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Research Article

Piecewise Constant Controlled Linear Fuzzy Systems

Andrej V. Plotnikov [a,b]*, Anastasiya V. Arsiry [b], Tatyana A. Komleva [c]
(a) *Department of Applied Mathematics, Odessa State Academy Civil Engineering and Architecture, Odessa, Ukraine*
(b) *Department of Optimal Control, Odessa National University, Odessa, Ukraine*
(c) *Department of Mathematics, Odessa State Academy Civil Engineering and Architecture, Odessa, Ukraine*

Abstract
In this work we prove that for any measurable admissible control $w(\cdot)$ and for any $\varepsilon > 0$ there exists piecewise constant admissible control $\bar{w}(\cdot)$ such that for fuzzy solutions of control fuzzy linear system are ε-closed.
Keywords : fuzzy system; control; linear; piecewise constant

1 Introduction

In recent years, the fuzzy set theory introduced by Zadeh [48] has emerged as an interesting and fascinating branch of pure and applied sciences. The applications of fuzzy set theory can be found in many branches of science as physical, mathematical, differential equations and engineering sciences. Recently there have been new advances in the theory of fuzzy differential equations [8, 9, 10, 11, 12, 15, 16, 20, 21, 22, 23, 28, 37, 44, 46], fuzzy integrodifferential equations [2, 5, 6, 7, 18, 42], differential inclusions with fuzzy right-hand side [1, 3, 4, 14, 20, 21, 34, 35] and fuzzy differential inclusions [36, 45, 47] as well as in the theory of control fuzzy differential equations [17, 26, 27, 29, 30], control fuzzy integrodifferential equations [19, 24, 25], control fuzzy differential inclusions [31, 32, 33], and control fuzzy integrodifferential inclusions [43].

In many engineering control systems piecewise constant controls, instead of measurable controls are applied. In this article we prove that for any measurable admissible control $w(\cdot)$ and for any $\varepsilon > 0$ there exists piecewise constant admissible control $\bar{w}(\cdot)$ such that for fuzzy solutions of control fuzzy linear system are ε-closed.

*Corresponding author. Email address: a-plotnikov@ukr.net

2 Preliminaries

Let $\mathcal{CC}(\mathbb{R}^n)$ be the family of all nonempty compact convex subsets of $\mathbb{R}^n$ with the Hausdorff metric $h(A,B) = \max\{\max_{a\in A}\min_{b\in B}\|a-b\|,\ \max_{b\in B}\min_{a\in A}\|a-b\|\}$, where $\|\cdot\|$ denotes the usual Euclidean norm in $\mathbb{R}^n$.

Let $\mathbb{E}^n$ be the family of mappings $x : \mathbb{R}^n \to [0,1]$ satisfying the following conditions:

(i) x is normal, i.e. there exists an $\xi_0 \in \mathbb{R}^n$ such that $x(\xi_0) = 1$;

(ii) x is fuzzy convex, i.e. $x(\lambda\xi + (1-\lambda)\zeta) \geq \min\{x(\xi), x(\zeta)\}$ whenever $\xi, \zeta \in \mathbb{R}^n$ and $\lambda \in [0,1]$;

(iii) x is upper semicontinuous, i.e. for any $\xi_0 \in \mathbb{R}^n$ and $\varepsilon > 0$ exists $\delta(\xi_0, \varepsilon) > 0$ such that $x(\xi) < x(\xi_0) + \varepsilon$ whenever $||\xi - \xi_0|| < \delta,\ \xi \in \mathbb{R}^n$;

(iv) the closure of the set $cl\{\xi \in \mathbb{R}^n : x(\xi) > 0\}$ is compact.

Let $\widehat{0}$ be the fuzzy mapping defined by $\widehat{0}(\xi) = 0$ if $\xi \neq 0$ and $\widehat{0}(0) = 1$.

