Available online at http://ijim.srbiau.ac.ir/
Int. J. Industrial Mathematics (ISSN 2008-5621)
Vol. 5, No. 4, 2013 Article ID IJIM-00350, 9 pages
Research Article

Groups performance ranking based on inefficiency sharing

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Abstract

In the real world there are groups which composed of independent units. The conventional data envelopment analysis(DEA) model treats groups as units, ignoring the operation of individual units within each group.The current paper, investigates parallel system network approach proposed by Kao and modifies it. As modified Kao' model is more eligible to recognize efficient groups, a new ranking method is proposed based on a model which calculates efficiencies with additional constraint that made model share constant inefficiency among groups.To show advantages, modifies model is applied to efficiency calculation of both artificial and real groups and results is compared with conventional DEA model and parallel system network model as well.Finally it is shown by tow numerical and empirical examples that efficient groups recognized by modified model how can be ranked according to proposed ranking model.

Keywords : Data Envelopment Analysis; Group Ranking; Network DEA; Parallel Systems Efficiency; Efficient Groups.

1 Introduction

Technique widely applied to measure relative efficiency of a set of competitive systems or decision making units (DMUs) which utilize same inputs to produce same outputs is data envelopment analysis (DEA). Suppose there are n DMUs, the j th DMU utilize m inputs $X_{ij}, i = 1, \dots, m$ to produce s outputs $Y_{rj}, r = 1, \dots, s$. This means inputs and outputs of each DMU can be expressed by two vector of X And Y. A pair of such vectors (X, Y)that can be considered as a point in $(m + s)$ dimensional linear vector space,is called an activity. Data envelopment analysis essentially is based on comparison within set of feasible activities. Set of feasible activities that is called production possibility set (PPS) can be postulated as follows [3]

1. The observed activities belong to PPS. (observation inclusion assumption)

2. If the activity (X, Y)belong to PPS then activity $(\lambda X, \lambda Y)$ belong to PPS. (constant return to scale assumption)

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3. For an activity (X, Y) in PPS, any activity $(\bar{X}, \bar{Y})$ with $\bar{Y} \leq Y$ and $\bar{X} \geq X$ is included in PPS.(plausibility assumption)

4. Any linear combination of activities in PPS belong to PPS. (convexity assumption)

PPS can be defined satisfying above assumption by

$$PPS = \left\{ (X, Y) \vdots X \geq \sum_{i=1}^{m} \lambda_j X_{ij}, Y \leq \sum_{r=1}^{s} \lambda_j Y_{rj} \right\} \tag{1.1}$$

Calculating efficiency of under evaluation group theoretically is equivalent to solving following program [3]

$$\text{Max } \theta_k = \frac{\sum_{r=1}^{s} u_r Y_{rk}}{\sum_{i=1}^{m} v_i X_{ik}} \tag{1.2}$$
$$\frac{\sum_{r=1}^{s} u_r Y_{rj}}{\sum_{i=m}^{m} v_i X_{ij}} \leq 1 \qquad u_r, v_i \geq 0$$

this model can be interpreted as a mechanism to find most favorable weights u_r and v_i to be applied to the r th output and i th input for under evaluation DMU. Note that objective function in above model is associated with one DMU and for calculating efficiency of each DMU model have to be run repeatedly.Possibility of improving can be investigated in aspects of reducing inputs and increasing outputs, separately according to input oriented and output oriented models respectively. A linear form of above model called CCR input oriented model can be presented as follows [3]

$$\text{Max } \theta_k = \sum_{r=1}^{s} u_r Y_{rk} \tag{1.3}$$
$$\sum_{i=1}^{m} v_i X_{ik} = 1$$
$$\sum_{r=1}^{s} u_r Y_{rj} - \sum_{i=m}^{m} v_i X_{ij} \leq 0$$
$$u_r, v_i \geq 0$$

Conventional DEA model consider DMUs as a whole when measuring efficiency and consequently ignores internal structure of those. Such approach to efficiency calculation has been criticized from view point of weakness in recognizing origins of inefficiency within DMU. Fare and grosskopf [4], Fare and et al [5], Lewis and Setxon [8], Prieto and Zoflo [9], Tone and Tsutsui [10] Kao [7] [6] attempt to take into consideration the internal structure of a whole system. This approach widely is called network DEA.

