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Research Article

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Introducing a new approach for comparing fuzzy quantities by Rank and Mode

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Abstract

Many methods for ranking fuzzy numbers have been proposed. The existing methods for ranking generalized fuzzy numbers based on the optimistic index (α) gives different values for comparing the numbers by using different values of the optimistic index (α) and it is the shortcoming of this method. So we introduce a new defuzzification using a crisp number for ordering and comparing the fuzzy numbers. The calculation of the proposed method is simpler and easier in comparison the other methods and it provides the correct ordering of generalized fuzzy numbers. It is shown that the proposed modification satisfies all the reasonable properties of fuzzy quantities (I) and (II) proposed by X. Wang, E. E. Kerre [18, 19].

Keywords : Fuzzy numbers; Fuzzy function; L-R type generalized fuzzy number.

1 Introduction

In decision analysis under the fuzzy environment ranking fuzzy numbers is a very important decision-making procedure. L-R fuzzy number as the most general form of fuzzy number has been used extensively. The purpose of this paper is to introduce a new method for ranking fuzzy numbers. Ranking fuzzy numbers were first proposed by Jain for decision making in fuzzy situations by representing the ill-defined quantity as a fuzzy set. Ranking fuzzy numbers is important in decision making, data analysis, artificial intelligence, economic systems and operations research. Jain [11, 12], Dubios and Prade [13] introduced the relevant concepts of fuzzy numbers. Bortlan and Degani [14] reviewed some methods to rank fuzzy numbers, Chen and Hwang [15] proposed fuzzy multiple attribute decision making, Choobineh and Li [16] proposed an index for ordering fuzzy numbers. The rest of this paper is organized as follows. Section 2 introduces basic concepts and definitions of fuzzy numbers. Section 3 proposes an approach to ranking fuzzy numbers based on RM method. Section 4 presents some numerical examples to illustrate the advantages of the proposed approach.

2 Preliminaries

In this section some basic definitions and arithmetic operations are reviewed.

Definition 2.1 *The characteristic function μ_A of a crisp set $A \subseteq X$ assigns a value either 0 or 1 to each member in X.*
This function can be generalized to a function μ_A such that the value assigned to the element of the universal set X fall with in a specified rang i.e. $\mu_A : X \to [0, 1]$. The assigned value indicate the membership grade of the element in the set A.
The function μ_A is called the membership function and the set $A = \{(X, \mu(x)) : x \in X\}$ defined by μ_A for each $x \in X$ is called a fuzzy set [1, 2, 3, 4, 5, 6].

Definition 2.2 *An extended fuzzy number A is described as any fuzzy subset of the universe set U with membership function μ_A defined as follows [7, 8, 9, 10, 21]:*

(a) μ_A is a continuous mapping from U to the closed interval $[0, w]$, $0 < w < 1$.

(b) $\mu_A = 0$, for all $x \in (-\infty, a_1]$.

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(c) μ_A *is strictly increasing on* $[a_1, a_2]$.

(d) $\mu_A(x) = w$, *for all* $x \in [a_2, a_3]$ *as* w *is constant and* $0 < w \leq 1$.

(e) μ_A *is strictly decreasing on* $[a_3, a_4]$.

(f) $\mu_A(x) = 0$, *for all* $x \in [a_4, \infty)$.

Definition 2.3 *A triangular fuzzy number* A *is a fuzzy number with a piecewise linear membership function* μ_A *defined by:*

$$\mu_A = \begin{cases} \frac{x-a_1}{a_2-a_1}, & a_1 \leq x \leq a_2, \\ \frac{a_2-x}{a_3-a_2}, & a_2 \leq x \leq a_3, \\ 0, & otherwise. \end{cases} \quad (2.1)$$

Which can e denoted as a triplet (a_1, a_2, a_3) *[14].*

Definition 2.4 *A trapezoidal fuzzy number* A *is a fuzzy number with a membership function* μ_A *defined by:*

$$\mu_A = \begin{cases} \frac{x-a_1}{a_2-a_1}, & a_1 \leq x \leq a_2, \\ 1, & a_2 \leq x \leq a_3, \\ \frac{a_2-x}{a_3-a_2}, & a_3 \leq x \leq a_4, \\ 0, & otherwise. \end{cases} \quad (2.2)$$