Definition 2.1. *The set $\{y \in \mathbb{R}^n : x(y) \geq \alpha\}$ is called the α - level $[x]^\alpha$ of a mapping $x \in \mathbb{E}^n$ for $0 < \alpha \leq 1$. The closure of the set $\{y \in \mathbb{R}^n : x(y) > 0\}$ is called the 0 - level $[x]^0$ of a mapping $x \in \mathbb{E}^n$.*

Define the metric $D : \mathbb{E}^n \times \mathbb{E}^n \to \mathbb{R}_+$ by the equation $D(x,y) = \sup_{\alpha\in[0,1]} h([x]^\alpha, [y]^\alpha)$.

Using the results of [40], we know that

(i) $(\mathbb{E}^n, D)$ is a complete metric space,

(ii) $D(x+z, y+z) = D(x,y)$ for all $x, y, z \in \mathbb{E}^n$,

(iii) $D(kx, ky) = |k|D(x,y)$ for all $x, y \in \mathbb{E}^n$, $k \in \mathbb{R}$.

Let A, B, C be in $\mathcal{CC}(\mathbb{R}^n)$. The set C is the Hukuhara difference of A and B, if $B + C = A$, i.e. $C = A \overset{H}{-} B$. From Rådström's Cancellation Lemma [41], it follows that if this difference exists, then it is unique.

Definition 2.2. *[13] A mapping $F : [0,T] \to \mathcal{CC}(\mathbb{R}^n)$ is differentiable in the sense of Hukuhara at $t \in [0,T]$ if for some $h > 0$ the Hukuhara differences*

$$F(t+\Delta t) \overset{H}{-} F(t),\ \ F(t) \overset{H}{-} F(t-\Delta t)$$

exists in $\mathcal{CC}(\mathbb{R}^n)$ for all $0 < \Delta t < h$ and there exists an $D_H F(t) \in \mathcal{CC}(\mathbb{R}^n)$ such that

$$\lim_{\Delta t \to 0+} h(\Delta t^{-1}(F(t+\Delta t) \overset{H}{-} F(t)), D_H F(t)) = 0$$

and

$$\lim_{\Delta t \to 0+} h(\Delta t^{-1}(F(t) \overset{H}{-} F(t-\Delta t)), D_H F(t)) = 0.$$

Here $D_H F(t)$ is called the Hukuhara derivative of $F(t)$ at t.

Definition 2.3. *[15] A mapping $x : [0,T] \to \mathbb{E}^n$ is called differentiable at $t \in [0,T]$ if, for any $\alpha \in [0,1]$, the set-valued mapping $x_\alpha(t) = [x(t)]^\alpha$ is is differentiable in the sense of Hukuhara at point t with $D_H x_\alpha(t)$ and the family $\{D_H x_\alpha(t) : \alpha \in [0,1]\}$ define a fuzzy number $\dot{x}(t) \in E^n$.*

If $x : [0,T] \to \mathbb{E}^n$ is differentiable at $t \in [0,T]$, then we say that $\dot{x}(t)$ is the fuzzy derivative of $x(\cdot)$ at the point $t \in [0,T]$.

Consider the fuzzy Cauchy problem

$$\dot{x} = A(t)x + g(t), \; x(0) = x_0, \tag{2.1}$$

where $A(t)$ is $n \times n$-dimensional matrix-valued function; $g(t)$ is the fuzzy map, $x_0 \in \mathbb{E}^n$.

Definition 2.4. *A fuzzy mapping $x : [0,T] \to \mathbb{E}^n$ is a solution to the problem (2.1) if and only if it is continuous and satisfies the integral equation $x(t) = x_0 + \int_0^t [A(s)x(s) + g(s)]ds$ for all $t \in [0,T]$.*

Theorem 2.1. *[37, 46] Let the following conditions are true:*

1) *$A(t)$ is measurable on $[0,T]$;*

2) *There exists $a > 0$ such that $\|A(t)\| \le a$ for almost every $t \in [0,T]$;*

3) *The fuzzy map $g(s)$ is measurable on $[0,T]$;*

4) *There exists $\bar{g}(t) \in L_2[0,T]$ such that $D\left(g(t), \widehat{0}\right) \le \bar{g}(t)$ almost everywhere on $t \in [0,T]$.*

Then problem (2.1) has on exactly one solution.