In real world there are cases that a DMU essentially is group of units. A typical example is a firm with several plants or a bank with many branches that each of them operates independently. Castelli et al [1] Cook and Green [2] defined this problem as an hierarchical structure. kao [7] especially concentrated on this case and presented a network DEA model for parallel systems. The structure of this paper is organized as follows. In the next Section parallel system network DEA proposed by Kao is investigated. In Section 3 modified version of model is proposed that is aimed to be more consistent with initial idea of DEA model. As modified model is more eligible to recognize groups efficient, a new ranking method based on calculating efficiency along with sharing inefficiency among groups is proposed in Section 4. In Section 5 for the purpose of comparing, efficiency scores calculated from conventional DEA model, parallel network DEA model, modified model are presented for two numerical examples. It is shown efficient groups how can be ranked according to proposed ranking method. Finally section 6 is allocated to conclusion.

2 Parallel system network DEA

Assuming n groups each of their inputs and outputs is the sum of all its units, Kao [7] presents general case of parallel system as a network. In this framework it is supposed each unit $p = 1, \ldots, q_k$ belong to k th group converts inputs $x_{ik}^{p}, i = 1, \ldots, m$ into outputs $y_{rk}^{p}, r = 1, \ldots, s$ independently, and sum of all inputs over p units $\sum_{p=1}^{q_k} x_{ik}^{p}$ and all outputs over p units $\sum_{p=1}^{q_k} y_{rk}^{p}$

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are the input Xik and output Y_{rk} of k th group respectively. First of all kao [7] suggests to measure efficiency from the view point of inefficiency since as a matter of fact inefficiency is a compliment of efficiency. According to model 1.3 efficiency of k th group is equivalent to $Max \sum_{r=1}^{s} u_r Y_{rk}$ and can be measured by $Min 1 - \sum_{r=1}^{s} u_r Y_{rk}$ which is equal to slack S_k in following program.

$$\text{Max } \theta_k = \sum_{r=1}^{s} u_r Y_{rk} \tag{2.4}$$
$$(1) \sum_{i=1}^{m} v_i X_{ik} = 1$$
$$(2) \sum_{r=1}^{s} u_r Y_{rk} - \sum_{i=m}^{m} v_i X_{ik} + S_k = 0$$
$$(3) \sum_{r=1}^{s} u_r Y_{rj} - \sum_{i=m}^{m} v_i X_{ij} + S_j = 0 \quad j \neq k$$
$$u_r, v_i \geq 0$$

Note that model (1.3) is equivalent to above program, just the constraint associated with under evaluation group has been written separately and slacks have been added to model. equivalence of $Min 1 - \sum_{r=1}^{s} u_r Y_{rk}$ to S_k easily can be shown by dividing constraint (2) by $\sum_{i=1}^{m} v_i X_{ik}$ that becomes

$$1 - \frac{\sum_{r=1}^{s} u_r Y_{rk}}{\sum_{i=1}^{m} v_i X_{ik}} = S_k \tag{2.5}$$

Note that according to constraint (1) denominator of above fraction is equal to one thus

$$1 - \sum_{r=1}^{s} u_r Y_{rk} = S_k \tag{2.6}$$

Substituting each input and output of under evaluation group by sum of those of all its units, kao presents constraint associated with k th group in model (2.4) as follows

$$\sum_{r=1}^{s} u_r \left(\sum_{p=1}^{q_k} y_{rk}^p \right) - \sum_{i=1}^{m} v_i \left(\sum_{p=1}^{q_k} x_{ik}^p \right) + S_k = 0 \tag{2.7}$$

This is equivalent to

$$\sum_{p=1}^{q_k} \left(\sum_{r=1}^{s} u_r y_{rk}^p - \sum_{i=1}^{m} v_i x_{ik}^p \right) + S_k = 0 \tag{2.8}$$

