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Fully fuzzy linear programming with inequality constraints

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Abstract

Fuzzy linear programming problem occur in many fields such as mathematical modeling, Control theory and Management sciences, etc. In this paper we focus on a kind of Linear Programming with fuzzy numbers and variables namely Fully Fuzzy Linear Programming (FFLP) problem, in which the constraints are in inequality forms. Then a new method is proposed to fine the fuzzy solution for solving (FFLP). Numerical examples are providing to illustrate the method.

Keywords : Fuzzy numbers; Linear programming; Fuzzy linear programming; Membership function; Ranking function.

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1 Introduction

THe first formulation of Fuzzy Linear Programming is proposed by Zimmermann [16]. Thereafter, many authors considered various types of the fuzzy linear programming problems and proposed several approaches for solving these problems [3, 5, 9, 12, 4, 13, 15]. In all of the above mentioned works, those cases of fuzzy linear programming have studied in which not all parts of the problem were assumed to be fuzzy, e.g., the right hand side, constraint coefficients

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or the objective function coefficients were fuzzy but the variable were not fuzzy. The fuzzy linear programming problems in which all the parameters as well as the variables are represented by fuzzy numbers are known as Fully Fuzzy Linear Programming (FFLP) problems. Some authors [1, 2, 7, 6] have proposed different methods for solving FFLP problems with inequality constraints. All of these methods are based on replacing fuzzy linear programming problems by crisp linear programming problems and then the obtained crisp linear programming is solved to find the fuzzy optimal solution of the FFLP problems with inequality constraints. In this paper we propose a new method for finding the fuzzy solution of FFLP problems with inequality constraints. This paper is organized in 5 sections. In the next section some basic definitions and arithmetic of fuzzy numbers are reviewed. In Section 3 formulation of FFLP problems and application of ranking function for solving FFLP problems are discussed. In Section 4 the new method is pro-

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posed. To illustrate the proposed method some numerical examples are presented in Section 5. Finally conclusion and some suggestion are given in Section 6.

2   Preliminaries

In this Section, we review some necessary notations and definitions of the fuzzy set theory in which will be used in this paper [8, 10, 11]:

2.1   Basic definitions

Definition 2.1 *Let* **R** *be the real line, then a fuzzy set A in* **R** *is defined to be a set of ordered pairs* $A = \{(x, \mu_A(x)) \mid x \in \mathbf{R}\}$, *where* $\mu_A(x)$ *is called the membership function for the fuzzy set. The membership function maps each element of* **R** *to a membership value between 0 and 1.*

Definition 2.2 *The support of a fuzzy set A is defined as fallow:*

$$supp(A) = \{x \in \mathbf{R} \mid \mu_A(x)\}.$$

Definition 2.3 *The core of a fuzzy set is the set of all points x in* **R** *with* $\mu_A(x) = 1$.

Definition 2.4 *A fuzzy set A is called normal if its core is nonempty. In other words, there is at least one point* $x \in \mathbf{R}$ *with* $\mu_A(x) = 1$.

Definition 2.5 *The*
α–*cut or* α–*level set of a fuzzy set is a crisp set defined by*

$$A_\alpha = \{x \in \mathbf{R} \mid \mu_A(x) > \alpha\}.$$

Definition 2.6 *A fuzzy set A on* **R** *is convex, if for any* $x, y \in \mathbf{R}$ *and* $\lambda \in [0, 1]$*, we have* $\mu_A(\lambda x + (1 - \lambda)y) \geq min\{\mu_A(x), \mu_A(y)\}$.

Definition 2.7 *A fuzzy number* $\overline{a}$ *is a fuzzy set on the real line that satisfies the condition of normality and convexity.*

Definition 2.8 *A fuzzy number* $\tilde{A} = \langle a, b, c \rangle$ *on* **R** *is said to be triangular fuzzy number, if its membership function is given as follows:*

$$\tilde{a}(x) = \begin{cases} \frac{x-a}{b-a}, & x \in [a, b] \\ \frac{x-c}{b-c}, & x \in [b, c] \\ 0, & o.w. \end{cases}$$

We denote a triangular fuzzy number $\tilde{a}$ *by three real numbers s, l and r as* $\tilde{a} = \langle a, b, c \rangle$*, whose meaning are defined in Figure 1. We also denote the set of all triangular fuzzy numbers with* **F(R)**.
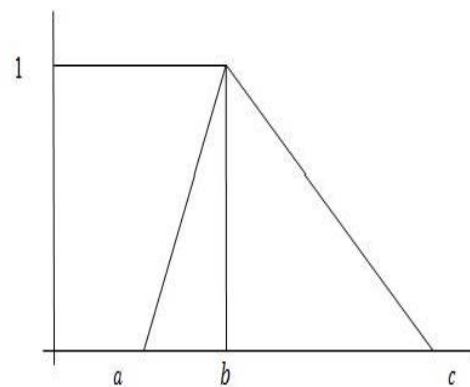

Figure 1: Triangular fuzzy numbers.

