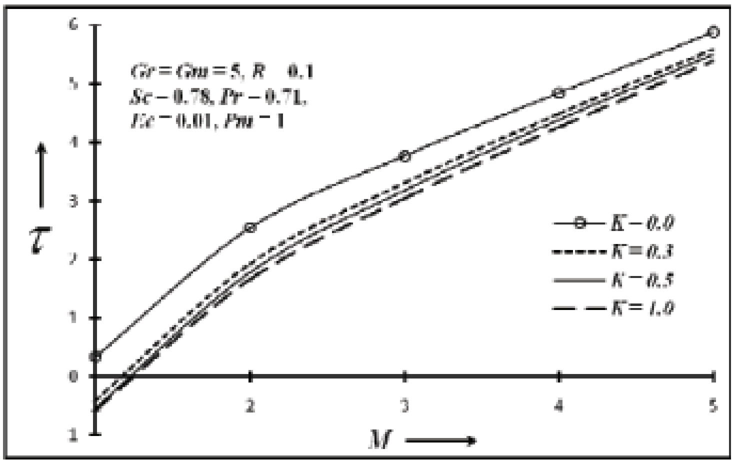

Fig. 12. Shear stress distribution versus Hartmann number for various chemical reaction parameters.

Table 3 presents the induced magnetic field (H) and temperature field (θ) for various

Eckert numbers (Ec). It is predicted that the fluid temperature increases due to viscous dissipation as (Ec) increases while the induced magnetic field shows a different behavior in which it decreases and then increases as (Ec) is increased from 0 to 0.05.

Table 3
Induced magnetic field (H) and temperature field (θ) for various Eckert numbers for When $Gr = 5 = Gm, Pr = 0.71, Pm = K = R = 0.1, M = 0.25$

⋆	H	H	H	⋆	θ	θ	θ
y	Ec = 0.00	Ec = 0.03	Ec = 0.05	y	Ec = 0.00	Ec = 0.03	Ec = 0.05
0	-0.04226	-0.08204	-0.01574	0	1.00000	1.00000	1.00000
1	-0.11541	-0.13119	-0.10489	1	0.05304	0.07427	0.08843
2	-0.12354	-0.12831	-0.12035	2	0.01222	0.01843	0.02256
3	-0.10944	-0.11071	-0.1.0859	3	0.00281	0.00435	0.00435
4	-0.09175	-0.09296	-0.09153	4	0.00065	0.00101	0.00125

Table 4 displays the effects of the Eckert number (Ec) and the Hartmann number (M) on the current density (J). It is seen that the current density increases with increases in either of the Hartmann number or the Eckert number.

Table 4
Current density (J) for $Gr = 5 = Gm$, $Pr = 0.71$, $Pm = K = R = 0.1$, $M = 0.25$

M	Ec = 0.00	Ec = 0.05	Ec = 0.10
0.1	0.02783	0.09170	0.18835
0.2	0.05444	0.19218	0.36436
0.3	0.07878	0.27390	0.51780
0.4	0.10021	0.34103	0.64206
0.5	0.11845	0.39227	0.73454

7 Conclusion

The problem of steady heat and mass transfer by mixed convection flow of a viscous, incompressible, electrically-conducting and radiating fluid which is an optically thin gray gas, along a vertical porous plate under the action of a transverse magnetic field was studied analytically. The governing equations for this problem were developed and the perturbation theory was used. A parametric study illustrating the effects of the various parameters on the flow, heat and mass transfer characteristics was performed. The values of the fluid velocity were reduced considerably with a rise in the radiation parameter whereas they increased as either of the Hartmann number or the magnetic Prandtl number was increased. The fluid temperature was found to be markedly boosted with an increase in either of the radiation parameter, chemical reaction parameter or the Eckert number. An increase in either of the magnetic Hartmann number or the chemical reaction parameter was found to escalate the induced magnetic field whereas an increase in either of the radiation parameter or the magnetic Prandtl number was shown to exert the opposite effect. Similarly, the current density and the shear stress were both considerably increased with increases in either of the Hartmann number, magnetic Prandtl number or the Eckert number.