Note that phrase in the parentheses represents transformation mechanism of p th unit. Kao denotes the slack associated with p th unit s_k^p and supposes the total slack of group $S_k$can be allocated to its q_k units as follows

$$S_k = \sum_{p=1}^{q_k} s_k^p \tag{2.9}$$

Thus last equation becomes

$$\sum_{p=1}^{q_k} \left(\sum_{r=1}^{s} u_r y_{rk}^p - \sum_{i=1}^{m} v_i x_{ik}^p + s_k^p \right) = 0 \tag{2.10}$$

Since each quantity in the parentheses is equal to zero kao derives a set of q_k constraints.

$$\sum_{r=1}^{s} u_r y_{rk}^p - \sum_{i=1}^{m} v_i x_{ik}^p + s_k^p = 0 \quad p = 1, \cdots, q_k \tag{2.11}$$

By the same token the constraints associated with each group rather than k th group is replaced by corresponding to its q_j units, consequently following program can be applied to measure inefficiency of kth group and its q_k units.

$$\text{Min } \sum_{p=1}^{q_k} s_k^p \tag{2.12}$$
$$\sum_{i=1}^{m} v_i X_{ik} = 1$$
$$\sum_{r=1}^{s} u_r y_{rk}^p - \sum_{i=1}^{m} v_i x_{ik}^p + s_k^p = 0$$
$$\sum_{r=1}^{s} u_r y_{rj}^p - \sum_{i=1}^{m} v_i x_{ij}^p + s_j^p = 0 j \neq k$$

In next Section this model is investigated from interpretation view point and a modified version is proposed.

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3 Modified model

In Kao's model summation of constraints corresponding to each group is equivalent to constraint associated to that, thus model implies group constraints. But in conventional DEA model each constraint is associated with one of an observed activities that based on observation inclusion assumption is a member of PPS. Substitution of constraints in this manner lets comparison be accomplished in PPS, constructed by units instead of by groups. As a matter of fact from interpretation viewpoint kao's model can be criticized arguing that the model reduce aim of evaluation that is comparison among groups to compare units.

Computational consequence of kao's model is weight allocation so that fraction of weighted outputs and inputs for each unit can't exceed one. Considering aim of evaluation that is comparison among groups not units constraints associated with units at least for them that don't belong to under evaluation group are not required at all. As an explanation note that DEA model is aimed to find most favorable weights in calculating efficiency of under evaluation DMU, considered here as a group and value corresponding to fraction of weighted outputs and inputs for units not belong to under evaluation group doesn't mean efficiency. Unfortunately hierarchical DEA model of Cook and Green [2] can be criticized in the same way. By replacing initial constraints corresponding to not under evaluation groups instead of those associated with units that belong them,Kao's model can be modified as follows

$$\begin{aligned} \text{Min} \; & \sum_{p=1}^{q_k} s_k^p \\ & \sum_{i=1}^{m} v_i X_{ik} = 1 \\ & \sum_{r=1}^{s} u_r y_{rk}^p - \sum_{i=1}^{m} v_i x_{ik}^p + s_k^p = 0 \quad p = 1, \cdots, q_k \\ & \sum_{r=1}^{s} u_r Y_{rj} - \sum_{i=m}^{m} v_i X_{ij} \leq 0 \quad j \neq k \\ & u_r, v_i \geq 0 \end{aligned} \tag{3.13}$$

Constraints in above modified model are weaker than those Kao's model, consequently efficiency scores will be larger than those in Kao's model. Therefore modified model is more eligible to recognize some of groups as efficient groups. Logically a ranking method can be assessed useful.

