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An Iterative Approach for Estimation of Efficiency by Weighted Distance

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Abstract

In group decision analysis, numerous approaches have been suggested in an attempt to solve the problem of aggregation of individual fuzzy opinion to form a group consensus as the basis of a group decision. If the inputs and outputs are fuzzy quantity, the decision making units can not be easily evaluated and ranked using the obtained efficiency scores. In this article, a kind of modified idea based on interactive method is introduced for ranking of decision making units with fuzzy data by weighted distance.

Keywords : Fuzzy number; Efficiency; Data envelopment analysis; Weighted distance.

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1 Introduction

More and more, modeling techniques, control problems and operation research algorithms have been designed to fuzzy data since the concept of fuzzy number and arithmetic operations with these numbers was introduced and investigated first by Zadeh. Data envelopment analysis was suggested by Charnes, Cooper and Rhodes [2], and built on the idea of Farrell [3] which is concerned with the estimation of technical efficiency and efficient frontiers. In some cases, we have to use imprecise input and output. To deal quantitatively with imprecision in decision progress, Bellman Zadeh [1] introduce the notion of fuzziness. Some researchers have proposed several fuzzy models to evaluate decision making units with fuzzy data and introduce a ranking approach with efficiency measure of the model [9]. In this paper, the researcher first introduces an approach with weighted distance for ranking of decision making units with crisp data. Second, this model is used for ranking of decision making units with fuzzy data.
This paper is organized as follows. In Section 2, the researcher recalls some fundamental

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results on fuzzy numbers. The proposed model is introduced in Section 3. An approach for ranking by using weighted distance is introduced in section 4. Interactive method is introduced in Section 5.

2 Preliminaries

The basic definition of a fuzzy number given in [5, 11, 12, 13, 14, 15] is as follows:

Definition 2.1. *A fuzzy number is a mapping* $\mu : \Re \rightarrow [0,1]$ *with the following properties:*

1. μ *is an upper semi-continuous function on* $\Re$,
2. $\mu(x) = 0$ *outside of some interval* $[a_1, b_2] \subset \Re$,
3. *There are real numbers* a_2, b_1 *such as* $a_1 \leq a_2 \leq b_1 \leq b_2$ *and*

   3.1 $\mu(x)$ *is a monotonic increasing function on* $[a_1,\ a_2]$,

   3.2 $\mu(x)$ *is a monotonic decreasing function on* $[b_1,\ b_2]$,

   3.3 $\mu(x) = 1$ *for all* x *in* $[a_2, b_1]$.

Definition 2.2. *A fuzzy number is a fuzzy set* A *on the real line* $\Re$ *such that*

$$\mu(x) = \begin{cases} f_A(x) & if \quad x \in [a_1, a_2], \\ 1 & if \quad x \in [a_2, a_3], \\ g_A(x) & if \quad x \in [a_3, a_4], \\ 0 & otherwise\ . \end{cases} \tag{2.1}$$

Such that $f_A(.)$ *is increasing function on* $[a_1, a_2]$ *and* $g_A(.)$ *is decreasing function on* $[a_3, a_4]$.

The α-cut of a fuzzy number A is defined as $[A]^\alpha = \{x \mid u(x) \geq \alpha\}$. Since $\mu(.)$ is upper semi-continuous then α-cuts are closed and bounded intervals and we represent by $[A]^\alpha = [f_A^{-1}(\alpha), g_A^{-1}(\alpha)]$.

Definition 2.3. *A fuzzy number* A *in parametric form is a pair* $(\underline{A}, \overline{A})$ *of functions* $\underline{A}(\alpha)$ *and* $\overline{A}(\alpha)$ *that* $0 \leq \alpha \leq 1$, *which satisfies the following requirements:*

1. $\underline{A}(\alpha)$ *is a bounded monotonic increasing left continuous function,*
2. $\overline{A}(\alpha)$ *is a bounded monotonic decreasing left continuous function,*
3. $\underline{A}\alpha) \leq \overline{A}(\alpha), 0 \leq \alpha \leq 1$.