3 The control fuzzy differential equation

Now we consider following control fuzzy differential equation

$$\dot{x} = A(t)x + B(t)w + f(t), \; x(0) = x_0 \tag{3.2}$$

where $w \in \mathbb{R}^m$ is the control, $B(t)$ is $n \times m$-dimensional matrix-valued function; $f : \mathbb{R}_+ \to \mathbb{E}^n$ is the fuzzy map.

Let $W : \mathbb{R}_+ \to \mathbb{R}^m$ be the measurable set-valued map.

Definition 3.1. *The set LW of all measurable single-valued branches of the set-valued map $W(t)$ is the set of the admissible controls.*

Obviously, the control fuzzy differential equation (3.2) turns into the ordinary fuzzy differential equation (2.1) if the control $\tilde{w}(\cdot) \in LW$ is fixed and $g(t) \equiv B(t)\tilde{w}(t) + f(t)$.

Let $x(t)$ denotes the fuzzy solution of the differential equation (2.1), then $x(t,w)$ denotes the fuzzy solution of the control differential equation (3.2) for the fixed $w(\cdot) \in LW$.

Definition 3.2. *The set $Y(T) = \{x(T,w) : w(\cdot) \in LW\}$ be called the attainable set of the system (3.2).*

Theorem 3.1. *[30] Let the following conditions are true:*

1) *$A(t)$ is measurable on $[0,T]$;*

2) *There exists $a>0$ such that $\|A(t)\| \le a$ for almost every $t \in [0,T]$;*

3) *$B(t)$ is measurable on $[0,T]$;*

4) *There exists $b>0$ such that $\|B(t)\| \le a$ for almost every $t \in [0,T]$;*

5) *The set-valued map $W : [0,T] \to \mathcal{CC}(\mathbb{R}^m)$ is measurable on $[0,T]$;*

6) *The fuzzy map $f : [0,T] \to \mathbb{E}^n$ is measurable on $[0,T]$;*

7) *There exist $v(\cdot) \in L_2[0,T]$ and $\bar{f}(\cdot) \in L_2[0,T]$ such that $h(W(t),\{0\}) \le v(t)$, $D(f(t),\widehat{0}) \le \bar{f}(t)$ almost everywhere on $[0,T]$.*

Then for every $w(\cdot) \in LW$ there exists the fuzzy solution $x(\cdot,w)$ on $[0,T]$ and the attainable set $Y(T)$ is compact and convex.

Let $U = \prod_{i=1}^{m} [u^i_{min}, u^i_{max}]$ and $W(t) \equiv U$ on $[0,T]$.

Now, we need to establish that for any measurable admissible control $w(\cdot)$ and for any $\varepsilon > 0$ there exists piecewise constant admissible control $\bar{w}(\cdot)$ such that for fuzzy solutions of system (3.2) holds $D(x(t,w), x(t,\bar{w})) < \varepsilon$ for all $t \in [0,T]$.

Theorem 3.2. *Let the conditions of the theorem 3.1 are true.*

Then for every $w(\cdot) \in LW$ there exists $\bar{w}(\cdot) \in LW$ such that

1) *$\bar{w}(t)$ is constant on every $\left[(i-1)\frac{T}{k}, i\frac{T}{k}\right)$, $i=\overline{1,k}$;*

2) *$\bar{w}_i(t) = \{(\bar{w}^1_i(t), ..., \bar{w}^m_i(t))^T \mid \bar{w}^j_i(t) \in \{u^j_{min}, u^j_{max}\}, i = \overline{1,k}, j = \overline{1,m}\}$ for every $t \in [0,T]$;*

3) *$D(x(t,w), x(t,\bar{w})) \le be^{aT}\frac{T}{2k}\|u_{max} - u_{min}\|$ for all $t \in [0,T]$,*

where $u_{min} = \left(u^1_{min}, ..., u^m_{min}\right)^T$, $u_{max} = \left(u^1_{max}, ..., u^m_{max}\right)^T$.

