Available online at http://ijim.srbiau.ac.ir
Int. J. Industrial Mathematics (ISSN 2008-5621)
Vol. 4, No. 4, Year 2012 Article ID IJIM-00261, 15 pages
Research Article

Effects of Chemical Reaction and Pressure Work on Free Convection over a Stretching Cone Embedded in a Porous Medium

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Abstract
Laminar free convection from a stretching cone embedded in the porous media with effects of pressure work, heat generation, thermal stratification and chemical reaction are considered. The governing partial differential equations have been transformed by a similarity transformation into a system of ordinary differential equations, which are solved numerically using a fourth order Runge-Kutta scheme with the shooting method. Solutions obtained in terms of local heat and mass transfer, velocity, temperature and concentration profiles for the values of physical parameters are displayed in both graphical and tabular forms.
Keywords : Chemical reaction; Heat generation or absorption; MHD; Pressure work; Thermal stratification

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1 Introduction

A great deal of interest has been generated in the area of two-dimensional boundary layer flow over a cone embedded in a porous medium. Diffusion of a chemically-reactive species from a stretching sheet is studied by Andersson et al. [1]. Anjali Devi and Kandasamy [2, 3] have analyzed the effects of chemical reaction, heat and mass transfer on laminar flow with or without MHD effects along a semi infinite horizontal plate. Muthucumaraswamy and Ganeshan [4, 5, 6] have studied the impulsive motion of a vertical plate with heat flux, mass flux, suction and diffusion of chemically reactive species. The

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study of heat generation in moving fluids is important in view of several physical problems such as those dealing with chemical reactions and those concerned with dissociating fluids. Also, there has been a considerable interest in studying the effect of a magnetic field on natural convection heat and mass transfer in porous media. Hering and Grosh [7] have studied the laminar natural convection from a non-isothermal cone and showed that similarity solutions exist when the cone wall temperature varies as a power function of distance along a cone ray. Later, Hering [8] has extended the analysis to investigate cases for low Prandtl numbers. Roy [9] has extended the study of Hering and Grosh [7] to treat the case of high Prandtl numbers. Alamgir [10] has used an integral method to study the overall heat transfer from vertical cones in laminar natural convection. Yih [11] has reported the effect of uniform lateral mass flux on free convection about a vertical cone embedded in a fluid-saturated porous medium. Hossain et al. [12] have studied non-Darcy natural convection heat and mass transfer along a vertical permeable cylinder embedded in a porous medium. Chamkha et al. [13] have studied simultaneous heat and mass transfer by natural convection about a vertical wedge and a cone embedded in a porous medium.

In many chemical engineering processes, chemical reactions take place between a foreign mass and the working fluid which moves due to the stretching of a surface. The order of the chemical reactions depends on several factors. One of the simplest chemical reactions is the first-order homogeneous reaction in which the rate of reaction is directly proportional to the species concentration. Muthucumaraswamy [14] has studied the effects of a chemical reaction on a moving isothermal vertical infinitely long surface with suction. Anjali Devi and Kandasamy [15] have studied the effects of chemical reaction, heat and mass transfer on non-linear MHD laminar boundary layer flow over a wedge with suction and injection. On the other hand, it should be noted here that the pressure work terms in the energy equation, except of the theoretical interest, has applications in glaciology, in granular material, in the infall of molten iron during gravitational differentiation of terrestrial planets, in the interaction between the crustand mantle during continental convergence and in the separation of oceanic crust from the descending oceanic lithosphere. Much work on these fields has been done by Yuen and his co-workers [16, 17, 18].
It is interest to study the effects of combined chemical reactions and pressure work in free convection flows. In the present work these effects are considered for the problem of free convection over a stretching cone embedded in a Darcian porous medium in the presence of heat generation or absorption and thermal stratification effects.