4 Proposed ranking method

It has been shown previously that inefficiency of under evaluation group in equivalent to slack S_k of constraint (3) in model (2.4). By dividing constraint (3) that is associated with other groups in that model by $\sum_{i=1}^{m} v_i X_{ij}$ inefficiencies of not under evaluation groups are calculated as follows

$$1 - \frac{\sum_{r=1}^{s} u_r Y_{rj}}{\sum_{i=m}^{m} v_i X_{ij}} = \frac{S_j}{\sum_{i=m}^{m} v_i X_{ij}} \tag{4.14}$$

Supposing summation of whole inefficiencies for all of groups be equal to one results in

$$\sum_{\substack{j=1 \\ j \neq k}}^{n} \frac{S_j}{\sum_{i=m}^{m} v_i X_{ij}} + S_k = 1 \tag{4.15}$$

Adding up above constraint to model leads to non-linearity, thus following change of variable is applied.

$$\sum_{i=m}^{m} v_i X_{ij} = \frac{1}{t_j} \tag{4.16}$$

Inequality of t_j to zero will be discussed later. Applying above change of variable is required thus

$$\sum_{\substack{j=1 \\ j \neq k}}^{n} t_j S_j + S_k = 1 \tag{4.17}$$

But there exist $\sum_{i=m}^{m} v_i X_{ij}$ in constraints associated with not under evaluation groups and applying above change of variable results in

$$\sum_{r=1}^{s} u_r t_j Y_{rj} + t_j S_j = 1 \tag{4.18}$$

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Thus adding up new constraint converts model (2.4) into following nonlinear program.

$$\text{Max } \theta_k = \sum_{r=1}^{s} u_r Y_{rk} \tag{4.19}$$
$$(1) \sum_{i=1}^{m} v_i X_{ik} = 1$$
$$(2) \sum_{r=1}^{s} u_r Y_{rk} - \sum_{i=m}^{m} v_i X_{ik} + S_k = 0$$
$$(3) \sum_{r=1}^{s} u_r t_j Y_{rj} + t_j S_j = 1 \quad j \neq k$$
$$(4) \sum_{\substack{j=1 \\ j\neq k}}^{n} t_j S_j + S_k = 1$$
$$u_r, v_i \geq 0$$

Note that discussed change of variable has been applied in constraint (3). Now it is required to assure $t_j \neq 0$. Suppose $t_j = 0$ thus left side of constraint (3) in above model becomes equal to zero while right side is equal to one and this is a contradiction thus $t_j = 0$ is impossible. Setting $t_j S_j = \acute{S}_j$ and $u_r t_j = u_{rj}$ above program converts into linear program but as can be considered model lets independent outputs weights allocation for all groups except under evaluation group. Such weight allocation as permits each group being evaluated according to its own output weights can be pointed as weakness of model mathematically, however from computational view point it doesn't matter, as constraint added to model restrict power choice of model in weights allocation strongly.

As mentioned above output weights u_r that is remained in constraint associated with k th group is not independent of u_{rj} that exists in other constraints.To make it independent u_r can be substituted by u_{rk} that is assumed equal to $u_r t_k$ but in spite of t_j inequality to zero for t_k is not required necessarily. Applying above changes model (4.19) converts into following program.

$$\text{Max } \theta_k = \sum_{r=1}^{s} u_{rk} Y_{rk} \tag{4.20}$$
$$\sum_{i=1}^{m} v_i X_{ik} = 1$$
$$\sum_{r=1}^{s} u_{rk} Y_{rk} - \sum_{i=m}^{m} v_i X_{ik} + S_k = 0$$
$$\sum_{r=1}^{s} u_{rj} Y_{rj} + \acute{S}_j = 1 \quad j \neq k$$
$$\sum_{\substack{j=1 \\ j\neq k}}^{n} \acute{S}_j + S_k = 1$$
$$u_r, v_i \geq 0$$

By the same token in modified Kao's model above program converts into following form

$$\text{Min } \sum_{p=1}^{q_k} s_k^p \tag{4.21}$$
$$(1) \sum_{i=1}^{m} v_i X_{ik} = 1$$
$$(2) \sum_{r=1}^{s} u_{rk} y_{rk}^p - \sum_{i=1}^{m} v_i x_{ik}^p + s_k^p = 0$$
$$(3) \sum_{r=1}^{s} u_{rj} Y_{rj} + \acute{S}_j = 1 \quad j \neq k$$
$$(4) \sum_{\substack{j=1 \\ j\neq k}}^{n} \acute{S}_j + \sum_{p=1}^{q_k} s_k^p = 1$$
$$u_r, v_i \geq 0$$

Note that $\sum_{p=1}^{q_k} s_k^p = S_k$ is considered in last constraint of model. Above model is able to calculate efficiency scores for groups provided that summation of group inefficiencies was assumed to be equal to one. Following theorems clarify properties of this model.