Definition 2.4. *The symmetric triangular fuzzy number* $A = (x_0, \sigma)$, *with defuzzifier* x_0 *and fuzziness* σ *is a fuzzy set where the membership function is as*

$$\mu(x) = \begin{cases} \frac{1}{\sigma}(x - x_0 + \sigma) & x_0 - \sigma \leq x \leq x_0, \\ \\ \frac{1}{\sigma}(x_0 - x + \beta) & x_0 \leq x \leq x_0 + \sigma, \\ \\ 0 & otherwise\ . \end{cases}$$

The parametric form of symmetric triangular fuzzy number is

$$\underline{A}(\alpha) = x_0 - \sigma + \sigma\alpha \quad , \quad \overline{A}(\alpha) = x_0 + \sigma - \sigma\alpha.$$

Definition 2.5. *For fuzzy set A Support function is defined as follows:*

$$supp(A) = \overline{\{x|\mu(x) > 0\}},$$

where $\overline{\{x|\mu(x) > 0\}}$ *is closure of set* $\{x|\mu(x) > 0\}$.

Definition 2.6. *[16], A function* $f : [0,1] \to [0,1]$ *symmetric around* $\frac{1}{2}$, *i.e.* $f(\frac{1}{2} - \alpha) = f(\frac{1}{2} + \alpha)$ *for all* $\alpha \in [0, \frac{1}{2}]$, *which reaches its minimum in* $\frac{1}{2}$, *is called the bi-symmetrical weighted function. Moreover, the bi-symmetrical weighted function is called regular if*

(1) $f(\frac{1}{2}) = 0$,

(2) $f(0) = f(1) = 1$,

(3) $\int_0^1 f(\alpha)d\alpha = \frac{1}{2}$.

Definition 2.7. *[14], For two arbitrary fuzzy numbers A and B with* α*-cuts* $[f_A^{-1}(\alpha), g_A^{-1}(\alpha)]$ *and* $[f_B^{-1}(\alpha), g_B^{-1}(\alpha)]$ *respectively, the quantity*

$$d(A,B) = [\int_0^1 f(\alpha)(f_A^{-1}(\alpha) - f_B^{-1}(\alpha))^2 d\alpha + \int_0^1 f(\alpha)(g_A^{-1}(\alpha) - g_B^{-1}(\alpha))^2 d\alpha]^{\frac{1}{2}} \tag{2.2}$$

is the weighted distance between A and B.

Definition 2.8. *[4, 16], Let A is an arbitrary fuzzy number, the expected interval and expected value of a fuzzy number A are noted by* $EI(A)$ *and* $EV(A)$ *respectively, and considered as follows,(with* $f(\alpha) = \alpha$*),*

$$\begin{aligned} EI(A) &= [E_1^A, E_2^A] = [2\int_0^1 \alpha f_A^{-1}(\alpha)d\alpha, 2\int_0^1 \alpha g_A^{-1}(\alpha)d\alpha], \\ EV(A) &= \frac{E_1^A + E_2^A}{2} = \int_0^1 \alpha f_A^{-1}(\alpha)d\alpha + \int_0^1 \alpha g_A^{-1}(\alpha)d\alpha \end{aligned} \tag{2.3}$$

If $A = (a_1, a_2, a_3, a_4)$ *is a trapezoidal fuzzy number then:*

$$EI(A) = [\frac{a_1 + 2a_2}{3}, \frac{2a_3 + a_4}{3}], \tag{2.4}$$

$$EV(A) = \frac{1}{3}(a_1 + 2a_2 + 2a_3 + a_4). \tag{2.5}$$

Proposition 2.1. *If A and B are two fuzzy numbers and* $\lambda,\ \nu \in \mathbb{R}$ *then:*

$$\begin{aligned} EI(\lambda A + \nu B) &= \lambda EI(A) + \nu EI(B), \\ EV(\lambda A + \nu B) &= \lambda EV(A) + \nu EV(B), \end{aligned} \tag{2.6}$$

