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Research Article

Solving fully fuzzy linear programming

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Abstract

In this paper, a new method is proposed to find the fuzzy optimal solution of fully fuzzy linear programming (abbreviated to FFLP) problems. Also, we employ linear programming (LP) with equality constraints to find a non-negative fuzzy number vector $\tilde{x}$ which satisfies $\tilde{A}\tilde{x} = \tilde{b}$, where $\tilde{A}$ is a fuzzy number matrix. Then we investigate the existence of a positive solution of fully fuzzy linear system (FFLS).

Keywords : Fuzzy sets; Linear programming; Fully fuzzy linear system.

1 Introduction

The concept of fuzzy numbers and fuzzy arithmetic operations were first introduced by Zadeh [36], Dubois *et al.* [12]. We refer the reader to [22, 10] for more information on fuzzy numbers and fuzzy arithmetic. Fuzzy systems are used to study a variety of problems ranging from fuzzy topological spaces [9] to control chaotic systems [17, 21, 37], fuzzy metric spaces [31, 16], fuzzy linear and nonlinear systems [1, 2, 4, 29, 30, 32] and particle physics [15, 28, 33, 27].

One of the major applications of fuzzy number arithmetic is treating fuzzy linear programming problems and fuzzy linear systems [5, 6], several problems in various areas such as economics, engineering and physics boil down to the solution of a linear system of equations. In many applications, at least some of the parameters of the system should be represented by fuzzy rather than crisp numbers. Thus, it is immensely important to develop numerical procedures that would appropriately treat fuzzy linear programming problems and fuzzy linear systems and solve them.

Bellman *et al.* [8] proposed the concept of decision making in fuzzy environment. Many researchers adopted this concept for solving fuzzy linear programming problems [34, 38, 10, 25, 20, 14]. However, in all of the above mentioned works, those cases of fuzzy linear programming have been studied in which not all parts of the problem were assumed to be fuzzy, e.g., only the right hand side or the objective function coefficients were fuzzy but the variables were not fuzzy.

Friedman *et al.* [18] introduced a general model for solving a fuzzy $n \times n$ linear system whose coefficient matrix is crisp and the right-hand side column is an arbitrary fuzzy number vector. They used the parametric form of fuzzy numbers and replaced the original fuzzy $n \times n$ linear system by a crisp $2n \times 2n$ linear system and studied duality in fuzzy linear systems $A\tilde{x} = B\tilde{x} + \tilde{y}$ where A and

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B are two real $n \times n$ matrices, the unknown vector $\tilde{x}$ and the constant $\tilde{y}$ are two vector consisting of n fuzzy numbers, in [19]. In [1, 2] the authors presented conjugate gradient, LU decomposition method for solving general fuzzy linear systems or symmetric fuzzy linear systems. Also, Wang *et al.* [35] presented an iterative algorithm for solving dual linear system of the form $\tilde{x} = A\tilde{x} + \tilde{u}$, where A is a real $n \times n$ matrix, the unknown vector $\tilde{x}$ and the constant $\tilde{u}$ are all vectors consisting of fuzzy numbers and Abbasbandy *et al.* [3] investigated the existence of a minimal solution of general dual fuzzy linear equation system of the form $A\tilde{x} + \tilde{f} = B\tilde{x} + \tilde{c}$, where A, B are two real $m \times n$ matrices, the unknown vector $\tilde{x}$ is a vector consisting of n fuzzy numbers and the constant $\tilde{f}$, $\tilde{c}$ are two vectors consisting of m fuzzy numbers. Recently, Dehghan *et al.* [11] considered fully fuzzy linear systems of the form $\tilde{A} \otimes \tilde{x} = \tilde{b}$ where $\tilde{A}$ is a positive fuzzy matrix, $\tilde{b}$ and $\tilde{x}$ are known and unknown positive fuzzy vectors.

In this paper the shortcomings of the existing methods [3, 11, 24] are pointed out and to overcome these shortcomings, a new method is proposed for finding the fuzzy solution of FFLP problems and FFLS.

2 Preliminaries

In this Section the basic notations used in fuzzy calculus are introduced. We start by defining the fuzzy number.