Theorem 4.1 *model is always feasible.*

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Table 1: Efficiency results of assumed Groups

groups	Conventional model	Kao's model	modified model
s1	0.98	0.74	0.84
s2	0.85	0.62	0.81
s3	1	0.75	0.90
s4	1	0.78	0.78
s5	0.93	0.64	0.71

Table 2: Efficiency results of MELLAT BANK supervisory sections

S	(1)	(2)	(3)	(4)	S	(1)	(2)	(3)	(4)
s1	1	0.48	0.77	0.83	s21	1	0.18	0.44	0.34
s2	1	0.16	0.63	0.75	s22	0.90	0.20	0.73	0.83
s3	1	0.19	0.63	0.66	s23	0.67	0.11	0.63	0.72
s4	0.94	0.18	0.57	0.70	s24	1	0.14	0.54	0.63
s5	1	0.29	0.64	0.63	s25	0.75	0.13	0.54	0.60
s6	1	0.35	0.63	0.64	s26	1	0.17	0.45	0.46
s7	1	0.28	1	0.67	s27	1	0.23	0.72	0.80
s8	0.84	0.18	0.84	1	s28	1	0.23	0.62	0.70
s9	1	0.89	1	1	s29	1	0.15	0.64	0.81
s10	0.79	0.15	0.66	0.65	s30	0.71	0.13	1	0.53
s11	1	0.15	0.57	0.53	s31	1	0.29	0.56	0.56
s12	1	0.23	0.50	0.74	s32	1	0.21	1	0.67
s13	0.85	0.24	0.50	0.51	s33	0.81	0.17	0.64	0.67
s14	1	0.12	0.78	0.82	s34	1	0.15	0.68	0.76
s15	1	0.13	0.53	0.60	s35	0.82	0.19	0.59	0.71
s16	0.85	0.15	0.74	0.76	s36	1	0.13	1	0.60
s17	1	0.15	0.72	0.94	s37	1	0.17	0.70	0.72
s18	0.89	0.16	0.58	0.59	s38	1	0.24	0.71	0.75
s19	1	0.25	0.71	0.82	s39	1	0.17	1	0.66
s20	0.82	0.14	0.50	0.72	s40	1	0.90	1	0.99

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It is enough to present a feasible solution. It is shown that following solution is feasible.

$$\underset{i\neq 1}{v_i} = 0, v_1 = \frac{1}{X_{1k}}, u_{rk} = 0 \quad \underset{j\neq 1}{u_{rj}} = 0, u_{1j} = \frac{1}{Y_{1j}}$$

X_{1k} and Y_{1j} represent any nonzero input and output of under evaluation group respectively and subscripts 1 for input and output are independent of each other. Above solution satisfies constraint(1)and setting in constraints (2) and (3) leads to following results:

$$s_k^p = \frac{x_{ik}^p}{X_{ik}} \quad p = 1, ..., q_k$$
$$\acute{S}_j = 0 \quad j = 1, ..., n$$

Consequently solution satisfies last constraint of model.

Theorem 4.2 *Objective function value is equal to or less than one.*

Feasible solution presented in last theorem leads to objective function value equal to one considering the direction of objective function that is minimazing theorem is proved.