The addition and scalar multiplication of fuzzy numbers are defined by the extension principle and can be equivalently represented as follows. For arbitrary fuzzy numbers $A = (\underline{A}, \overline{A})$ and $B = (\underline{B}, \overline{B})$, this article defines addition $(A + B)$ and multiplication by scalar $k > 0$ as

$$(\underline{A+B})(\alpha) = \underline{A}(\alpha) + \underline{B}(\alpha) \quad , \quad (\overline{A+B})(\alpha) = \overline{A}(\alpha) + \overline{B}(\alpha), \tag{2.7}$$

$$(\underline{kA})(\alpha) = k\underline{A}(\alpha) \quad , \quad (\overline{kA})(\alpha) = k\overline{A}(\alpha). \tag{2.8}$$

To emphasis the collection of all fuzzy numbers with addition and multiplication as defined by (2.2) and (2.3) denoted by F, which is a convex cone.

3 The proposed model

In this section, the researcher introduce ranking model based on weighted distance in data envelopment analysis. This article assumes that the DMU_p is extreme efficient [12, 6]. By omitting (X_p, Y_p) from T_c (PPS of CCR model), the researcher defines the production possibility set T'_c as follows, [7]:

$$T'_c = \{(X, Y) \mid \ X \geq \sum_{j=1, j\neq p}^{n} w_j X_j \ , \ Y \leq \sum_{j=1, j\neq p}^{n} w_j Y_j, \ w_j \geq 0, j = 1, \cdots, n, j \neq p\}. \tag{3.9}$$

Where

$$T_c = \{(X, Y) \mid \ X \geq \sum_{j=1}^{n} w_j X_j \ , \ Y \leq \sum_{j=1, j\neq p}^{n} w_j Y_j, \ w_j \geq 0, j = 1, \cdots, n\}. \tag{3.10}$$

To obtain the ranking score of DMU_p, this article considers the following model:

$$\begin{aligned}
min \quad & \Gamma^p_c(X, Y) = \textstyle\sum_{i=1}^{m} f(\alpha)(x_i - x_{ip})^2 + \sum_{r=1}^{s} f(\alpha)(y_r - y_{rp})^2 \\
s.t \quad & \textstyle\sum_{j=1, j\neq p}^{n} w_j x_{ij} \leq x_i, \qquad i = 1 \cdots, m, \\
& \textstyle\sum_{j=1, j\neq p}^{n} w_j y_{rj} \geq y_r, \qquad i = 1 \cdots, s, \\
& x_i \geq 0, \qquad i = 1 \cdots, m, \\
& y_r \geq 0, \qquad r = 1 \cdots, s, \\
& w_j \geq 0, \qquad j = 1 \cdots, n, j \neq p.
\end{aligned} \tag{3.11}$$

Where $X = (x_1, \ldots, x_n)$, $Y = (y_1, \ldots, y_n)$ and $\lambda = (\lambda_1, \ldots, \lambda_n)$ are the variables of the model (3.11) and $\Gamma^p_c(X, Y)$ is the weighted distance (X_p, Y_p) from (X, Y) by weighted distance, also $f(\alpha)$ is regular weighted function. Quadratic programming represents a special class of nonlinear programming in which the objective function is quadratic and the constraints are linear. The KKT conditions of a quadratic programming problem reduce to a linear complementary problem. Thus the complementary pivoting algorithm can be used for solving a quadratic programming problem.