2 Mathematical Analysis

A steady, two-dimensional, boundary-layer convective flow of an incompressible, viscous and electrically-conducting fluid along a stretched cone embedded in porous media in the presence of a uniform magnetic field, heat and mass transfer, pressure work and chemical reaction is considered. The fluid properties are assumed to be constant except the density term in the buoyancy terms of the momentum equations and the chemical reaction is homogeneous and of first order taking place in the flow. Under the usual boundary layer and Buossinesq assumptions, the governing equations are given by:

$$\frac{\partial(ru)}{\partial x} + \frac{\partial(rv)}{\partial y} = 0, \tag{2.1}$$

$$\Big(1+\frac{k_1\sigma\mu_e^2H_0^2}{\mu}\Big)\frac{\partial u}{\partial y}=\frac{k_1 g\cos(\Omega)}{\nu}\Big(\beta_T\frac{\partial T}{\partial y}+\beta_C\frac{\partial C}{\partial y}\Big), \tag{2.2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho C_P}\frac{\partial^2 T}{\partial y^2}+\frac{\beta_T T_f}{\rho C_P}u\frac{dP}{dx}+\frac{Q_0}{\rho C_P}\Big(T-T_\infty\Big), \tag{2.3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2}-k_c\Big(C-C_\infty\Big), \tag{2.4}$$

where, u and v are the velocity components in the x and y directions, T is the temperature, C is the concentration. σ, μ_e and H_0 are electrical conductivity of the fluid, magnetic permeability and magnetic field intensity, respectively. r is the radius of the cone's surface ($r = x\sin(\Omega)$). ρ is the fluid density, k_1 is the permeability of porous medium, k and C_P are the thermal conductivity and specific heat of the fluid, respectively. μ and v are dynamic and kinematic viscosities. g is the acceleration due to gravity. D, k_c and P are the mass diffusivity, the rate of chemical reaction and pressure, respectively. β_T is the thermal expansion coefficient, β_C is the concentration expansion coefficient, Q_0 is heat generation or absorption constant, C_∞ and T_∞ are the free stream dimensional concentration and temperature, respectively and T_f is the film temperature.
The fluid pressure consists of the hydrostatic (P_h)and motion pressure (P_m):

$$P=P_h+P_m \tag{2.5}$$

The motion pressure is considered small compared to the hydrostatic pressure and is therefore ignored [19]. The hydrostatic pressure is given by:

$$\frac{dP_h}{dx}=-\rho g \tag{2.6}$$

Under these conditions the energy equation takes the form:

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho C_P}\frac{\partial^2 T}{\partial y^2}-\frac{\beta_T T_f}{C_P}ug+\frac{Q_0}{\rho C_P}\Big(T-T_\infty\Big), \tag{2.7}$$

The boundary conditions for this problem can be written as:

$$\begin{aligned} &v=0, T=T_W, C=C_W \quad at \quad y=0\\ &u=0, T=T_\infty, C=C_\infty \quad as \quad y\longrightarrow\infty \end{aligned} \tag{2.8}$$

The following transformation can be introduced:

$$\begin{aligned} \psi=r(x)\Big(vxU(x)\Big)^{\frac{1}{2}}f(\eta) &\quad \eta=\Big(\tfrac{U(x)}{vx}\Big)^{\frac{1}{2}}y,\\ \theta(\eta,x)=\tfrac{(T-T_\infty)}{(T_W-T_\infty)} &\quad (T_W-T_\infty)=sx^n,\\ \phi(\eta,x)=\tfrac{(C-C_\infty)}{(C_W-C_\infty)} &\quad (C_W-C_\infty)=s_1x^{n_1}, U(x)=ax, \end{aligned} \tag{2.9}$$

where n is a constant and is called the thermal stratification parameter such that $0\preceq n\prec 1$. Also, a is a dimensional constant. s, s_1 and n_1 are all constant. It should be noted that the parameter n is equal to $\frac{m_1}{1+m_1}$ of Nakayama and Koyama [20] where m_1 is a constant.

The modified stream function, which is related to the components of the velocity fields, is introduced by the equations

$$ru = \frac{\partial \psi}{\partial y},\ rv = -\frac{\partial \psi}{\partial x}, \tag{2.10}$$

It can be easily verified that the continuity Eq. 2.1 is identically satisfied. Introducing the relations 2.9 into the equations 2.2, 2.4 and 2.7, we obtain the following dimensionless ordinary differential equations:

$$(1+M)f'' - Ra\Big(\theta' + N\phi'\Big) = 0, \tag{2.11}$$

$$\theta'' + Pr\Big(2f\theta' - nf'\theta + Q\theta - \varepsilon f'\Big) = 0, \tag{2.12}$$