Proof. We have any $w(\cdot) \in LW$ and any $k \in N$. Let $W_i = (W^1_i, ..., W^m_i)^T$, where $W^j_i = \int_0^{i\frac{T}{k}} w^j(s)ds$, $i = \overline{1,k}$, $j = \overline{1,m}$.

Obviously, $W^j_{i+1} - W^j_i = \int_{\frac{iT}{k}}^{\frac{(i+1)T}{k}} w^j(s)ds$, $u^j_{min}\frac{T}{k} \le W^j_{i+1} - W^j_i \le u^j_{max}\frac{T}{k}$, $j = \overline{1,m}$, and

$$\|W_{i+1} - W_i\| \le \|u_{max} - u_{min}\|\frac{T}{k}.$$

Now we take

$$\bar{w}(t) = \begin{cases} \bar{w}_1, & t \in [0, \frac{T}{k}), \\ \vdots & \vdots \\ \bar{w}_{k-1}, & t \in [\frac{(k-2)T}{k}, \frac{(k-1)T}{k}), \\ \bar{w}_k, & t \in [\frac{(k-1)T}{k}, T], \end{cases}$$

such that

1) $\bar{w}_1 = (\bar{w}_1^1, ..., \bar{w}_1^m)^T$, where $\bar{w}_1^j = \begin{cases} u_{max}^j, & \text{if } W_1^j \geq \frac{T}{2k}(u_{max}^j + u_{min}^j), \\ u_{min}^j, & \text{if } W_1^j < \frac{T}{2k}(u_{max}^j + u_{min}^j), \end{cases}$ $j = \overline{1,m}$;

2) $\bar{w}_i = (\bar{w}_i^1, ..., \bar{w}_i^m)^T$, $i = \overline{2,k}$,

where $\bar{w}_i^j = \begin{cases} u_{max}^j, & \text{if } W_i^j - \sum\limits_{l=1}^{i-1} \bar{w}_j^l \frac{T}{k} \geq \frac{T}{2k}(u_{max}^j + u_{min}^j), \\ u_{min}^j, & \text{if } W_i^j - \sum\limits_{l=1}^{i-1} \bar{w}_j^l \frac{T}{k} < \frac{T}{2k}(u_{max}^j + u_{min}^j), \end{cases}$ $j = \overline{1,m}$;

Obviously, for $i = 1$ and $j = \overline{1,m}$ we have

a) if $\bar{w}_1^j = u_{max}^j$, when $-\frac{T}{2k}(u_{max}^j - u_{min}^j) \leq W_1^j - \bar{w}_1^j \frac{T}{k} \leq 0$,

b) if $\bar{w}_1^j = u_{min}^j$, when $\frac{T}{2k}(u_{max}^j - u_{min}^j) > W_1^j - \bar{w}_1^j \frac{T}{k} \geq 0$.

Hence we obtain $|W_1^j - \bar{w}_1^j| \leq \frac{T}{2k}(u_{max}^j - u_{min}^j)$, $j = \overline{1,m}$, and $\|W_1 - \bar{w}_1\| \leq \frac{T}{2k}\|u_{max} - u_{min}\|$.

Thus, by induction, we obtain that, for $i = \overline{2,k}$

$$|W_i^j - \sum_{l=1}^{i} \bar{w}_l^j \frac{T}{k}| \leq \frac{T}{2k}(u_{max}^j - u_{min}^j), \ j = \overline{1,m},$$
$$\text{and} \tag{3.3}$$
$$\|W_i - \sum_{l=1}^{i} \bar{w}_l \frac{T}{k}\| \leq \frac{T}{2k}\|u_{max} - u_{min}\|.$$

Therefore, if $t_i = \frac{iT}{k}$, $i = \overline{1,k}$; then $\left\| \int_0^{t_i} w(s)ds - \int_0^{t_i} \bar{w}(s)ds \right\| \leq \frac{T}{2k}\|u_{max} - u_{min}\|$.

