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Research Article

The dynamic effect in parallel production systems; An illustration with iranian banks

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Abstract

In the real world, there are production network which are composed of a set of the production processes, so that each production process have several interdependent subunits. In this paper, we consider dynamic networks which are composed of a set of production systems with parallel subunits, so that each subunit at any period uses of exogenous inputs and pervious period outputs to product the final outputs and intermediate outputs. Then, we will proposed a model that focuses on the evaluating the performance of production systems across time and the calculating of the efficiency of the whole system and subunits at each period.

Keywords : DEA; Dynamic network; Parallel Production System; Statistical Analysis.

1 Introduction

A production network can be described as a collection of production processes performed by several interdependent sub decision-making units. Dynamic network model will consider behavior of DMUs across time and will evaluate the performance of the production systems across time. Performance evaluation is an important task for a decision making unit (DMU) [1] in order to find its weaknesses and for subsequent improvements. In the dynamic network world [10], it import occurs for each DMU across time period. Fare et al. (2000) [10] suggested a model for solving minimum potential input over different time period. The proposed model by them evaluated efficiency of the whole system at each period. But, if the production systems across time has some parallel subunit [2, 3, 5, 6, 12], so that sum of inputs-outputs of each subunits is equal to inputs-outputs whole system, then Fare's model cannot evaluate subunits performance during time period. The main purpose of this paper is considering dynamic network model for production systems with parallel subunits, so that each subunit at any period uses exogenous inputs and pervious period outputs for producing final outputs and intermediate outputs. Also, a model will be proposed which evaluate the whole systems of efficiency and the relevant subunits during time period.

Finally, the paper unfolds as follows: in the next section, the dynamic network model proposed by fare et.al. (2000) [10] and the parallel model proposed by Kordrostami et.al. (2010) [12] are reminded briefly. In section three, the dynamic network model is generalized for parallel production system and evaluated efficiency subunits during time period, then analyzing of the proposed model is presented using a simple example. The

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application of the study presented on the data obtained from Iran's bank using the proposed model, in section four. In section five conclusions will be presented.

2 The Related Models

We use these two kinds of model, the dynamic network model [7, 8, 9, 10], and the parallel production systems [2, 3, 4, 5, 6, 11, 12] which are used in this paper. We introduce these models briefly as follows.

2.1 The Dynamic Network Model

This section is based on Fare and Grosskopf [8, 10], which is our basic model for estimation of optimal private and public investment. The dynamics of our technology are modeled as the choice of consuming total output in period of production or instead diverting some current production toward adding to the next period's capital stock. They use a discrete formulation of time and employ an activity analysis (DEA)model as our production technology. It has three time periods $t-1, t, t+1$ and that there is a technology $p^T, T = t-1, t, t+1$In addition at each T there are some exogenous inputs x^T and final outputs y^T . Final output is that part of total production y^Tthat is not allocated to private ${}^i y^T$; i.e.

$$y^T = ^i y^T + ^f y^T \tag{2.1}$$

It may now sketch Fare model as a network, using the above notation.

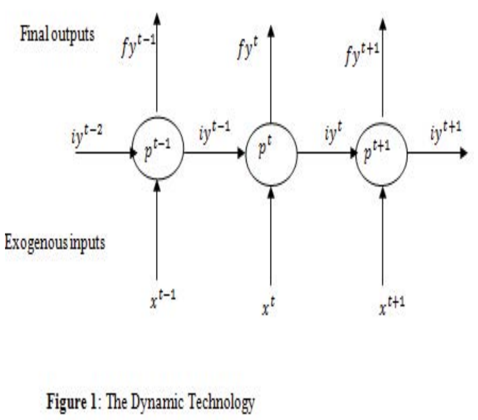

Figure 1: The Dinamic thecnology.