$$\phi'' + Sc\Big(2f\phi' - \gamma\phi - n_1 f'\phi\Big) = 0, \tag{2.13}$$

where a prime denotes a differentiation with respect to η and $Ra = \frac{k_1 g \beta_T (T_W - T\infty)\cos\Omega}{vU}$ is the Rayleigh number, $Pr = \frac{\mu C_P}{k}$ is the Prandtl number, $Sc = \frac{v}{D}$ is the Schmidt number, $M = \frac{k_1 \sigma \mu_e^2 H_0^2}{\mu}$ is the magnetic field parameter, $\gamma = \frac{k_c}{a}$ is the chemical reaction parameter, $\varepsilon = \frac{\beta_T T_f g U}{a C_P (T_W - T_\infty)}$ is the pressure work parameter and $Q = \frac{Q_o}{\rho C_P a}$ is the heat generation or absorption parameter.
The corresponding dimensionless boundary conditions are given by:

$$f(0) = 0, \theta(0) = 1, \phi(0) = 1, \tag{2.14}$$

$$f'(\infty) = 0, \theta(\infty) = 0, \phi(\infty) = 0, \tag{2.15}$$

Of special significance and physical interest in this problem are the local Nusselt and Sherwood numbers which are defined by:

$$Nu = \frac{-x}{(T_W - T_\infty)} \frac{\partial T}{\partial y}\Big|_{y=0} = -(Re)^{\frac{1}{2}}\theta'(0) \text{ or } \frac{Nu}{(Re)^{\frac{1}{2}}} = -\theta'(0), \tag{2.16}$$

$$Sh = \frac{-x}{(C_W - C_\infty)} \frac{\partial C}{\partial y}\Big|_{y=0} = -(Re)^{\frac{1}{2}}\phi'(0) \text{ or } \frac{Sh}{(Re)^{\frac{1}{2}}} = -\phi'(0), \tag{2.17}$$

where $Re = U^2/av$ is the Reynold's number.

3 Numerical Method

The above system of equations (2.11)-(2.13) subject to the boundary conditions (2.14) have been solved numerically for various values of parameters by using the fourth-order Runge-Kutta integration method. In the present problem, a solution was considered to be converged if the newly calculated values of f, θ and ϕ differed from their previous guessed values within a tolerance of $E \prec 10^-5$. The numerical results were found to be dependent on η_∞ and the step size $\triangle\eta$. We have used $\triangle\eta = 0.05$ and $\eta_\infty = 4$, which gave accurate results without causing numerical oscillations in the values f, f', θ, θ', ϕ and ϕ'.

4 Results and Discussion

In order to get a clear insight of the physical problem, numerical results are displayed with the help of graphical illustrations. A representative set of results is shown in Figures 2-20.

Figures 2-4 show the effects of the pressure work parameter ε on the velocity, temperature and concentration profiles, respectively. It is clear that an increase of the pressure work parameter leads to decreases in both the velocity and temperature profiles and a relatively small increase in the concentration profiles. These behaviours in the velocity, temperature and concentration take place with insignificant changes in their boundary layer thicknesses.

Figures 5-7 depict the effects of heat generation or absorption parameter Q on the velocity, temperature and concentration profiles, respectively. Increasing the heat generation or absorption parameter Q has the tendency to increase the thermal state of the fluid. This increase in the fluid temperature causes more induced flow along the cone through the thermal buoyancy effect. However, these increases in both the velocity and temperature profiles are accompanied by a slight decrease in the concentration profiles as the heat generation or absorption parameter increases. Also, the hydrodynamic and thermal boundary layer thicknesses increase with insignificant decrease in the concentration boundary layer thickness as Q increases.

Figures 8-10 present the effects of the magnetic field parameter M on the velocity, temperature and concentration profiles, respectively. Application of a transverse magnetic field produces a darg-like force called the Lorentz force acting in the direction opposite to flow. This causes the velocity to decrease and the temperature and species concentration to increase as the magnetic field parameter M increases. It should be noted that increasing M causes significant decreases in the wall velocity owing the stretching action of the cone.

Figures 11-13 illustrate the influence of the chemical reaction parameter γ and the Schmidt number Sc on the velocity, temperature and concentration profiles in the boundary layer, respectively. Increasing the chemical reaction parameter produces a decrease in the species concentration. In turn, this causes the concentration buoyancy effects to decrease as γ increases. Consequently, less flow is induced along the cone resulting in decreases in the fluid velocity in the boundary layer. On the other hand, the fluid temperature increases as γ increases. In addition, the concentration boundary layer thickness decreases as γ increases. Moreover, the Schmidt number is an important parameter in heat and mass transfer processes as it characterizes the ratio of thicknesses of the viscous and concentration boundary layers. Its effect on the species concentration has similarities to the Prandtl number effect on the temperature. That is, increases in the values of Sc cause the species concentration and its boundary layer thickness to decrease resulting in less induced flow and higher fluid temperatures. This is depicted in the decreases in the velocity and species concentration and increases in the fluid temperature as Sc increases. These behaviors are clearly evident in Figures 11-13.