Now, we take $t \in \left(\frac{(i-1)T}{k}, \frac{iT}{k}\right)$. Then

$$\left\| \int_0^t w(s)ds - \int_0^t \bar{w}(s)ds \right\| \leq \left\| W_{i-1} - \sum_{l=1}^{i-1} \bar{w}_l \frac{T}{k} + \int_{\frac{(i-1)T}{k}}^{t} (w(s) - \bar{w}_i)ds \right\|.$$

As for all $j = \overline{1,m}$

$$W_i^j - \sum_{l=1}^{i} \bar{w}_l^j \frac{T}{k} \geq W_{i-1}^j - \sum_{l=1}^{i-1} \bar{w}_l^j \frac{T}{k} + \int_{\frac{(i-1)T}{k}}^{t} (w^j(s) - \bar{w}_i^j)ds \geq W_{i-1}^j - \sum_{l=1}^{i-1} \bar{w}_l^j \frac{T}{k};$$

then

$$\left\| \int_0^t w(s)ds - \int_0^t \bar{w}(s)ds \right\| \leq \max\{\|W_i - \sum_{l=1}^{i} \bar{w}_l \frac{T}{k}\|, \|W_{i-1}^j - \sum_{l=1}^{i-1} \bar{w}_l^j \frac{T}{k}\|\}.$$

By (3.3), we get for all $t \in [0, T]$

$$\left\| \int_0^t w(s)ds - \int_0^t \bar{w}(s)ds \right\| \leq \frac{T}{2k}\|u_{max} - u_{min}\|. \tag{3.4}$$

Now, applying definition 2.4 and conditions of the theorem, we obtain

$$D(x(t,w), x(t,\bar{w})) =$$

$$= D(\int_0^t [A(s)x(s,w) + B(s)w(s)]ds, \int_0^t [A(s)x(s,\bar{w}) + B(s)\bar{w}(s)]ds) \leq$$

$$\leq \int_0^t D(A(s)x(s,w), A(s)x(s,\bar{w}))ds + \| \int_0^t B(s)w(s)ds - \int_0^t B(s)\bar{w}(s)ds\| \leq$$

$$\leq a \int_0^t D(x(s,w), x(s,\bar{w}))ds + b \left\| \int_0^t w(s)ds - \int_0^t \bar{w}(s)ds \right\|.$$

Using Gronwall-Bellman's inequality, we obtain

$$D(x(t,w), x(t,\bar{w})) \leq be^{aT} \left\| \int_0^t w(s)ds - \int_0^t \bar{w}(s)ds \right\|.$$

By (3.4), we have $D(x(t,w), x(t,\bar{w})) \leq be^{aT} \frac{T}{2k} \|u_{max} - u_{min}\|$. Theorem is proved. □

Remark 3.1. *Obviously, if we take* $k > be^{aT} \frac{T}{2\varepsilon} \|u_{max} - u_{min}\|$; *then* $D(x(t,w), x(t,\bar{w})) < \varepsilon$ *for all* $t \in [0,T]$.

4 Conclusion

We remark that this result helps to build ε-optimal piecewise constant controls for optimal control fuzzy system (fuzzy Mayer problem [26], fuzzy time-optimal problem [30, 31, 32, 33] and other).

We can as will receive that for any measurable admissible control $w(\cdot)$ and for any $\varepsilon > 0$ there exists piecewise constant admissible control $\bar{w}(\cdot)$ such that for fuzzy R-solutions of control linear differential inclusion with fuzzy right-hand side

$$\dot{x} \in A(t)x + B(t)w + f(t), \ x(0) = x_0 \tag{4.5}$$

holds $D(X(t,w), X(t,\bar{w})) < \varepsilon$ for all $t \in [0,T]$, where $x \in R^n$, $x_0 \in R^n$, $\dot{x} = \frac{dx}{dt}$, $X(\cdot,w)$ is fuzzy R-solution of system (4.5).

Also, the given result as can be received if to take the generalized derivative [9, 38, 39].

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