Definition 2.9 *A fuzzy triangular fuzzy number* $\tilde{A} = \langle a, b, c \rangle$ *is sad to be nonnegative triangular fuzzy number iff* $a \geq 0$.

Definition 2.10 *Tow triangular fuzzy numbers* $\tilde{A} = \langle a, b, c \rangle$ *and* $\tilde{B} = \langle e, f, g \rangle$ *are equal if and only if* $a = e$, $b = f$ *and* $c = g$.

Definition 2.11 *A ranking function is a function* $\Re : F(R) \to R$ *where* $F(R)$ *is a set of fuzzy numbers defined on set of real numbers, which maps each fuzzy number into the real line, where a natural order exists. Let* $\tilde{A} = \langle a, b, c \rangle$ *be a triangular fuzzy number then* $\Re(\tilde{A}) = \frac{a+2b+c}{4}$.

2.2 Fuzzy Arithmetic

In this Subsection arithmetic operations between two triangular fuzzy numbers defined on universal set of real numbers R, are reviewed: Let $\tilde{A} = \langle a, b, c \rangle$ and $\tilde{B} = \langle e, f, g \rangle$ be two triangular fuzzy numbers. Then:

$$\tilde{A} + \tilde{B} = \langle a + e, b + f, c + g \rangle$$

$$-\tilde{A} = \langle -c, -b, -a \rangle,$$

$$\tilde{A} - \tilde{B} = \langle a - g, b - f, c - e \rangle,$$

$$\tilde{a} \leq \tilde{b} \text{ if and only if } a \leq e, \quad b \leq f, \ c \leq g,$$

and for any $\tilde{B} = \langle e, f, g \rangle \geq 0$ we define:

$$\tilde{A} \times \tilde{B} \simeq \begin{cases} \langle ae, bf, cg \rangle, & a \geq 0 \\ \langle ag, bf, cg \rangle, & a < 0, \ c \geq 0 \\ \langle ae, bf, ca \rangle, & c < 0 \end{cases}$$

3 Linear Programming

In this Section we first define the model and then the application of ranking function for solving FFLP problems is presented.

3.1 Definition of Model

Linear programming is one of the most frequently applied operations research technique. A linear programming problem may be defined as the problem of maximizing or minimizing a linear function subject to linear constraints. The constraints may be equality or inequality. Any linear programming model representing real world situations involves a lot of parameters whose values are assigned by experts. However both experts and decision maker frequently do not precisely know the value of those parameters. It is useful to consider the knowledge of experts about the parameters as fuzzy data. In the conventional approach value of the parameters of linear programming models must be well defined and precise. However, in real world environment, this is not a realistic assumption. In the real life problems there may exists uncertainty about the parameters. In such a situation the parameters of linear programming problems may be represented as fuzzy numbers. The fuzzy linear programming problems in which all the parameters as well as the variables are represented by fuzzy numbers are known as Fully Fuzzy Linear Programming (FFLP) problems. In this paper we deal with the solution of the problems with inequality constraints. Fully fuzzy linear programming FFLP problems with m fuzzy inequality constraints and n fuzzy variables may be formulated as follows:

$$(p_1): \ maximize \ (or \ minimize) \ \tilde{C}^T \otimes \tilde{X}$$

$$s.t. \begin{cases} \tilde{A} \otimes \tilde{X} \leq \tilde{b} \\ \tilde{X} \quad is \ nonnegative \ fuzzy \ number \end{cases}$$

where $\tilde{C}^T = \left[\tilde{C}_j \right]_{1 \times n}^T$, $\tilde{A} = [\tilde{a}_{ij}]_{m \times n}$, $\tilde{b} = \left[\tilde{b}_i \right]_{m \times 1}$ and $\tilde{X} = [\tilde{x}_j]_{1 \times n}$.