Figures 14-16 elucidate the effects of the of buoyancy ratio parameter N on the velocity, temperature and concentration profiles, respectively. It is clear that an increase in the buoyancy ratio parameter N causes an increase in the velocity profiles. On the other hand, an increase in the buoyancy ratio parameter leads to decreases in both the temperature and the concentration profiles.

Figure 17 shows the effects of the thermal stratification parameter n on the temperature profiles. It is seen that as thermal stratification parameter n increases, the temperature of the fluid decreases. Similarly, Fig. 18 displays the effects of the constant exponent value on the concentration profiles. It is also observed that increasing the exponent n_1 leads to decreases in the concentration profiles.

Figures 19 and 20 elucidate the effects of the pressure work parameter ε for three values of Prandtl number Pr on the local Nusselt and Sherwood numbers, respectively. It is shown that increasing the pressure work parameter ε produces an increase in the rate of heat transfer (or local Nusselt number) while it causes a decrease in the rate of mass transfer (or local Sherwood number). Moreover, the rate of heat transfer increases while the rate of mass transfer decreases as the Prandtl number Pr increases. These behaviours are clear from Figures 19 and 20.

Table 1 presents the effects of the chemical reaction, heat generation or absorption parameter, magnetic field parameter and the Schmidt number on the local Nusselt and Sherwood numbers. It is seen that as magnetic field parameter increases, both the local Nusselt number and local Sherwood number decrease. In addition, the local Nusselt number decreases as either of the heat generation or absorption parameter, chemical reaction parameter or the Schmidt number increases. On the other hand, the local Sherwood number increases as either of the heat generation or absorption parameter, chemical reaction parameter or the Schmidt number increases.

Table 2 illustrates the effects of the buoyancy ratio parameter N, Rayleigh number Ra, thermal stratification parameter n and constant exponent value n_1 on the local Nusselt and Sherwood numbers. It is observed that the local Nusselt number increases as either of the thermal stratification parameter, Rayleigh number and or the buoyancy ratio parameter increases, while it decreases as the wall concentration exponent n_1 increases. In addition, the local Sherwood number increases as either of n_1, Ra or N increases, while it decreases as the thermal stratification parameter n increases.

5 Conclusions

A study of the effects of pressure work, chemical reaction and heat generation or absorption on laminar MHD free convective heat and mass transfer over a stretching cone embedded in a porous media was performed. Numerical solution of the governing equations was obtained using the fourth-order Runge-Kutta method. Tabulated and graphical representation of the results for the velocity, temperature and solute concentration for various values of material parameters were reported. The conclusions of this study can be summarized as follows:

- The velocity, temperature and concentration fields are appreciably influenced by the pressure work parameter, magnetic field, heat generation or absorption parameter, chemical reaction parameter and Schmidt number.
- The rate of heat transfer or local Nusselt number increased as either of the pressure work parameter, Prandtl number, thermal stratification parameter, Rayleigh number or the buoyancy ratio parameter increased while it decreased as either of the heat generation or absorption coefficient, chemical reaction parameter, wall concentration exponent or the magnetic field parameter increased.
- The rate of mass transfer or local Sherwood number increased as either of the wall

concentration exponent, Rayleigh number, buoyancy ratio, heat generation or absorption parameter, chemical reaction parameter or the Schmidt number increased while it decreased as the pressure work parameter, magnetic field parameter, Prandtl number or the thermal stratification parameter increased.