Which can be denoted as a quarter (a_1, a_2, a_3, a_4) *[15].*

Definition 2.5 *A fuzzy number* $A = (a, b, c, d; w)_{LR}$ *is said to be a* $L - R$ *type generalized fuzzy number if its membership function is given by*

$$\mu_A(x) = \begin{cases} wL(\frac{x-a}{b-a}), & a < x < b, \\ w, & b \leq x \leq c, \\ wR(\frac{x-c}{d-c}), & c < x < d, \end{cases} \quad (2.3)$$

where L *and* R *are references functions [20].*

Definition 2.6 *A* $L - R$ *type generalized fuzzy number* $A = (a, b, c, d; w)_{LR}$ *is said to be a generalized trapezoidal fuzzy number if its membership function is given by*

$$\mu_A(x) = \begin{cases} w(\frac{x-a}{b-a}), & a < x < b, \\ w, & b \leq x \leq c, \\ w(\frac{x-c}{d-c}), & c < x < d, \end{cases} \quad (2.4)$$

2.1 Arithmetic operations

In this subsection, arithmetic operations between two $L - R$ type generalized fuzzy numbers, defined on universal set of real numbers, are reviewed [20].

Let $A_1 = (a_1, b_1, c_1, d_1; w_1)_{LR}$, $A_2 = (a_2, b_2, c_2, d_2; w_2)_{LR}$, be two $L - R$ type generalized fuzzy numbers and $A_3 = (a_3, b_3, c_3, d_3; w_3)_{RL}$, be a $R - L$ type generalized fuzzy number then

(i) $A_1 \oplus A_2 = (a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2; \min(w_1, w_2))$.

(ii) $A_1 \ominus A_3 = (a_1 - d_3, b_1 - c_3, c_1 - b_3, d_1 - d_3; \min(w_1, w_3))$.

(iii) $\lambda A_1 = \begin{cases} (\lambda a_1, \lambda b_1, \lambda c_1, \lambda d_1; w_1)_{LR}, & \lambda \geq 0, \\ (\lambda d_1, \lambda c_1, \lambda b_1, \lambda a_1; w_1)_{RL}, & \lambda < 0. \end{cases}$

3 Proposed approach

In this section, we will propose the ranking of fuzzy numbers associated with the RM method and the minimum value of a_1, a_2.
An efficient approach for comparing the fuzzy numbers is by the use of ranking function [20], $\Re : F(R) \to R$, where $F(R)$ is a set of fuzzy numbers defined on set of real numbers, which maps each fuzzy number into the real line, where a natural order exists i.e.,

(i) $A \succ B$ if and only if $\Re(A) > \Re(B)$,

(ii) $A \prec B$ if and only if $\Re(A) < \Re(B)$,

(iii) $A \sim B$ if and only if $\Re(A) = \Re(B)$.

Remark 3.1 *For all fuzzy numbers* A, B, C *and* D *we have [18]*

(i) $A \succ B \Longrightarrow A \oplus C \succ B \oplus C$,

(ii) $A \prec B \Longrightarrow A \ominus C \prec B \ominus C$,

(iii) $A \sim B \Longrightarrow A \oplus C \sim B \oplus C$,

(iv) $A \succ B, C \succ D \Longrightarrow A \oplus C \succ B \oplus D$.

3.1 Method to find values of RM(A), RM(B)[20]

Let $A = (a_1, b_1, c_1, d_1; w_1)_{(L_1R_1)}$ and $B = (a_2, b_2, c_2, d_2; w_2)_{(L_2R_2)}$ be two L-R type generalized fuzzy numbers then use the following steps to find the values of $RM(A)$ and $RM(B)$

Step 1. Find $w = \min(w_1, w_2)$.

Step 2. Let the membership function of A and B be $\mu_A(x)$ and $\mu_B(x)$, respectively, where,

$$\mu_A(x) = \begin{cases} 0, & -\infty < x \leq a_1, \\ w_1 L_1(\frac{b_1-x}{b_1-a_1}), & a_1 \leq x < b_1, \\ w_1, & b_1 \leq x \leq c_1, \\ w_1 R_1(\frac{x-c_1}{d_1-c_1}), & c_1 < x \leq d_1, \\ 0, & d_1 \leq x < \infty, \end{cases}$$

and

$$\mu_B(x)=\begin{cases}0, & -\infty<x\le a_2,\\ w_2L_2(\frac{b_2-x}{b_2-a_2}), & a_2\le x<b_2,\\ w_2, & b_2\le x\le c_2,\\ w_2R_2(\frac{x-c_2}{d_2-c_2}), & c_2<x\le d_2,\\ 0, & d_2\le x<\infty,\end{cases}$$

where $\mu_A(x)\in[0,w_1]$ and $\mu_B(x)\in[0,w_2]$.

Let λ represent the membership value, which is common for both A and B, then $\lambda\in[0,w]$, where $w=\min(w_1,w_2)$.