4 Comparison In Fuzzy DEA

In this section, the researcher supposes that inputs and outputs of DMUs are fuzzy numbers. Therefore,

$$\tilde{T}'_c = \{(X,Y) \mid X \geq \sum_{j=1, j\neq p}^{n} w_j \tilde{X}_j \ , \ Y \leq \sum_{j=1, j\neq p}^{n} w_j \tilde{Y}_j, \ w_j \geq 0, j = 1, \cdots, n, j \neq p\} \quad (4.12)$$

Weighted distance model with Eqs. (2.2) can be extended to the following model:

$$\begin{aligned} min \quad \Gamma_c^p(X,Y) = & \textstyle\sum_{i=1}^m \left(\int_0^1 f(\alpha)(f_{x_i}^{-1}(\alpha) - f_{x_{ip}}^{-1}(\alpha))^2 d\alpha + \int_0^1 f(\alpha)(g_{x_i}^{-1}(\alpha) - g_{x_{ip}}^{-1}(\alpha))^2 d\alpha \right) \\ & + \textstyle\sum_{r=1}^s \left(\int_0^1 f(\alpha)(f_{y_r}^{-1}(\alpha) - f_{y_{rp}}^{-1}(\alpha))^2 d\alpha + \int_0^1 f(\alpha)(g_{y_r}^{-1}(\alpha) - g_{y_{rp}}^{-1}(\alpha))^2 d\alpha \right) \\ s.t \quad & (X,Y) \in \tilde{T}'_c. \end{aligned} \quad (4.13)$$

Where $X = (x_1, \ldots, x_n)$, $Y = (y_1, \ldots, y_n)$ and $\lambda = (\lambda_1, \ldots, \lambda_n)$ are the variables of the model (4.13), that all components of vectors X and Y for all DMUs are non-negative and each DMU has at least one strictly positive input and output.

For solving the model (4.13), we have some following definitions:

Definition 4.1. *[8], For any pair of fuzzy numbers A and B the degree in A is bigger than B has the following form:*

$$\mu_M(A,B) = \begin{cases} 0 & if \quad E_2^A - E_1^B < 0, \\ \frac{E_2^A - E_1^B}{E_2^A - E_1^A + E_2^B - E_1^B} & if \quad 0 \in [E_1^A - E_2^B, E_2^A - E_1^B], \\ 1 & if \quad E_1^A - E_2^B > 0. \end{cases} \quad (4.14)$$

Where $[E_1^A, E_2^A]$ and $[E_1^B, E_2^B]$ are the expected intervals of A and B. When $\mu_M(A,B) = \frac{1}{2}$, we will say that A and B are different. When $\mu_M(A,B) \geq \alpha$, we will say that A is bigger than, or equal to B at least in degree α and we will represent it by $A \geq_\alpha B$.

Definition 4.2. *Given a production possibility $(X,Y) \in \tilde{T}'_c$, we will say that it is product in degree α in $\tilde{T}'_c$ if:*

$$min \left\{ \begin{array}{ll} \mu_M(x_i, \sum_{j=1, j\neq p}^n w_j \tilde{x}_{ij}) \ , & \mu_M(\sum_{j=1, j\neq p}^n w_j \tilde{y}_{rj}, y_r) \\ \mu_M(x_i, \tilde{x}_{ip}) \ , & \mu_M(\tilde{y}_{rp}, y_r) \\ i = 1, \ldots, m & r = 1, \ldots, s \end{array} \right\} = \alpha. \quad (4.15)$$

That is to say

$$\begin{aligned} x_i \geq_\alpha \textstyle\sum_{j=1, j\neq p}^n w_j \tilde{x}_{ij} \quad , \quad i = 1, \ldots, m, \\ y_r \leq_\alpha \textstyle\sum_{j=1, j\neq p}^n w_j \tilde{y}_{rj} \quad , \quad r = 1, \ldots, s. \end{aligned} \quad (4.16)$$

There is:

$$x_i \geq \sum_{j=1,j\neq p}^{n} w_j(\alpha E_2^{x_{ij}} + (1-\alpha)E_1^{x_{ij}}) \quad , \quad i=1,\ldots,m,$$
$$y_r \leq \sum_{j=1,j\neq p}^{n} w_j(\alpha E_1^{y_{rj}} + (1-\alpha)E_2^{y_{rj}}) \quad , \quad r=1,\ldots,s. \tag{4.17}$$

(For more details see [12]).