3.2 Application of Ranking Function for Solving FFLP Problems

The fuzzy optimal solution of FFLP problem (p_1) will be a fuzzy number $\tilde{X}$ if it satisfies the following characteristics:

(i) $\tilde{X}$, is nonnegative fuzzy number

(ii) $\tilde{A} \oplus \tilde{X} \leq \tilde{b}$

(iii) If there exist any nonnegative fuzzy number $\tilde{X}'$

such that $\tilde{A} \oplus \tilde{X}' \leq \tilde{b}$, then $\Re(\tilde{C}^T \otimes \tilde{X}) > \Re(\tilde{C}^T \otimes \tilde{X}')$ (in case of maximizing problem) and $\Re(\tilde{C}^T \otimes \tilde{X}) < \Re(\tilde{C}^T \otimes \tilde{X}')$ (in case of minimizing problem).

Remark 3.1 *Let $\tilde{X}$ be a fuzzy optimal solution of FFLP problem (p_1). if there exists a fuzzy number $\tilde{Y}$ such that,*

(i) *$\tilde{Y}$, is non negative fuzzy number*

(ii) *$\tilde{A} \oplus \tilde{Y} \leq \tilde{b}$ fuzzy set,*

(iii) *$\Re\left(\tilde{C}^T \otimes \tilde{X}\right) = \Re\left(\tilde{C}^T \otimes \tilde{Y}\right)$*

then $\tilde{Y}$ is said to be an alternative fuzzy optimal solution of (p_1).

4 Proposed Method to Find the Fuzzy Optimal Solution of FFLP Problems

In this Section a new method is proposed to find the fuzzy optimal solution of FFLP problem [1]. The Steps of the proposed method are as follows:

Step 1: Substituting $\tilde{C}^T = \left[\tilde{C}_j\right]^T_{1 \times n}$, $\tilde{A} = [\tilde{a}_{ij}]_{m \times n}$, $\tilde{b} = \left[\tilde{b}_i\right]_{m \times 1}$ and $\tilde{X} = [\tilde{x}_j]_{1 \times n}$, the FFLP problem may be written as follows:

$$maximize \ (or \ minimize) \ \sum_{j=1}^n \tilde{C}_j \otimes \tilde{x}_j$$

$$s.t. \begin{cases} \sum_{j=1}^n \tilde{a}_{ij} \otimes \tilde{x}_j \leq \tilde{b}_i, \ \forall i=1,\ldots,m \\ \\ \tilde{x}_j \ \ is \ nonnegative \ fuzzy \ number \end{cases}$$
$$(4.1)$$

Step 2: If all the parameters $\tilde{C}_j$, $\tilde{a}_{ij}$, $\tilde{b}_i$ and $\tilde{x}_j$, are represented by triangular fuzzy numbers $\langle p_j, q_j, r_j \rangle$, $\langle a_{ij}, b_{ij}, c_{ij} \rangle$, $\langle b_i, g_i, h_i \rangle$ and $\langle x_j, y_j, z_j \rangle$ respectively then the FFLP problem obtained in Step 1, may be written as:

$$max \ (or \ min) \ \sum_{j=1}^n \langle p_j, q_j, r_j \rangle \otimes \langle x_j, y_j, z_j \rangle$$

$$s.t. \begin{cases} \sum_{j=1}^n \langle a_{ij}, b_{ij}, c_{ij} \rangle \otimes \langle x_j, y_j, z_j \rangle \\ \quad\quad \leq \langle b_i, g_i, h_i \rangle, \ \ \forall i=1,\ldots,m \\ \\ \langle x_j, y_j, z_j \rangle \ \ is \ nonnegative \ fuzzy \ number \end{cases}$$
$$(4.2)$$

Step 3: Assuming $\langle a_{ij}, b_{ij}, c_{ij} \rangle \otimes \langle x_j, y_j, z_j \rangle = \langle m_{ij}, n_{ij}, o_{ij} \rangle$, the FFLP obtained in Step 2, may be written as follows:

$$max \ (or \ min) \ \Re\left(\sum_{j=1}^n \langle p_j, q_j, r_j \rangle \otimes \langle x_j, y_j, z_j \rangle \right)$$

$$s.t. \begin{cases} \sum_{j=1}^n \langle m_{ij}, n_{ij}, o_{ij} \rangle \\ \quad\quad \leq \langle b_i, g_i, h_i \rangle, \ \forall i=1,\ldots,m \\ \\ \langle x_j, y_j, z_j \rangle \ \ is \ nonnegative \ fuzzy \ number \end{cases}$$
$$(4.3)$$