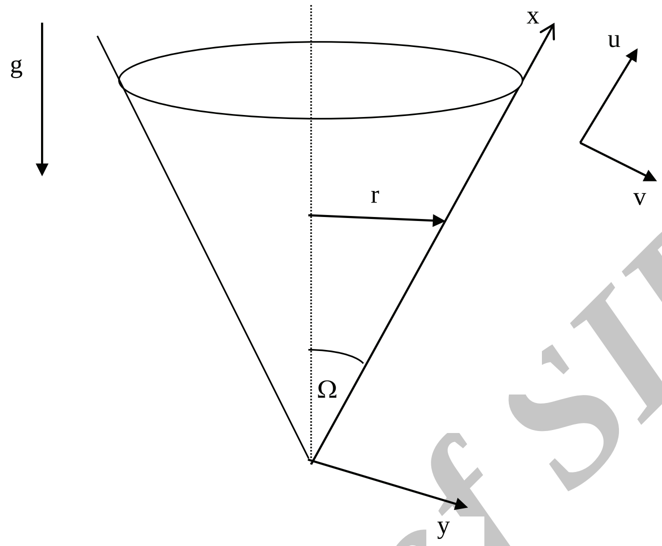

Fig. 1. Flow model and physical coordinate system

Table 1
Local Nusselt number and local Sherwood number for various values of Sc, γ, Q and M for $N = 1$, $Ra = 1$, $n = 0.5$, $n_1 = 0.5$, $Pr = 0.71$ and $\varepsilon = 0.3$.

Sc	γ	Q	M	$Nu/(Re)^{1/2}$	$Sh/(Re)^{1/2}$
0.62	1.0	0.2	0.4	1.1034	1.25194
			1	0.89076	1.13681
			4	0.48651	0.94932
			10	0.27169	0.86818
0.62	1.0	-0.4	1.0	1.11527	1.12550
		-0.2		1.04597	1.12886
		0.0		0.97151	1.13260
		0.2		0.89076	1.13681
		0.5		0.7545	1.14424
		1.0		0.6506	1.15015
0.62	0.0	0.2	1.0	0.9307	0.81027
	0.5			0.90814	0.9870
	1.0			0.89076	1.13681
	2.0			0.86516	1.38762
0.22	1.0	0.2	1.0	0.96562	0.66414
0.6				0.89339	1.11728
0.94				0.85703	1.41597

Table 2
Local Nusselt number and local Sherwood number for various values of N, Ra, n, n_1 for $Sc = 0.62$, $\gamma = 1.0$, $Q = 0.2$, $M = 1$, $Pr = 0.71$ and $\varepsilon = 0.3$.

N	Ra	n	n_1	$Nu/(Re)^{1/2}$	$Sh/(Re)^{1/2}$
1.0	1.0	0.5	0.0	0.89785	1.04196
			0.2	0.89494	1.08056
			0.6	0.88941	1.15515
			1.0	0.88422	1.22659
1.0	1.0	0.0	0.5	0.7608	1.14181
		0.2		0.81422	1.13974
		0.5		0.89076	1.13681
		1.0		1.01006	1.13237
1.0	0.2	0.5	0.5	0.29218	0.87536
	0.5			0.56809	0.98356
	1.0			0.89076	1.13681
	1.2			0.99508	1.19201
0.0	1.0	0.5	0.5	0.60519	0.99401
0.5				0.75451	1.06695
1.0				0.89076	1.13681
1.5				1.01558	1.20378