Now, given that it has $j = 1, ..., n$ observations of $i = 1, ..., m$ inputs $(x_1, ..., x_m)$ and $r = 1, ..., s$ outputs $(y_1, ..., y_s)$ in each period t. for each observation j_o they estimate its dynamic efficiency by solving the following problem: Min $\sum^T \theta^T$

$$\begin{array}{ll}
{}^f y_o^T + ^i y_o^T \le \sum_{j=1}^n z_j^T y_j^T, & for\ all\ T, \\
\sum_{j=1}^n z_j^T x_j^T \le \theta^T x_o^T, & for\ all\ T, \\
\sum_{j=1}^n z_j^{T i} y_j^T \le \theta^{t i} y_o^T, & T = t-2, \\
\sum_{j=1}^n z_j^{T i} y_j^T \le {}^i y_o^T, & T = t+1, \\
\theta^T \le 1 & \\
z_j^T \ge 0, & j = 1, ..., n.
\end{array}$$

$In the above model \theta^T$is efficiency score for each period, where minimum the inputs over time period. In this model, they have restricted the annual efficiency score to be less than or equal to one which means that inputs cannot be increased in any given period below the observed level.

2.2 The Parallel Production System

In the real world there are cases that a DMU is composed of a set of components, and each utilizes the same input to produce the same outputs. A typical example is a firm with several plants, each operations independently. Each of the firm's inputs and outputs is sum of those of all its plants. The general case is a parallel production system p with $K = 1, , k$ production units,as depicted in Fig. 2, where each production unit $K = 1, ..., k$ converts inputs $X_{ip}^K, i = 1, ..., m$ into outputs $Y_{rp}^K, r = 1, ..., s$ independently. The sums of all X_{ip}^K over K,$\sum_{K=1}^k X_{ip}^K$ and all Y_{rp}^K over$K$,$\sum_{K=1}^k Y_{rp}^K$ are the input X_{ip} and output Y_{rp} of system, respectively. Kao (2008)[6] in his

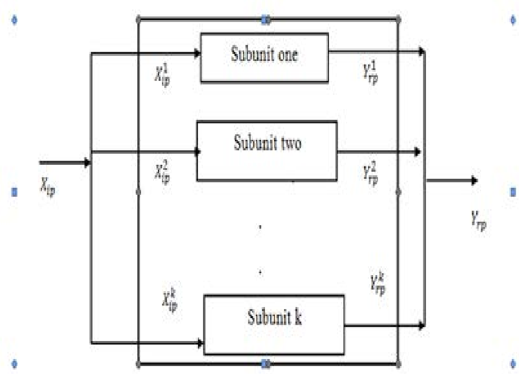

Figure 2: The parallel production system, where a DMUp has k production units

paper investigated the production system with

parallel production units. A parallel DEA model is developed to calculate the efficiency of whole system as well as the efficiencies of individual production units. Based on the results of the study decision marker is able to reallocate resources to different production units in the system in order to improve it.
kordrostami et.al. (2010) [12] production possibility set (PPS) of subunit t under the various returns to scale (VRS) is as follows:

$$T_v^{(k)} = \{(X^{(t)}, Y^{(t)}) : \sum_{j=1}^{n} \lambda_j X_j^{(k)} \le X^{(k)},$$

$$\sum_{j=1}^{n} \lambda_j Y_j^{(k)} \ge Y^{(k)}, \sum_{j=1}^{n} \lambda_j = 1, \lambda_j \ge 0\}$$

To evaluate the technical efficiency of DMUP, we solve the following mathematical program:

$$Min \sum_{k=1}^{K} w_k \theta_k$$

$$\sum_{j=1}^{n} \lambda_j X_j^{(k)} \le \theta_k X_p^{(k)}, \quad k = 1, ..., K,$$

$$\sum_{j=1}^{n} \lambda_j Y_j^{(k)} \ge Y_p^{(k)}, \quad k = 1, ..., K,$$

$$\sum_{j=1}^{n} \lambda_j = 1$$

$$\theta_k \le 1$$

$$z_j^T \ge 0, \quad j = 1, ..., n.$$

The objective function is the weighted sum of

$$E_p^k = \theta_k, (k = 1, ..., K).$$

W_k s are the user-defined multipliers, and we have $\sum_{k=1}^{K} W_k = 1$. In this model E_p^k is efficiency score of the subunit k, and also E_p is efficiency score of the whole system, i.e. $E_p = \sum_{k=1}^{K} W_k(\theta_k) = \sum_{k=1}^{K} W_k E_p^k$. It is easy to show the feasibility and boundedness of LP (2-3).