Let $wL_1(\frac{b_1-x}{b_1-a_1})=\lambda$ and

$$\begin{aligned} wR_1(\frac{x-c_1}{d_1-c_1}) &= \lambda \\ &\Rightarrow \frac{b_1-x}{b_1-a_1} \\ &= L_1^{-1}(\frac{\lambda}{w}) \\ &\Rightarrow x \\ &= b_1-(b_1-a_1)L_1^{-1}(\frac{\lambda}{w}) \end{aligned}$$

and

$$\begin{aligned} (\frac{x-c_1}{d_1-c_1}) &= R_1^{-1}(\frac{\lambda}{w}) \\ &\Rightarrow x \\ &= c_1-(d_1-c_1)R_1^{-1}(\frac{\lambda}{w}) \end{aligned}$$

and now

$$\Re(A)=\alpha\int_0^w(b_1-(b_1-a_1)L_1^{-1}(\frac{\lambda}{w}))d\lambda + (1-\alpha)\int_0^w(c_1-(d_1-c_1)R_1^{-1}(\frac{\lambda}{w}))d\lambda.$$

Let

$$\frac{\lambda}{w}=h\Rightarrow d\lambda=wdh$$

.

$$\Re(A)=w\alpha\int_0^1(b_1-(b_1-a_1)L_1^{-1}(h))dh + w(1-\alpha)\int_0^1(c_1-(d_1-c_1)R_1^{-1}(h))dh.$$

Similarity,

$$\Re(B)=w\alpha\int_0^1(b_2-(b_2-a_2)L_2^{-1}(h))dh + w(1-\alpha)\int_0^1(c_2-(d_2-c_2)R_2^{-1}(h))dh.$$

Step 3. If $\Re(A)\neq\Re(B)$ then $RM(A)=\Re(A)$ and $RM(B)=\Re(B)$ otherwise

$$\begin{aligned} RM(A) &= mode(A) \\ &= \alpha\int_0^w b_1dx+(1-\alpha)\int_0^w c_1dx \\ &= w\alpha b_1+(1-\alpha)wc_1, \end{aligned}$$

and

$$\begin{aligned} RM(B) &= mode(B) \\ &= alpha\int_0^w b_2dx+(1-\alpha)\int_0^w c_2dx \\ &= w\alpha b_2+(1-\alpha)wc_2, \end{aligned}$$

For generalized trapezoidal numbers $A=(a,b,c,d;w)$,

$$\Re(A)=\frac{w}{2}\alpha(a+b)+\frac{w}{2}(1-\alpha)(c+d),$$

and for triangular fuzzy numbers $A=(a,b,c;w)$,

$$\Re(A)=\frac{w}{2}\alpha(a+b)+\frac{w}{2}(1-\alpha)(b+c).$$

3.2 Method to find values of $RM_a(A)$ and $RM_a(B)$

In this subsection, we will propose a new method for ranking $L-R$ type generalized fuzzy numbers. So use the following steps to find the values of $RM_a(A)$ and $RM_a(B)$.

Step 1. Find $a=\min(a_1,a_2)$.

Step 2. Let $RM_a(A)=RM(A)-a$ and similarity,

Step 3. $RM_a(B)=RM(B)-a$.

(i) $A\succ B$ if and only if $RM_a(A)>RM_a(B)$.

(ii) $A\prec B$ if and only if $RM_a(A)<RM_a(B)$.

(iii) $A\sim B$ if and only if $RM_a(A)=RM_a(B)$.

We consider the following axioms for the ordering approach [18, 19].
$\mathcal{A}_1$: For an arbitrary finite subset Γ of E and $A\in\Gamma$, $A\succeq A$.
$\mathcal{A}_2$: For an arbitrary finite subset Γ of E and $(A,B)\in\Gamma^2$, $A\succeq B$ and $B\succeq A$, we should have $A\sim B$.
$\mathcal{A}_3$: For an arbitrary finite subset Γ of E and $(A,B,C)\in\Gamma^3$, $A\succeq B$ and $B\succeq C$, we should have $A\succeq C$.