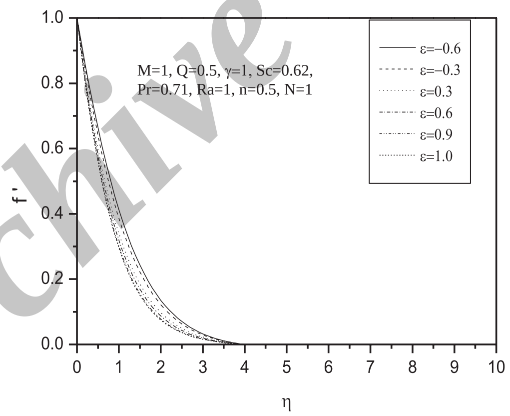

Fig. 2. Effects of pressure work parameter on the velocity profiles

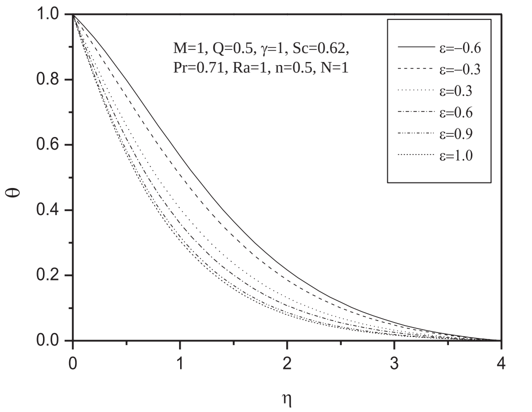

Fig. 3. Effects of pressure work parameter on the temperature profiles

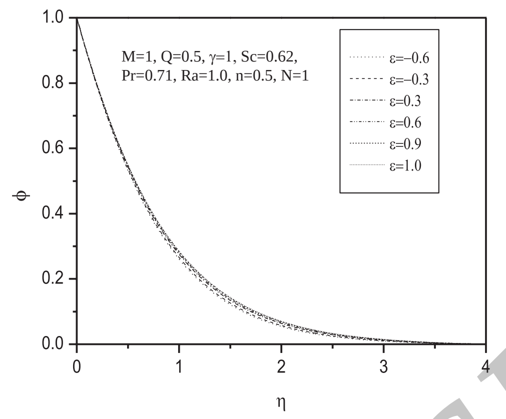

Fig. 4. Effects of pressure work parameter on the concentration profiles

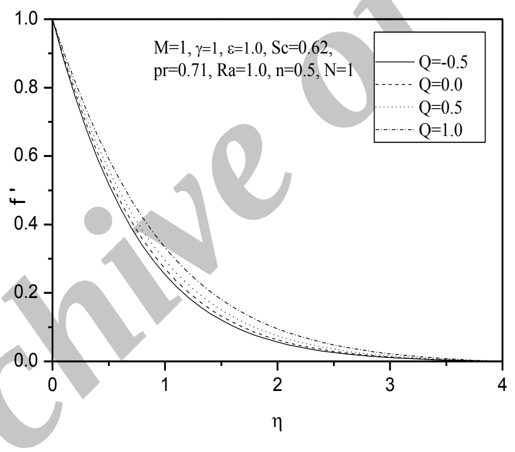

Fig. 5. Effects of heat generation or absorption parameter on the velocity profiles

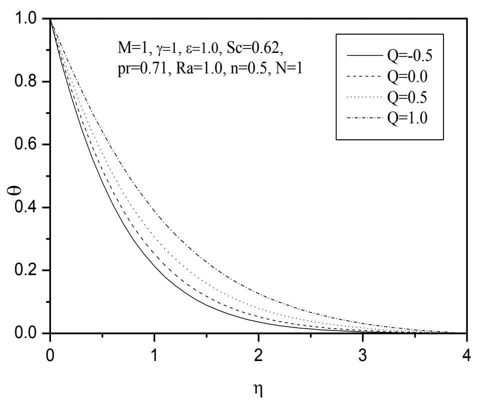

Fig. 6. Effects of heat generation or absorption parameter on the temperature profiles

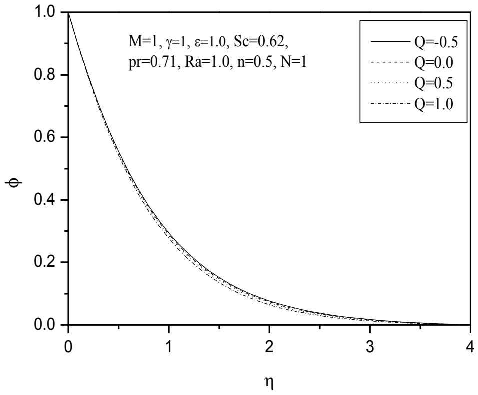

Fig. 7. Effects of heat generation or absorption parameter on the concentration profiles