3 The Dynamic Network Model for Parallel Production Systems

According to, the model Far is calculated total system performance within the time period, But if the production systems are parallel subunit, then the model far cannot calculate the performance of subunits within that time period. These are the disadvantages of this model.
So this section will provide a model to will pay calculate the total system performance and its subunits.
The general case is a parallel production system p with $K = 1, ..., k$ production units at three time period$t-1, t, t+1$, as depicted in Fig. 3, where each production unit $K = 1, ..., k$ was comprised of exogenous inputs $X_{ip}^{(K,T)}, i = 1, ..., m$ and pervious period output as next period input $iY_{rp}^{(K,T-1)}, r = 1, ..., s$ for produce final outputs $fY_{rp}^{(K,T)}, r = 1, ..., s$, and intermediate output $iY_{rp}^{(K,T)}, r = 1, ..., s$. The sums of all $X_ip^{(K,T)}$ over$K$, $\sum_{K=1}^{k} X_{ip}^{(K,T)}, \forall T$ and all $iY_{rp}^(K,T), r = 1, ..., s$ over K, $\sum_{K=1}^{k} iY_{rp}^{(K,T)}$, $\forall T$ and all $fY_{rp}^{(K,T)}$ over$K$, $\sum_{K=1}^{k} fY_{rp}^{(K,T)}$,$\forall T$ are the exogenous input X_{ip}^{T} and intermediate output iY_{rp}^{T} and final output fY_{rp}^{T} of system, respectively. Production possibility set of

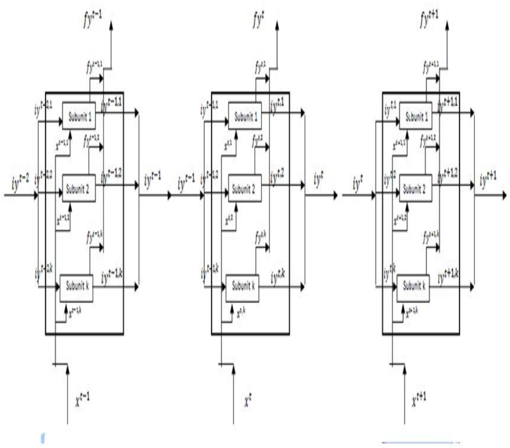

Figure 3: The dynamic technology for parallel production systems

subunit K over three time period is a follows:

p(X^t-1,K,X^t,K,Xt+1,K,iYt-2,K)={(fY^t-1,K,fY^t,K,(fY^t+1,K+iYt+1,K)):

$\sum_{j=1}^{n} z_j^{t-1}(fY_{rj}^{t-1,K} + iY_{rj}^{t-1,K}) \ge fYrp^{t-1,K} + iY_{rp}^{t-1,K}$ $\forall r, K$

$\sum_{j=1}^{n} z_j^{t-1}(iY_{rj}^{t-2,K}) \le iY_{rp}^{t-2,K}$ $\forall r, K$

$\sum_{j=1}^{n} z_j^{t-1}(X_{ij}^{t-1,K}) \le X_{ip}^{t-1,K}$ $\forall i, K$

Table 1: Data Set.

DMU	Exogenous input	Intermediate output	Final output
A			
$a_1.t_1$	11	22	25
$a_2.t_1$	18	9	10
$a_3.t_1$	9	30	50
$a_1.t_2$	50	42	45
$a_2.t_2$	15	24	21
$a_3.t_2$	24	47	48
$a_1.t_3$	29	9	10
$a_2.t_3$	35	10	12
$a_3.t_3$	16	11	15
B			
$b_1.t_1$	4	9	10
$b_2.t_1$	11	15	30
$b_3.t_1$	8	9	18
$b_1.t_2$	40	28	38
$b_2.t_2$	35	8	20
$b_3.t_2$	30	24	26
$b_1.t_3$	2	18	20
$b_2.t_3$	14	12	15
$b_3.t_3$	4	16	19

$z_j^{t-1} \geq 0$

$\sum_{j=1}^{n} z_j^t (fY_{rj}^{t,K} + iY_{rj}^{t,K}) \geq fYrp^{t,K} + iY_{rp}^{t,K} \quad \forall r, K$