Step 4: Using arithmetic operations defined in Subsection 2.2 and Definition 2.4 the fuzzy linear programming problem obtained in Step 3, is convert into the following problem:

$$max \ (or \ min) \ \Re\left(\sum_{j=1}^n \langle p_j, q_j, r_j \rangle \otimes \langle x_j, y_j, z_j \rangle \right)$$

$$s.t. \begin{cases} \sum_{j=1}^{n} m_{ij} \le b_i, \ \forall i = 1, \dots, m \\[2mm] \sum_{j=1}^{n} n_{ij} \le g_i, \ \forall i = 1, \dots, m \\[2mm] \sum_{j=1}^{n} o_{ij} \le h_i, \ \forall i = 1, \dots, m \\[2mm] y_j - x_j \ge 0, z_j - y_j \ge 0 \end{cases} \quad (4.4)$$

Step 5: Find the optimal solution x_j, y_j and z_j by solving the problem 4.4 in Step 4.

Step 6: Find the fuzzy optimal solution by putting the values of x_j, y_j and z_j in $\tilde{x}_j = \langle x_j, y_j, z_j \rangle$.

Step 7: Find the fuzzy optimal value by putting $\tilde{x}_j$ in $\sum_{j=1}^{n} \tilde{C}_j \otimes \tilde{x}_j$.

5 Numerical Examples

In this Section, we utilize some examples to illustrate the solution method proposed in this paper. Then compare our method to Kumar, Kaur and Singh approach [7].

Example 5.1 *Let us consider the following FFLP problem and solve it by the proposed method:*

$$\max \ Z = (\langle 1, 2, 3 \rangle \otimes \tilde{x}_1 \oplus \langle 2, 3, 4 \rangle \otimes \tilde{x}_2)$$

$$s.t. \begin{cases} \langle 0, 1, 2 \rangle \otimes \tilde{x}_1 \oplus \langle 1, 2, 3 \rangle \otimes \tilde{x}_2 \preccurlyeq \langle 1, 10, 27 \rangle \\[2mm] \langle 1, 2, 3 \rangle \otimes \tilde{x}_1 \oplus \langle 0, 1, 2 \rangle \otimes \tilde{x}_2 \preccurlyeq \langle 2, 11, 28 \rangle \end{cases}$$
$$(5.5)$$

Where $\tilde{x}_1$ and $\tilde{x}_2$ are non-negative triangular fuzzy numbers. Let $\tilde{x}_1 = \langle x_1, y_1, z_1 \rangle$ and $\tilde{x}_2 = \langle x_2, y_2, z_2 \rangle$. Then the given FFLP problem may be written as follows:

$$\max Z =$$

$$(\langle 1, 2, 3 \rangle \otimes \langle x_1, y_1, z_1 \rangle \oplus \langle 2, 3, 4 \rangle \otimes \langle x_2, y_2, z_2 \rangle)$$

$$s.t. \begin{cases} \langle 0, 1, 2 \rangle \otimes \langle x_1, y_1, z_1 \rangle \oplus \langle 1, 2, 3 \rangle \\ \qquad \otimes \langle x_2, y_2, z_2 \rangle \preccurlyeq \langle 1, 10, 27 \rangle \\[2mm] \langle 1, 2, 3 \rangle \otimes \langle x_1, y_1, z_1 \rangle \oplus \langle 0, 1, 2 \rangle \\ \qquad \otimes \langle x_2, y_2, z_2 \rangle \preccurlyeq \langle 2, 11, 28 \rangle \end{cases}$$
$$(5.6)$$

According to Step 3 of proposed method, the above problem can be written as follows:

$$\max Z = \Re \left(\langle x_1, 2y_1, 3z_1 \rangle \oplus \langle 2x_2, 3y_2, 4z_2 \rangle \right)$$

$$s.t. \begin{cases} \langle 0, y_1, 2z_1 \rangle \oplus \langle x_2, 2y_2, 3z_2 \rangle \preccurlyeq \langle 1, 10, 27 \rangle \\[2mm] \langle x_1, 2y_1, 3z_1 \rangle \oplus \langle 0, y_2, 2z_2 \rangle \preccurlyeq \langle 2, 11, 28 \rangle \end{cases}$$
$$(5.7)$$