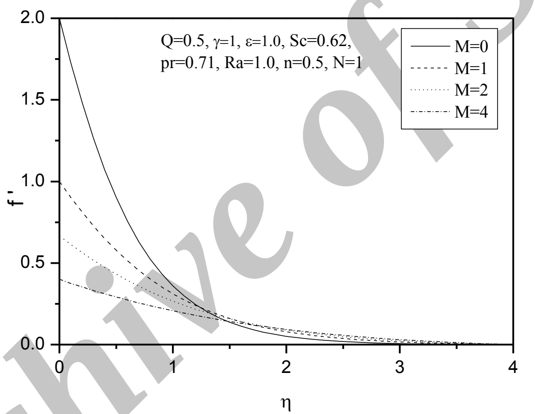

Fig. 8. Effects of magnetic field parameter on the velocity profiles

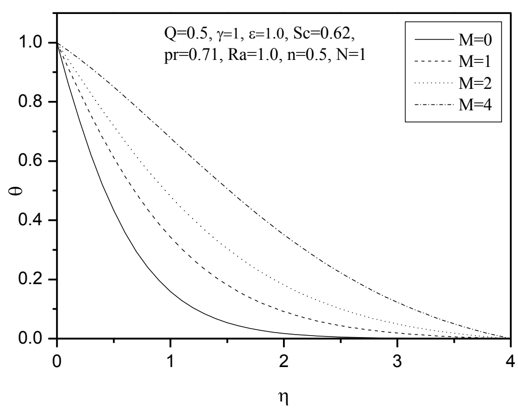

Fig. 9. Effects of magnetic field parameter on the temperature profiles

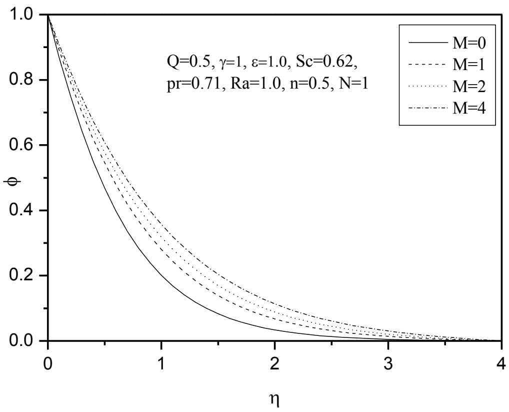

Fig. 10. Effects of magnetic field parameter on the concentration profiles

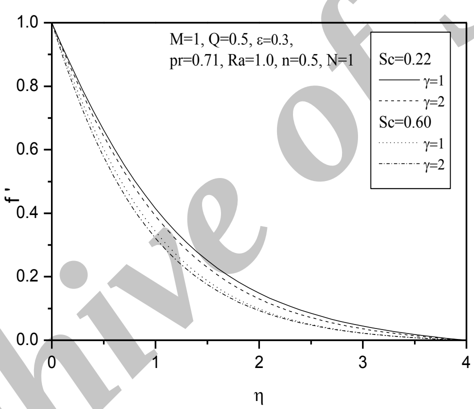

Fig. 11. Effects of chemical reaction parameter and Schmidt number on the velocity profiles

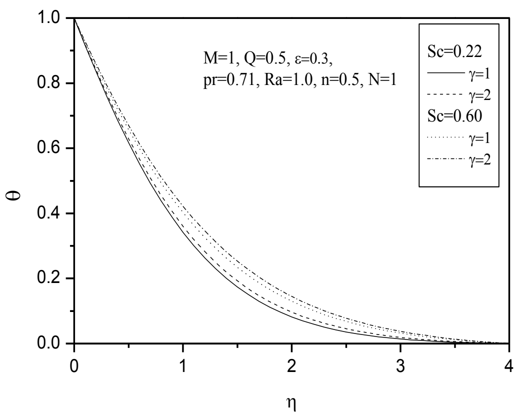

Fig. 12. Effects of chemical reaction parameter and Schmidt number on the temperature profiles

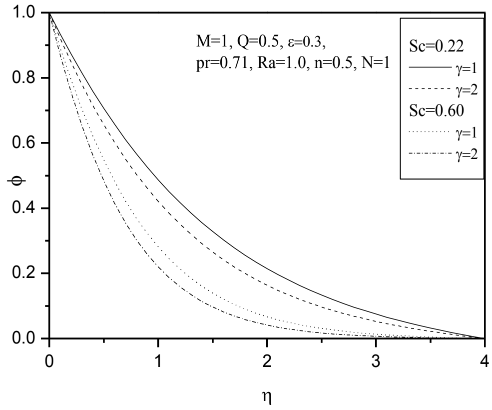

Fig. 13. Effects of chemical reaction parameter and Schmidt number on the concentration profiles