$\sum_{j=1}^{n} z_j^t (iY_{rj}^{t-1,K}) \leq iY_{rp}^{t-1,K} \quad \forall r, K$

$\sum_{j=1}^{n} z_j^t (X_{ij}^{t,K}) \leq X_{ip}^{t,K} \quad \forall i, K$

$z_j^t \geq 0$

$\sum_{j=1}^{n} z_j^{t+1} (fY_{rj}^{t+1,K} + iY_{rj}^{t+1,K}) \geq fYrp^{t+1,K} + iY_{rp}^{t+1,K} \quad \forall r, K$

$\sum_{j=1}^{n} z_j^{t+1} (iY_{rj}^{t,K}) \leq iY_{rp}^{t,K} \quad \forall r, K$

$\sum_{j=1}^{n} z_j^{t+1} (X_{ij}^{t+1,K}) \leq X_{ip}^{t+1,K} \quad \forall i, K$

$z_j^{t+1} \geq 0\}$

Also, $iY_{rj}^{K,T}$ is as intermediate output period t that is use as input period$t+1$, then we defined the depreciation rate $\delta(0 \leq \delta \leq 1)$ and write

$$ic_j^{T,K} = ic_j^{T-1,K}(1-\delta) + iY^{T-1,K} \qquad j,\ T$$

For each observation p we can estimate dynamic efficiency for parallel production system by solving the following problem which generalizes the model to many periods. As our objective,we choose to minimize the exogenous input and intermediate output over at each period , scaled individually, namely $\theta_K^T\, \forall T, K$. θ_K^T is efficiency score of each subunit across time periods. Thus, we have

$\text{Min} \sum_T \sum_K W_K^T \theta_K^T$

$fY_{rp}^{T,K} + iY_{rp}^{T,K} \leq \sum_{j=1}^{n} z_j^T (iY_{rj}^{T,K} + fY_{rj}^{T,K}), \qquad \forall T, K, r,$

$\sum_{j=1}^{n} z_j^T ic_{rj}^{T,K} \leq \theta_K^T (ic_{rp}^{T-1,K}(1-\delta) + iY_{rp}^{T-1,K}, \qquad \forall T, K, r,$

$\sum j = 1^n z_j^T X_{ij}^{T,K} \leq \theta_K^T X_{ip}^{T,K}, \qquad \forall T, K, i,$

$\theta_K^T \leq 1, \qquad \forall T, K,$

$\sum_{j=1}^{n} z_j^T = 1, \qquad \forall T,$

$z_j^T \geq 0, \qquad forall j, T.$

Objective function is the weighed sum of θ_K^T . Weights are given by user and we have

$\sum_{K=1}^{K} W_K^T = 1.$

In the above model, we restricted efficiency of subunits at each period score to be less than or equal to one, which means that exogenous input and intermediate output cannot be increased at any given period below the observed level.

3.1 A simple example

We consider, two decision making units with three parallel subunits over three time periods, so that it has one exogenous input, intermediate output and final output. Table 1 shows the input-output data set. Then, result of running proposed model (3-5) is shown in Table 2. In this example was put$W_1^T = 0.2, W_2^T = 0.3, W_3^T = 0.5$, according to considered weights we observe that unit A is efficient over first and second periods and unit B is efficient over all three period. It must be pointed that efficiency of the whole system over different periods is equal to weighted sum of efficiency subunits of this system, thus the DMU is efficient over each period when all subunits have been efficient over those periods. The cause of inefficiency of unit A over third periods is all subunits this period. Also subunits one, two and three are efficient during first and second periods, but are inefficient during third period.

4 Applied study

We consider ten areas in banks Iran at three six-month (2009-2010), each area including three branches, so that each branch includes one exogenous input; personnel two output: resource and usages; resources as intermediate output of each period and input of next period, and usages as final output.
It is should be mentionable that sum the of input-output of each branch across time period is equal to input-output of this area. We provide a statistical result of data banks of Iran with three branches during three six-month periods in Table 3.
Now, we apply the model (3-5) for this data, and compute efficiency score of branches and efficiency score of areas during three six-month periods. (See table 4)
In this example, we put $w_1^T = 0.5, w_2^T = 0.2, w_3^T = 0.3$. According to the results areas one, two, three, four, eight, nine and ten are efficient over three six-month periods. In this areas all branches one, two and three were efficient.
Also area five is inefficient over first six-month period, the cause of inefficiency of this area over first six-month period is branches one and three, branch two has no role at inefficiency of this area, this area is efficient over first, second six-month periods and all branches are efficient. Branches one and three of this area are inefficient over first period and then efficient at next periods, branch two is efficient over three six-month periods, therefore branches one, two and three of area five over second, third periods are more efficient in proportion to first period.
Area six over first, third six-month periods are inefficient and efficient over second period, the cause of inefficiency of this area is all branches over first, second periods, the branches one, two and three are inefficient over first and third periods, but efficient over second period, therefore all branches over area six are more efficient over second six-month in proportion to first, third periods.
Area seven is efficient over first six-month period and inefficient over second and three six-month periods, the cause of inefficiency of this area is branches two and three, branch one has no role at inefficiency of this area. Branch one of this area is efficient over three six-month periods, but branches two and three are more efficient over first six-month period in proportion to next periods.
Therefore, it is possible the area was inefficient across time period but has efficient branches. If all branches were efficient over time periods then those area are efficient, also if all branches were inefficient over time periods then those area are inefficient.