References

[1] M.M. Alom, I. M. Rafiqul and F. Rahman, Steady heat and mass transfer by mixed convection flow from a vertical porous plate with induced magnetic field, constant heat and mass fluxes, Thammasat Int. J. Sc. Tech. 13 (2008) 1-13.

[2] S. Ahmed , The study of heat and mass transfer on free convective three-dimensional unsteady flows over a porous vertical plate, Journal of Energy, Heat and Mass Transfer 31 (2009) 89-110.

[3] S. Ahmed, I. C. Liu, Mixed convective three-dimensional heat and mass transfer flow with transversely periodic suction velocity, Int. J. Applied Mathematics and Mechanics 6 (2010) 58-73.

[4] O. Anwar Bg, A. Y. Bakier, V. R. Prasad, J. Zueco and S. K. Ghosh, Nonsimilar, laminar, steady, electrically-conducting forced convection liquid metal boundary layer flow with induced magnetic field effects, Int. J. Thermal Sciences 48 (2009) 1596-1606.

[5] A.R. Bestman, S. K. Adiepong, Unsteady hydromagnetic free convective flow with radiative heat transfer in a rotating fluid, Astrophysics and Space Science 143 (1988) 73-80.

[6] A.J. Chamkha, Thermal radiation and buoyancy effects on hydrodynamic flow over an accelerated permeable surface with heat source and sink, Int. J. Engineering Science 38 (15) (2000) 1699-1712.

[7] A.J. Chamkha, C. Issa , Effects of heat generation/absorption and thermophoresis on hydromagnetic flow with heat and mass transfer over a flat surface, Int. J. Numer. Methods Heat Fluid Flow 10 (2000) 432-448.

[8] A.J. Chamkha, MHD flow of a uniformly stretched vertical permeable surface in the presence of heat generation/absorption and a chemical reaction, Int. Communication in Heat and Mass Transfer 30 (2003) 413-422.

[9] W.G. England, A. F. Emery, Thermal radiation effects on the laminar free convection boundary layer of an absorbing gas, J. of Heat Transfer 91 (1969) 37-44.

[10] M. B. Glauert, The boundary layer on a magnetized plate, J. Fluid Mechanics 12 (1962) 625-638.

[11] N.G. Kafoussias, N. Nanousis, and G. A. Georgantopoulos, Free convection effects on the Stokes problem for an infinite vertical limiting surface with constant suction Astrophysics and Space Science 64 (1979) 391-399.

[12] R. Muthucumaraswamy, T. Kulaivel, Chemical reaction effects on moving infinite vertical plate with uniform heat flux and variable mass diffusion, Forschung im-Ingenieurwesen 68 (2) (2003) 101-104.

[13] H. Naroua, P. C. Ram, Finite-element analysis of natural convection flow in a rotating fluid with radiative heat transfer, J. of Magnetohydrodynamics and Plasma Research 7 (1998) 257-274.

[14] A.A. Raptis, V. M. Soundalgekar, Steady laminar free convection flow of an electrically conducting fluid along a porous hot vertical plate in presence of heat source/sink, ZAMM 64 (1984) 127-130.

[15] A. Raptis, C. V. Massalas, Magnetohydrodynamic flow past a plate by the presence of radiation, Heat and Mass transfer 34 (1998) 107-109.

[16] A. Raptis, C. Perdikis, and A. Leontitsis, Effects of radiation in an optically thin gray gas flowing past a vertical infinite plate in the presence of a magnetic field, Heat and mass Transfer 39 (2003) 771-773.