$\mathcal{A}_4$: For an arbitrary finite subset Γ of E and $(A, B) \in \Gamma^2$, $\inf supp(A) > \sup supp(B)$ we should have $A \succeq B$.
$\mathcal{A}'_4$: For an arbitrary finite subset Γ of E and $(A, B) \in \Gamma^2$, $\inf supp(A) > \sup supp(B)$ we should have $A \succeq B$.
$\mathcal{A}_5$: Let Γ and Γ' be two arbitrary finite subset of E in which A and B are in $\Gamma \bigcap \Gamma'$. We obtain the ranking order $A \succ B$ by defuzzification $RM_a(A)$ on Γ' if only and if $A \succ B$ by $RM_a(A)$ on Γ.
$\mathcal{A}_6$: Let A, B, $A + B$ and $B + C$ be elements of E. If $A \succeq B$ then $A + C \succeq B + C$.
$\mathcal{A}'_6$: Let A, B, A+B and B+C be elements of E. If $A \succeq B$ then $A + C \succ B + C$.
In addition for the axioms $A_1, A_2, , A_6$ we can consider two following properties:
$\mathcal{A}_7$: For an arbitrary finite subset Γ of E and $A \in \Gamma$, the defuzzification must belong to its support function.
$\mathcal{A}_8$: For an arbitrary finite subset Γ of E and $A \in \Gamma$, distance between A and its defuzzifiation will be minimized.

Defuzzification

$$RM_a(A) : E \to R$$

is called the best approximation operator for any $A \in E$ if it has axioms $A_1, A_2, ..., A_8$.

4 Numerical example

In this section, some examples are used to illustrate the proposed approach to ranking $L - R$ fuzzy numbers.

Example 4.1 *Let* $A = (0.1, 0.3, 0.3, 0.5; 1)$ *and* $B = (0.2, 0.3, 0.3, 0.4; 1)$ *be two generalized trapezoidal fuzzy numbers.*
Find $a = \min(0.1, 0.2) = 0.1$.
For $\alpha = 0$, $RM(A) = 0.4$, $RM(B) = 0.35$, $RM_a(A) = 0.4 - 0.1 = 0.3$ *and* $RM_a(B) = 0.35 - 0.1 = 0.25$. *So* $A \succ B$.

For $\alpha = 1$, $RM(A) = 0.2$, $RM(B) = 0.25$, $RM_a(A) = 0.2 - 0.1 = 0.1$ *and* $RM_a(B) = 0.25 - 0.1 = 0.15$. *So* $A \prec B$.

For $\alpha = \frac{1}{2}$, $RM(A) = 0.3$, $RM(B) = 0.3$, $RM_a(A) = 0.3 - 0.1 = 0.2$ *and* $RM_a(B) = 0.3 - 0.1 = 0.2$. *So* $A \sim B$.

Example 4.2 *Consider the three triangular fuzzy numbers* $A = (6, 1, 1)$, $B = (6, 0.1, 1)$ *and* $C = (6, 0, 1)$. *By using our method,* $RM(A) = 1 + \frac{5}{2}\alpha$, $RM(B) = \frac{5}{2}\alpha + 0.55$, $RM(C) = \frac{5}{2}\alpha + \frac{1}{2}$ *and with* $a = \min(6, 6, 6) = 6$, *we have* $RM_a(A) = \frac{5}{2}\alpha - 5$, $RM_a(B) = \frac{5}{2}\alpha - 5.45$, $RM_a(C) = \frac{5}{2}\alpha - 5.5$. *So* $A \prec B \prec C$.

Example 4.3 *Let* $A = (0.2, 0.4, 0.6, 0.8; 0.35)$ *and* $B = (0.1, 0.2, 0.3, 0.4; 0.7)$ *be two generalized trapezoidal fuzzy numbers.*
For $a = \min(0.2, 0.1) = 0.1$, *and* $\alpha = 0$, *then* $RM(A) = 0.245$, $RM(B) = 0.1225$, $RM_a(A) = 0.245 - 0.1 = 0.145$ *and* $RM_a(B) = 0.1225 - 0.1 = 0.0225$. *So* $A \succ B$.
Also, for $\alpha = 1$ *we have,* $RM(A) = 0.105$, $RM(B) = 0.0525$, $RM_a(A) = 0.105 - 0.1 = 0.005$ *and* $RM_a(B) = 0.0525 - 0.1 = -0.0475$. *So* $A \succ B$.
For $\alpha = \frac{1}{2}$, $RM(A) = 0.175$, $RM(B) = 0.0875$, $RM_a(A) = 0.175 - 0.1 = 0.075$ *and* $RM_a(B) = 0.0875 - 0.1 = -0.0125$. *So* $A \succ B$.

5 Conclusion

This paper presents a new approach for ranking $L - R$ fuzzy numbers. The examples given in this paper illustrate that the proposed approach gives the correct ordering of fuzzy numbers. Comparing with the existing approaches, it is efficient and simple. For the validation of the proposed ranking function, it is shown that this ranking function satisfies all the reasonable properties of fuzzy quantities (I) and (II) proposed by X.Wang, E.E. Kerre [18, 19].

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