4.1 Statistical analysis

The average efficiency score for the 30 branches over three six-month periods are listed in Table 5. Also, according to this table statistical chart is presented in Fig. 4. In Fig. 4, we observe efficiency branch one over second six-month period is better than first and third periods, also branch two over first and second periods has better performance than third period. In branch three efficiency of second six-month period are better than first and third periods.

Table 2: The result of model(3-5).

	Efficiency subunits at 1st period	Efficiency subunits at 2nd period	Efficiency subunits at 3rd period	Efficiency unit at 1st period	Efficiency unit at 2nd period	Efficiency unit at 3rd period
DMU	θ_K^1	θ_K^2	θ_K^3	θ^1	θ^2	θ^3
A				1.0000	1.0000	0.6879
a_1	1.0000	1.0000	0.7143			
a_2	1.0000	1.0000	0.4000			
a_3	1.0000	1.0000	0.8500			
B				1.0000	1.0000	1.0000
a_1	1.0000	1.0000	1.0000			
a_2	1.0000	1.0000	1.0000			
a_3	1.0000	1.0000	1.0000			

Therfore branch one over second period is more efficient in proportion to other branches over three six-month periods.

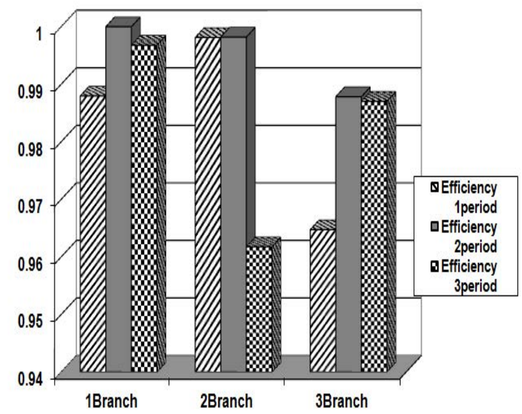

Figure 4: Average efficiency branches at three periods for each branch.

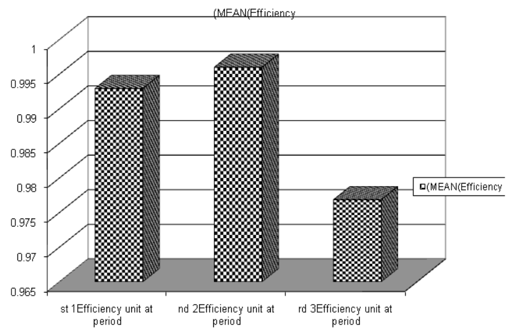

Figure 5: Average efficiency of scores whole areas for any periods.

The average efficiency scores of each area over three six-month periods and average efficiency score of whole areas for any six-month period are listed in table

According to the average efficiency score of whole areas for any periods are presented statistical chart in Fig. 5.

In Fig. 5, ten areas at second six-month period are more efficient in proportion to first and third six-month periods. Based on average efficiency score of areas over whole periods is drawn statistical chart in Fig. 6.

We observe area seven over whole six-month period is more inefficient in proportion to areas five and six and other areas are efficient over whole six-month period, in Fig.

Finally, the statistical chart of average efficiency score branches for any period is drawn in Fig. 7.