Definition 4.3. *A production possibility* $(X_o, Y_o)^\alpha \in \tilde{T}'_c$ *is an* α*-acceptable optimal solution of model (4.13) if it is an optimal solution of the following model:*

$$\begin{aligned} min \quad & \Gamma_c^p(X,Y)_\alpha = \sum_{i=1}^{m}\left(\int_0^1 f(\alpha)(f_{x_i}^{-1}(\alpha) - f_{x_{ip}}^{-1}(\alpha))^2 d\alpha + \int_0^1 f(\alpha)(g_{x_i}^{-1}(\alpha) - g_{x_{ip}}^{-1}(\alpha))^2 d\alpha\right) \\ & + \sum_{r=1}^{s}\left(\int_0^1 f(\alpha)(f_{y_r}^{-1}(\alpha) - f_{y_{rp}}^{-1}(\alpha))^2 d\alpha + \int_0^1 f(\alpha)(g_{y_r}^{-1}(\alpha) - g_{y_{rp}}^{-1}(\alpha))^2 d\alpha\right) \\ s.t \quad & (X,Y) \in \tilde{T}_c'^{\alpha}. \end{aligned} \tag{4.18}$$

Where

$$\tilde{T}_c'^{\alpha} = \{(X,Y) \mid \ X \geq_\alpha \sum_{j=1,j\neq p}^{n} w_j \tilde{X}_j \ , \ Y \leq_\alpha \sum_{j=1,j\neq p}^{n} w_j \tilde{Y}_j, \ w_j \geq 0, j=1,\cdots,n\} \tag{4.19}$$

Proposition 4.1. *If* $\alpha_1 < \alpha_2$ *then* $\tilde{T}_c'^{\alpha_2} \subseteq \tilde{T}_c'^{\alpha_1}$.

We write model (4.18) as follows:

$$\begin{aligned} min \quad & \Gamma_c^p(X,Y)_\alpha = \sum_{i=1}^{m}\left(\int_0^1 f(\alpha)(f_{x_i}^{-1}(\alpha) - f_{x_{ip}}^{-1}(\alpha))^2 d\alpha + \int_0^1 f(\alpha)(g_{x_i}^{-1}(\alpha) - g_{x_{ip}}^{-1}(\alpha))^2 d\alpha\right) \\ & + \sum_{r=1}^{s}\left(\int_0^1 f(\alpha)(f_{y_r}^{-1}(\alpha) - f_{y_{rp}}^{-1}(\alpha))^2 d\alpha + \int_0^1 f(\alpha)(g_{y_r}^{-1}(\alpha) - g_{y_{rp}}^{-1}(\alpha))^2 d\alpha\right) \\ s.t \quad & x_i \geq \sum_{j=1,j\neq p}^{n} w_j(\alpha E_2^{x_{ij}} + (1-\alpha)E_1^{x_{ij}}) \quad i=1,\ldots,m, \\ & y_r \leq \sum_{j=1,j\neq p}^{n} w_j(\alpha E_1^{y_{rj}} + (1-\alpha)E_2^{y_{rj}}) \quad r=1,\ldots,s, \\ & y_r \geq 0 \qquad r=1,\ldots,s, \\ & w_j \geq 0 \qquad j=1,\ldots,n. \end{aligned} \tag{4.20}$$

Model (4.20) is a crisp α-parametric model. Therefore this article can solve it by the interactive method. Now this study is going to explain the interactive method.

Table 1
Fuzzy data of DMUs in fuzzy data in Example 4.1.

DMUs	Input	α-cut	Output	α-cut
A	(11,12,14)	[11+α,14-α]	(10,10,10)	[10,10]
B	(30,30,30)	[30,30]	(12,13,14,16)	[12+α,16-2α]
C	(40,40,40)	[40,40]	(11,11,11)	[11,11]
D	(45,47,52,55)	[45+2α,55-3α]	(12,15,19,22)	[12+3α,22-3α]

Table 2
The optimal values of α-parametric model.