Theorem 4.3 *Efficiency attributed to j th group by model used for calculating efficiency of k th groups is equal to or less than one.*

As $\acute{S}_j$ in Constraint(3) of model (4.21) is assumed greater than or equal to zero following inequality

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is resulted.

$$\sum_{r=1}^{s} u_{rj} Y_{rj} \leq 1 \tag{4.22}$$

According to definition, efficiency of j th group can be presented as follows

$$\theta_j = \frac{\sum_{r=1}^{s} u_r Y_{rj}}{\sum_{i=1}^{m} v_i X_{ij}} \tag{4.23}$$

By changing of variable as mentioned in equation (4.16) efficiency of j th group becomes

$$\theta_j = \sum_{r=1}^{s} u_r t_j Y_{rj} \tag{4.24}$$

Note that $u_r t_j = u_{rj}$is assumed, thus according to inequality (4.22) theorem is proved.

5 Numerical examples

First of all modified model is examined by applying to calculate efficiencies of some assumed groups and comparing results with efficiency calculated from conventional and kao 's model. Assumed groups constructed with data presented in DEA example of GAMS library to be accessible for viewers. Data table in that example consists of twenty units, two inputs and three outputs. Units for the purpose of group's efficiencies evaluation are allocated to five groups according to following pattern. Units' 1-4, 5-8, 9-11, 12-16, 17-20 is assumed that belong to group 1-5 respectively. Table 1 presents the results

Data associated with forty supervisory sections of MELLAT bank in Iran as real example is applied to show differences among conventional DEA, Kao's model, modified Kao's model results. Proposed ranking model is applied to rank efficient groups recognized by modified parallel system network model as well. Each supervisory section consists of several branches which work around country. Evaluation criteria consist of three inputs included Personnel cost, Cash Dividends Payable, Balance of facilities and four outputs included Deposits, Facilities and two kind of incomes. Results are stated in Table 2 according to following pattern, the columns (1),(2),(3),(4) represent efficiencies calculated from conventional DEA model, parallel network DEA model, modified parallel network DEA and proposed ranking model respectively. As can be seen efficiency scores calculated from modified model is larger than those calculated from kao's model, in addition 6 supervisory sections recognized efficient by modified model while according to Kao's model there isn't any efficient supervisory sections. As an explanation it can be point out that in modified model all units belong to under evaluation groups attend to be representatives of their group but in spite of initial version other groups are not represented by all their units. Hence under evaluation group has more opportunity to be evaluated optimistically.

Table 3 summarizes efficiencies and ranks calculated from proposed ranking model for efficient supervisory sections recognized by modified model.

6 Conclusion

The conventional DEA model treats the group as whole, ignoring the performance of their component in calculating relative efficiency of a set of groups. While many network DEA model have been criticized due to allocating different weights in evaluation of units belong to same group or ignoring relationship between group as a whole and its component, Kao's model does't suffer from these weaknesses. In this paper parallel network DEA model proposed by Kao has been investigated and from interpretation viewpoint has been criticized. It is argued that DEA essentially is based on comparison within PPS composed of observed activities. When the aim of comparison is evaluation of a set of groups, utilizing a different PPS constructed by units instead of groups leads to difficulty in interpretation and computational consequence is underestimating groups' efficiencies. Modified model was proposed to avoid this difficulty. Both version of model are aimed to find most favorable weights for all of units belong to under evaluation group but in modified model groups are compared with each other and just under evaluation group is represented by all of its

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Table 3: Ranking results

Supervisory	s7	s9	s30	s32	s36	s39	s40
Efficiency	0.67	1	0.53	0.67	0.60	0.66	0.99
Rank	3or4	1	7	3or4	6	5	2

units, while in initial version all units represent their groups and are compared with each other. In other words modified model permits just under evaluation group to be represented by all of its units, but initial version gives this opportunity to all groups. Consequently Constraints of modified model are stronger than those of conventional DEA model but they are weaker than Kao's parallel model; therefore the efficiency scores calculated from modified model are smaller than those calculated from conventional model but larger than those calculated from Kao's model. Thus modified model can be asses as a more consistent model with optimistic property of DEA. As modified model calculated larger efficiency scores, it is more eligible to recognize some groups as efficient groups.Therefore a ranking method was proposed based on new concept of sharing whole inefficiency.

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