Using the ranking function that is addressed in Section 2.1, the FFLP obtained in current step, may be written as follows:

$$\max Z = \Re \ \langle x_1 + 2x_2, 2y_1 + 3y_2, 3z_1 + 4z_2 \rangle$$

$$s.t. \begin{cases} \langle x_2, y_1 + 2y_2, 2z_1 + 3z_2 \rangle \preccurlyeq \langle 1, 10, 27 \rangle \\[2mm] \langle x_1, 2y_1 + y_2, 3z_1 + 2z_2 \rangle \preccurlyeq \langle 2, 11, 28 \rangle \end{cases}$$
$$(5.8)$$

The problem (5.9) and the fuzzy arithmetic that is mentioned in Section 2.2 yield:

$$\max Z = \frac{x_1 + 2x_2 + 4y_1 + 6y_2 + 3z_1 + 4z_2}{4}$$

$$s.t. \begin{cases} x_2 \leq 1 \\ y_1 + 2y_2 \leq 10 \\ 2z_1 + 3z_2 \leq 27 \\ x_1 \leq 2 \\ 2y_1 + y_2 \leq 11 \\ 3z_1 + 2z_2 \leq 28 \\ x_1, x_2, y_1, y_2, z_1, z_2 \geq 0 \end{cases} \quad (5.9)$$

This problem is a conventional linear programming problem. The optimal solution of this problem is obtained as follows:

$$\langle x_1, y_1, z_1 \rangle = \langle 2, 4, 6 \rangle$$
$$\langle x_2, y_2, z_2 \rangle = \langle 1, 3, 5 \rangle$$

Thus $\tilde{x}_1 = \langle 2, 4, 6 \rangle$ and $\tilde{x}_2 = \langle 1, 3, 5 \rangle$ is the fuzzy optimal solution of FFLP problem (5.5). By substituting these fuzzy numbers in objective function the fuzzy optimal value is obtained as follows:

$$Z^* = \langle 1, 2, 3 \rangle \otimes \langle 2, 4, 6 \rangle \oplus \langle 2, 3, 4 \rangle \otimes \langle 1, 3, 5 \rangle$$
$$= \langle 2, 8, 18 \rangle \oplus \langle 2, 9, 20 \rangle = \langle 4, 17, 38 \rangle$$

Therefore $Z^* = \langle 4, 17, 38 \rangle$ is the fuzzy optimal solution of problem (5.5).

Example 5.2 *Now consider FFLP problem that is defined as follows:*

$$\max Z = (\langle 1, 6, 9 \rangle \otimes \tilde{x}_1 \oplus \langle 2, 2, 8 \rangle \otimes \tilde{x}_2)$$

$$s.t. \begin{cases} \langle 0, 1, 1 \rangle \otimes \tilde{x}_1 \oplus \langle 2, 2, 3 \rangle \otimes \tilde{x}_2 \succcurlyeq \langle 4, 7, 14 \rangle \\ \langle 2, 2, 3 \rangle \otimes \tilde{x}_1 \oplus \langle -1, 4, 4 \rangle \otimes \tilde{x}_2 \preccurlyeq \langle -4, 14, 22 \rangle \\ \langle 2, 3, 4 \rangle \otimes \tilde{x}_1 \ominus \langle 1, 2, 3 \rangle \otimes \tilde{x}_2 = \langle -12, -3, 6 \rangle \end{cases} \quad (5.10)$$

Where $\tilde{x}_1$ and $\tilde{x}_2$ are non-negative triangular fuzzy numbers. Let $\tilde{x}_1 = \langle x_1, y_1, z_1 \rangle$ and $\tilde{x}_2 = \langle x_2, y_2, z_2 \rangle$. Then the given FFLP problem may be written as follows:

$$\max Z =$$

$$(\langle 1, 6, 9 \rangle \otimes \langle x_1, y_1, z_1 \rangle \oplus \langle 2, 2, 8 \rangle \otimes \langle x_2, y_2, z_2 \rangle)$$

$$s.t. \begin{cases} \langle 0, 1, 1 \rangle \otimes \langle x_1, y_1, z_1 \rangle \oplus \langle 2, 2, 3 \rangle \\ \quad \otimes \langle x_2, y_2, z_2 \rangle \succcurlyeq \langle 4, 7, 14 \rangle \\ \langle 2, 2, 3 \rangle \otimes \langle x_1, y_1, z_1 \rangle \oplus \langle -1, 4, 4 \rangle \\ \quad \otimes \langle x_2, y_2, z_2 \rangle \preccurlyeq \langle -4, 14, 22 \rangle \\ \langle 2, 3, 4 \rangle \otimes \langle x_1, y_1, z_1 \rangle \ominus \langle 1, 2, 3 \rangle \\ \quad \otimes \langle x_2, y_2, z_2 \rangle = \langle -12, -3, 6 \rangle \end{cases} \quad (5.11)$$