8 Appendix

$$(\xi = \frac{Pr + \sqrt{Pr^2 + RPr}}{2}), (\lambda = \frac{1 + Pm + \sqrt{(1 - Pm)^2 + 4M^2 Pm}}{2}),$$

$$(\eta = \frac{Sc + \sqrt{Sc^2 + 4KSc}}{2}), (A_1 = \frac{Gr(\xi - Pm)}{-\xi^3 + (1 + Pm)\xi^2 + \xi Pm(M^2 - 1)}),$$

$$(A_2 = \frac{Gr(\eta - Pm)}{-\eta^3 + (1 + Pm)\eta^2 + \eta Pm(M^2 - 1)}), (A_3 = -(1 + A_1 + A_2),$$

$$(A_4 = \frac{MA_1 Pm}{\xi - Pm}), (A_5 = \frac{MA_2 Pm}{\eta - Pm}), (A_6 = \frac{MA_3 Pm}{\lambda - Pm}),$$

$$(A_7 = -(A_4 + A_5 + A_6), (D_1 = \frac{-Pr\xi^2 A_1^2}{4\xi^2 - 2\xi Pr - PrR/4}),$$

$$(D_2 = \frac{-Pr\eta^2 A_2^2}{4\eta^2 - 2\eta Pr - PrR/4}), (D_3 = \frac{-Pr\lambda^2 A_3^2}{4\lambda^2 - 2\lambda Pr - PrR/4}),$$

$$(D_4 = \frac{-2Pr\xi\eta A_1 A_2}{(\xi + \eta)^2 - (\xi + \eta)Pr - PrR/4}), (D_5 = \frac{-2Pr\xi\lambda A_1 A_3}{(\xi + \lambda)^2 - (\xi + \lambda)Pr - PrR/4}),$$

$$(D_6 = \frac{-2Pr\lambda\eta A_2 A_3}{(\lambda + \eta)^2 - (\lambda + \eta)Pr - PrR/4}), (D_7 = -(D_1 + D_2 + D_3 + D_4 + D_5 + D_6)),$$

$$(E_1 = \frac{GrMPmD_1}{-8\xi^3 + 4(1 + Pm)\xi^2 + 2\xi Pm(M^2 - 1)}),$$

$$(E_2 = \frac{GrMPmD_2}{-8\eta^3 + 4(1 + Pm)\eta^2 + 2\eta Pm(M^2 - 1)}),$$

$$(E_3 = \frac{GrMPmD_3}{-8\lambda^3 + 4(1 + Pm)\lambda^2 + 2\lambda Pm(M^2 - 1)}),$$

$$(E_4 = \frac{GrMPmD_4}{-(\xi+\eta)^3 + (1+Pm)(\xi+\eta)^2 + (\xi+\eta)Pm(M^2-1)}),$$

$$(E_5 = \frac{GrMPmD_5}{-(\xi+\lambda)^3 + (1+Pm)(\xi+\lambda)^2 + (\xi+\lambda)Pm(M^2-1)}),$$

$$(E_6 = \frac{GrMPmD_6}{-(\eta+\lambda)^3 + (1+Pm)(\eta+\lambda)^2 + (\eta+\lambda)Pm(M^2-1)}),$$

$$(E_7 = \frac{GrMPmD_7}{-\xi^3 + (1+Pm)\xi^2 + \xi Pm(M^2-1)}), (E_8 = -(E_1+E_2+E_3+E_4+E_5+E_6+E_7)),$$

$$(F_1 = \frac{2M\xi E_1 - GrD_1}{2\xi(2\xi-1)}), (F_2 = \frac{2M\xi E_2 - GrD_2}{2\eta(2\eta-1)}), (F_3 = \frac{2M\xi E_3 - GrD_3}{2\lambda(2\lambda-1)}),$$

$$(F_4 = \frac{2M(\xi+\eta)E_4 - GrD_4}{(\xi+\eta)(\xi+\eta-1)}), (F_5 = \frac{2M(\xi+\lambda)E_5 - GrD_5}{(\xi+\lambda)(\xi+\lambda-1)}),$$

$$(F_6 = \frac{2M(\eta+\lambda)E_6 - GrD_6}{(\eta+\lambda)(\eta+\lambda-1)}), (F_7 = \frac{M\xi E_7 - GrD_7}{\xi(\xi-1)}),$$

$$(F_8 = \frac{M\lambda E_8}{\lambda(\lambda-1)}), (F_9 = -(F_1+F_2+F_3+F_4+F_5+F_6+F_7+F_8))$$