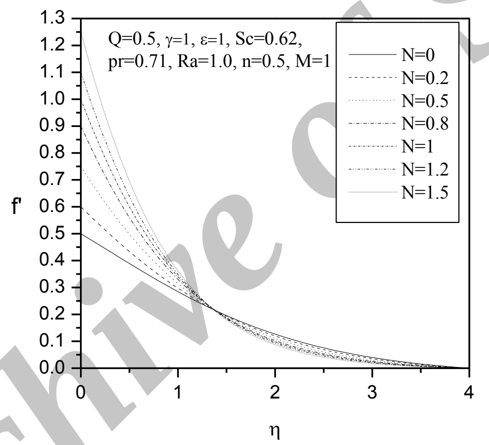

Fig. 14. Effects of buoyancy ratio parameter on the velocity profiles

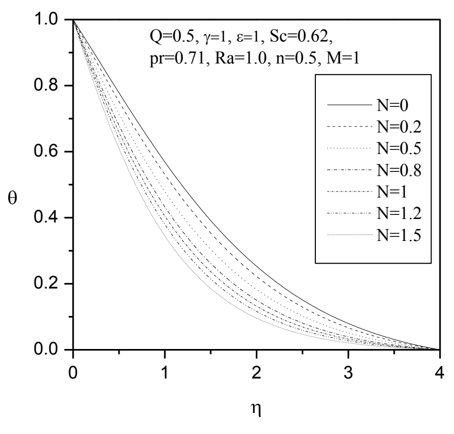

Fig. 15. Effects of buoyancy ratio parameter on the temperature profiles

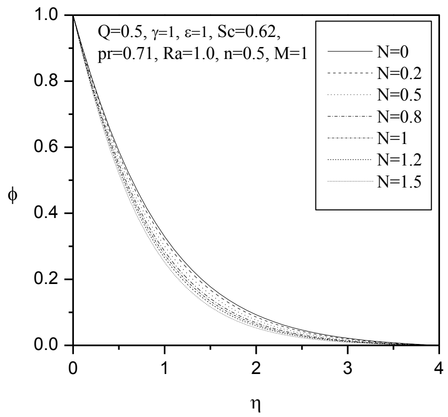

Fig. 16. Effects of buoyancy ratio parameter on the concentration profiles

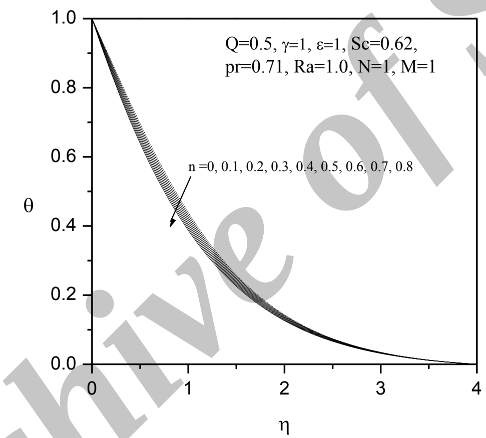

Fig. 17. Effects of thermal stratification parameter n on the temperature profiles

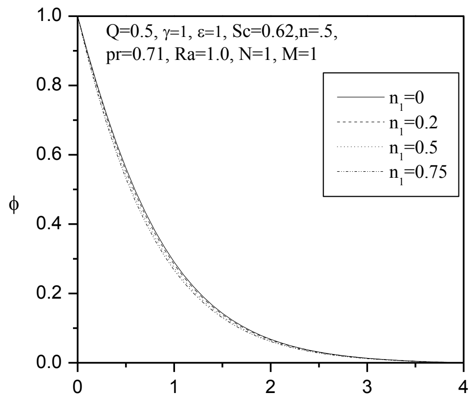

Fig. 18. Effects of wall concentration exponent n_1 on the concentration profiles

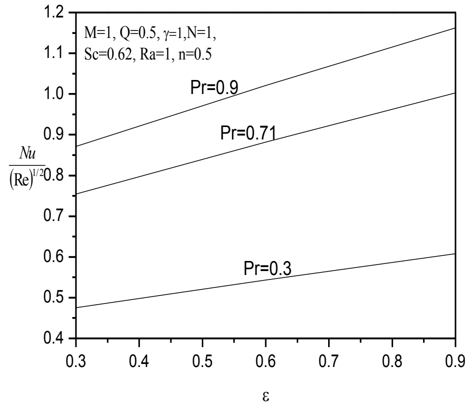

Fig. 19. Effects of pressure work and Prandtl number on Nusselt number

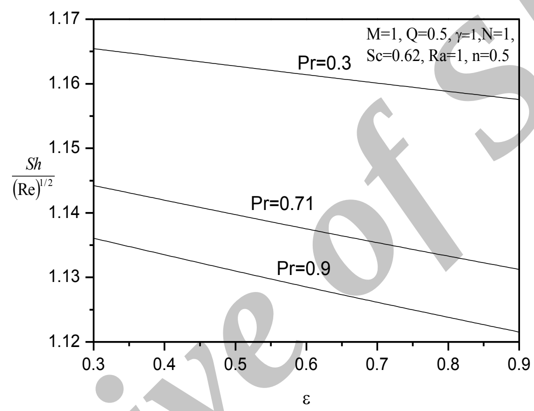

Fig. 20. Effects of pressure work and Prandtl number on Sherwood number

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