α	Γ_c^A	Γ_c^B	Γ_c^C	Γ_c^D	$(Ranking)_\alpha$
0.0	25.5	3.5	0	55.2	$(C \prec B \prec A \prec D)_{0.0}$
0.1	27.0	3.5	0	55.2	$(C \prec B \prec A \prec D)_{0.1}$
0.2	28.5	3.5	0	55.2	$(C \prec B \prec A \prec D)_{0.2}$
0.3	30.0	3.5	0	55.2	$(C \prec B \prec A \prec D)_{0.3}$
0.4	31.6	3.5	0	55.2	$(C \prec B \prec A \prec D)_{0.4}$
0.5	33.3	3.5	0	55.2	$(C \prec B \prec A \prec D)_{0.5}$
0.6	35.0	3.5	0	55.2	$(C \prec B \prec A \prec D)_{0.6}$
0.7	36.7	3.5	0	55.2	$(C \prec B \prec A \prec D)_{0.7}$
0.8	38.6	3.5	0	55.2	$(C \prec B \prec A \prec D)_{0.8}$
0.9	40.4	3.5	0	55.2	$(C \prec B \prec A \prec D)_{0.9}$
1.0	42.3	3.5	0	55.2	$(C \prec B \prec A \prec D)_{1.0}$

4.1 Interactive Method

Regarding to proposition (4.1), to obtain the nearest (X,Y) of $\tilde{T}_c'$ implies a lesser degree of production possibility. Then the decision-maker runs in to two conflicting objectives: to find the nearest (X,Y) and to improve the degree of production possibility. Following Kaufman [11] the researcher considers 11 scales, which allow for different choice of decision-maker idea in (4.20) model.

1: $\alpha = 0.0$ unacceptable solution
2: $\alpha = 0.1$ Practically unacceptable solution
3: $\alpha = 0.2$ Almost unacceptable solution
4: $\alpha = 0.3$ Very unacceptable solution
5: $\alpha = 0.4$ Quite unacceptable solution
6: $\alpha = 0.5$ Neither acceptable nor unacceptable solution
7: $\alpha = 0.6$ Quite acceptable solution
8: $\alpha = 0.7$ Very acceptable solution
9: $\alpha = 0.8$ Almost acceptable solution
10: $\alpha = 0.9$ Practically acceptable solution
11: $\alpha = 1.0$ Completely acceptable solution

We choice the α_0 is the minimum acceptable degree with decision-maker idea. Then, this article solving the (4.20) α-parametric model for each α_k that $k = 1, \ldots, (10 - 10\alpha_0)$. We obtain the α_k-acceptable optimal fuzzy value of objective function of original model (4.13) with α_k-acceptable solution of model (4.20) in model (4.13).

4.2 Example

Example 4.1. *We will consider a simple example was introduced in [10] with its data listed in Table 1. These DMUs are evaluated by proposed model in 4.13 with different α_k. The α-parametric model is as follows:*

$$\begin{array}{lll} min & \int_0^1 \alpha(x - 11 - \alpha)d\alpha + 2\int_0^1 \alpha(y - 10)d\alpha + \int_0^1 \alpha(x - 14 + 2\alpha)d\alpha & \\ s.t & x \geq 30w_B + 40w_C + w_D(53.5 - 7.5\alpha) & \\ & y \leq w_B(15 - 2.5\alpha) + 11w_C + w_D(20.5 - 7\alpha) & \\ & x \geq 0, & \\ & y \geq 0, & \\ & w_B, w_C, w_D \geq 0. & \end{array} \tag{4.21}$$

The α-parametric model for B, C and D can be showed similarly. The results is shown in Table 2.

5 Conclusion

In the present article a modified approach based on weighted distance is introduced for ranking of decision making units with fuzzy data. The method is based on the interactive method. α-acceptable optimal solution of proposed model for $\alpha \geq \frac{1}{2}$ is an acceptable solution. For any decision making unit, the score of ranking is obtained by solving α-parametric model (4.20).

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