According to Step 3 of proposed method, the above problem can be written as follows:

$$\max Z = \Re \ (\langle x_1, 6y_1, 9z_1 \rangle \oplus \langle 2x_2, 2y_2, 8z_2 \rangle)$$

$$s.t. \begin{cases} \langle 0, y_1, z_1 \rangle \oplus \langle 2x_2, 2y_2, 3z_2 \rangle \succcurlyeq \langle 4, 7, 14 \rangle \\ \langle 2, 2, 3 \rangle \otimes \langle x_1, y_1, z_1 \rangle \oplus \langle -1, 4, 4 \rangle \\ \quad \otimes \langle 2x_1, 2y_1, 3z_1 \rangle \oplus \langle -1z_2, 4y_2, 4z_2 \rangle \\ \qquad \preccurlyeq \langle -4, 14, 22 \rangle \\ \langle 2x_1, 3y_1, 4z_1 \rangle \ominus \langle x_2, 2y_2, 3z_2 \rangle \\ \qquad = \langle -12, -3, 6 \rangle \end{cases} \quad (5.12)$$

Using the ranking function that is addressed in Section (2.1) and fuzzy arithmetic that is mentioned in Section (2.2), the FFLP obtained in current step, may be written as follows:

$$\max Z = \frac{x_1 + 2x_2 + 12y_1 + 4y_2 + 9z_1 + 8z_2}{4}$$

$$s.t. \begin{cases} 2x_2 \geq 4 \\ y_1 + 2y_2 \geq 7 \\ y_1 + 2y_2 \geq 7 \\ z_1 + 3z_2 \geq 14 \\ 2x_1 - z_2 \leq -4 \\ 2y_1 + 4y_2 \leq 14 \\ 3z_1 + 4z_2 \leq 22 \\ 2x_1 - 3z_2 = -12 \\ 3y_1 - 2y_2 = -3 \\ 4z_1 - x_2 = 6 \\ x_1, x_2, y_1, y_2, z_1, z_2 \geq 0 \end{cases} \qquad (5.13)$$

This problem is a conventional linear programming problem. The optimal solution of this problem is obtained as follows:

$$\langle x_1, y_1, z_1 \rangle = \langle 0, 1, 2 \rangle$$

$$\langle x_2, y_2, z_2 \rangle = \langle 2, 3, 4 \rangle$$

Thus $\tilde{x}_1 = \langle 0, 1, 2 \rangle$ and $\tilde{x}_2 = \langle 2, 3, 4 \rangle$ is the fuzzy optimal solution of FFLP problem (5.10). By substituting these fuzzy numbers in objective function the fuzzy optimal value is obtained as follows:

$$Z^* = \langle 1, 6, 9 \rangle \otimes \langle 0, 1, 2 \rangle \oplus \langle 2, 2, 8 \rangle \otimes \langle 2, 3, 4 \rangle$$

$$= \langle 0, 6, 18 \rangle \oplus \langle 4, 6, 32 \rangle = \langle 4, 12, 50 \rangle$$

Therefore $Z^* = \langle 4, 12, 50 \rangle$ is the fuzzy optimal solution of problem (5.10). In [7], authors solve the FFLP problem (5.5) and (5.10), by a different approach and derived the same results. They convert each of these FFLP problem with inequality constraints to a FFLP problem with equality constraints by introducing slack variable s_i. Then these crisp linear programming problems should be solved to find the fuzzy optimal solution of the FFLP problems with inequality constraints 5-1 and 5-2, respectively.

6 Conclusion

In this paper a new method is proposed to find the fuzzy optimal solution of FFLP problems with inequality constraint by representing all parameters as triangular fuzzy numbers. By using the proposed method the fuzzy optimal solution of FFLP with inequality constraints, occurring in real life situation, can be easily obtained. To illustrate the proposed method some numerical example are solved and the obtained results are compared with other approaches.

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