With respect to efficiency of first period, branch two shows better performance than branches three and one. In terms of efficiency of second period, branches one and two are better performance than branch three, on the other hand over efficiency of third period branch one shows better performance than branches one and three. Based on these finding, bank management should pay more attention to inefficient branches in order to improve their overall branch network efficiency over three six-month periods.

Table 3: Descriptive Statistics.

		Range	Minimum	Maximum	Mean
Inputs and outputs of branch 1 at three six-month periods	personnel at 1st period	13.00	6.00	19.00	13.9000
	personnel at 2nd period	14.00	6.00	20.00	13.6000
	personnel at 3rd period	14.00	5.00	19.00	13.8000
	resources at 1st period	154,394.00	51,087.00	205,481.00	126,532.7000
	resources at 2nd period	124,831.00	54,840.00	179,671.00	119,343.8000
	resources at 3rd period	122,846.00	59,392.00	182,238.00	120,139.9000
	usage at 1nd period	133,978.00	21,915.00	155,893.00	73,548.3000
	usage at 2stperiod	126,823.00	19,776.00	146,599.00	71,692.8000
	usage at 3rd period	128,529.00	19,215.00	147,744.00	70,426.6000
Inputs and outputs of branch 2 at three six-month periods	personnel at 1st period	13.00	5.00	18.00	11.2000
	personnel at 2nd period	13.00	4.00	17.00	9.4000
	personnel at 3rd period	12.00	5.00	17.00	9.9000
	resources at 1st period	75,079.00	32,013.00	107,092.00	69,066.2000
	resources at 2nd period	82,938.00	35,112.00	118,050.00	71,202.6000
	resources at 3rd period	103,803.00	35,879.00	139,682.00	85,310.2000
	usage at 1nd period	42,979.00	27,756.00	70,735.00	44,179.9000
	usage at 2stperiod	44,952.00	25,658.00	70,610.00	44,498.6000
	usage at 3rd period	41,303.00	29,891.00	71,194.00	49,155.4000
Inputs and outputs of branch 3 at three six-month periods	personnel at 1st period	5.00	3.00	8.00	6.3000
	personnel at 2nd period	7.00	3.00	10.00	6.6000
	personnel at 3rd period	6.00	3.00	9.00	6.1000
	resources at 1st period	50,891.00	15,825.00	66,716.00	45,144.6000
	resources at 2nd period	42,913.00	25,548.00	68,461.00	45,924.7000
	resources at 3rd period	37,650.00	30,433.00	68,083.00	47,604.4000
	usage at 1nd period	56,026.00	7,972.00	63,998.00	28,732.6000
	usage at 2stperiod	54,183.00	9,061.00	63,244.00	29,638.1000
	usage at 3rd period	58,354.00	9,112.00	67,466.00	30,679.3000

Table 4: The results of model(3-5).

	Efficiency branches at 1st period	Efficiency branches at 2ndt period	Efficiency branches at 3rd period	Efficiency areas at 1st period	Efficiency areas at2nd period	Efficiency areas at 3rd period
areas	θ_K^1	θ_K^2	θ_K^3	θ^1	θ^2	θ^3
1				1.0000	1.0000	1.0000
Branch1	1.0000	1.0000	1.0000			
Branch 2	1.0000	1.0000	1.0000			
Branch 3	1.0000	1.0000	1.0000			
2				1.0000	1.0000	1.0000
Branch1	1.0000	1.0000	1.0000			
Branch 2	1.0000	1.0000	1.0000			
Branch 3	1.0000	1.0000	1.0000			
3				1.0000	1.0000	1.0000
Branch1	1.0000	1.0000	1.0000			
Branch 2	1.0000	1.0000	1.0000			
Branch 3	1.0000	1.0000	1.0000			
4				1.0000	1.0000	1.0000
Branch1	1.0000	1.0000	1.0000			
Branch 2	1.0000	1.0000	1.0000			
Branch 3	1.0000	1.0000	1.0000			
5				0.9710	1.0000	1.0000
Branch1	0.9702	1.0000	1.0000			
Branch 2	1.0000	1.0000	1.0000			
Branch 3	0.9531	1.0000	1.0000			
6				0.95835	1.0000	0.9423
Branch1	0.9096	1.0000	0.9680			
Branch 2	0.9813	1.0000	0.8952			
Branch 3	0.6943	1.0000	0.9310			
7				1.0000	0.96000	0.8263
Branch1	1.0000	1.0000	1.0000			
Branch 2	1.0000	0.9821	0.7225			
Branch 3	1.0000	0.8784	0.9394			

Continue Table 4.

	Efficiency branches at 1st period	Efficiency branches at 2ndt period	Efficiency branches at 3rd period	Efficiency areas at 1st period	Efficiency areas at2nd period	Efficiency areas at 3rd period
areas	θ_K^1	θ_K^2	θ_K^3	θ^1	θ^2	θ^3
8				1.0000	1.0000	1.0000
Branch1	1.0000	1.0000	1.0000			
Branch 2	1.0000	1.0000	1.0000			
Branch 3	1.0000	1.0000	1.0000			
9				1.0000	1.0000	1.0000
Branch1	1.0000	1.0000	1.0000			
Branch 2	1.0000	1.0000	1.0000			
Branch 3	1.0000	1.0000	1.0000			
10				1.0000	1.0000	1.0000
Branch1	1.0000	1.0000	1.0000			
Branch 2	1.0000	1.0000	1.0000			
Branch 3	1.0000	1.0000	1.0000			

Table 5: Average efficiency branches at three periods-whole branch.

branches	Efficiency period1	Efficiency period2	Efficiency period3
Branch 1	0.98798	1.0000	0.9968
Branch 2	0.99813	0.99821	0.96177
Branch 3	0.96474	0.98784	0.98704

Table 6: Average efficiency of areas at three six-month periods-whole period .

	areas	MEAN	
	1	1	
	2	1	
	3	1	
	4	1	
	5	0.990333	
	6	0.966883	
	7	0.928767	
	8	1	
	9	1	
	10	1	
Average efficiency of whole areas for any six-month period			
	efficiency of 1st	efficiency of 2nd	efficiency of 3rd
MEAN	0.992935	0.996	0.97686

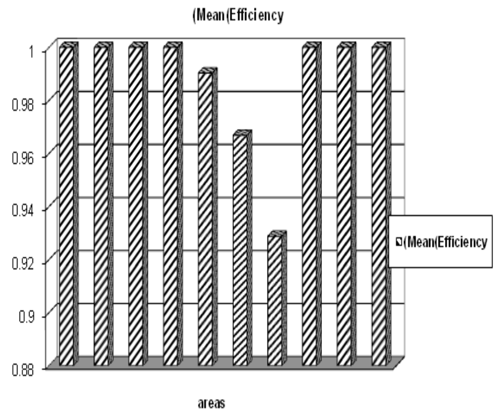

Figure 6: Average efficiency scores of areas-whole periods.

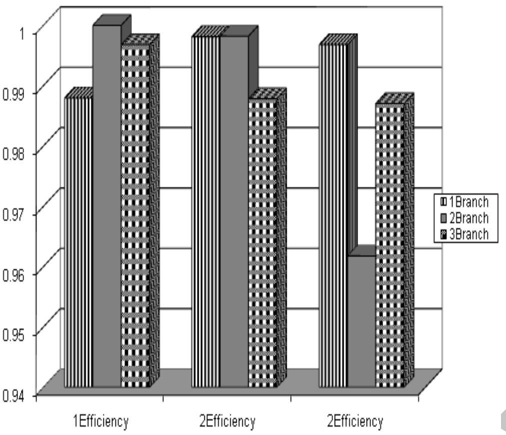

Figure 7: Average efficiency score branches for any period.

5 Conclusion

The dynamics of our technology are modeled as the choice of consuming total output in period of production or instead diverting some current production toward adding to the next period capital stock, at each T there are some exogenous inputs x^T and final outputs fy^T . final output is that part of total production y^T tat is not allocated to private iy^T investments;i.e.,

$$y^T = fy^T + iy^T$$

. For this purpose nine area of guilan Iran bank was taken in three six-month periods (2009-2010), so that each area including one exogenous input: personnel two output: resource and usages; resources as intermediate output of each period and input of next period, usages as final output, and efficiency of each area at each time period is evaluated by Fre et al. model. Therefor, by considering the subject which be studied at this paper. It was being understood that although some of the areas of bank reach to efficiency level but inefficiency at any periods was increasing in some of the areas.It must be pointed that's possible an area was be efficient in a period but would not be at next, so it's possible an area was be efficient in a period and keep it too.

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