Definition 2.1 *A fuzzy number is a fuzzy set* $u : \mathbb{R}^1 \longrightarrow I = [0,1]$ *such that*

(i) $u(x)$ *is upper semi-continuous,*

(ii) $u(x) = 0$ *outside some interval* $[a, d]$,

(iii) *There are real numbers b and c,* $a \leq b \leq c \leq d$, *for which*

1. $u(x)$ *is monotonically increasing on* $[a, b]$,

2. $u(x)$ *is monotonically decreasing on* $[c, d]$,

3. $u(x) = 1, b \leq x \leq c$.

The set of all the fuzzy numbers (as given in definition 1) is denoted by E^1.

A popular fuzzy number is the triangular fuzzy number $\tilde{u} = (u_m, u_l, u_r)$ where u_m denotes the modal value and the real values $u_l > 0$ and $u_r > 0$ represent the left and right spread, respectively. The membership function of a triangular fuzzy number is defined by:

$$u(x) = \begin{cases} \frac{x-u_m}{u_l} + 1, & u_m - u_l \leq x \leq u_m, \\ \frac{u_m-x}{u_r} + 1, & u_m \leq x \leq u_m + u_r, \\ 0, & otherwise. \end{cases}$$

Definition 2.2 *A fuzzy number* $\tilde{u}$ *is said to be an LR fuzzy number if*

$$\tilde{u}(x) = \begin{cases} L(\frac{u-x}{\alpha}), & x \leq u, \ \alpha > 0, \\ R(\frac{x-u}{\beta}), & x \geq u, \ \beta > 0, \end{cases}$$

where a is the mean value of $\tilde{u}$ *and* α *and* β *are left and right spreads, respectively; and the function L(.), which is called left shape function, satisfying:*
(1) L(x)=L(-x),
(2) L(0)=1 and L(1)=0,
(3) L(x) is nonincreasing on $[0, \infty)$.
The definition of a right shape function R(.) is usually similar to that of L(.).

The definition of a right shape function R(.) is usually similar to that of L(.).
The mean value, left and right spreads, and the shape functions of an LR fuzzy number $\tilde{u}$ are symbolically shown as $\tilde{u} = (u, \alpha, \beta)_{LR}$. Triangular fuzzy numbers are fuzzy numbers in LR representation where the reference functions L and R are linear.

Definition 2.3 *A fuzzy number* $\tilde{u}$ *is called positive (negative), denoted by* $\tilde{u} > 0$ $(\tilde{u} < 0)$*, if its membership function* $u(x)$ *satisfies* $u(x) = 0$, $\forall x < 0$ $(\forall x > 0)$.

Definition 2.4 *A matrix* $\tilde{A} = (\tilde{a_{ij}})$ *is called a fuzzy matrix, if each element of* $\tilde{A}$ *is a fuzzy number [13].*

Let each element of $\tilde{A}$ be a LR fuzzy number. We may represent $\tilde{A} = (\tilde{a_{ij}})$ that $\tilde{a_{ij}} = (a_{ij}, m_{ij}, n_{ij})_{LR}$,with new notation $\tilde{A} = (A, M, N)$, where A, M and N are three crisp matrices, with the same size of $\tilde{A}$, such that$A = (a_{ij})$, $M = (m_{ij})$, and $N = (n_{ij})$ are called the *center matrix* and the *right* and *left spread matrices,* respectively.

Definition 2.5 *Let $\tilde{u}$, $\tilde{v}$ be two fuzzy numbers of LR type:*

$$\tilde{u} = (u, \theta, \lambda)_{LR},\ \tilde{v} = (v, \phi, \eta)_{LR}$$

then

1. $(u, \theta, \lambda)_{LR} \oplus (v, \phi, \eta)_{LR} = (u + v, \theta + \phi, \lambda + \eta)_{LR}$.
2. $-(u, \theta, \lambda)_{LR} = (-u, \lambda, \theta)_{RL}$.
3. $(u, \theta, \lambda)_{LR} \ominus (v, \phi, \eta)_{RL} = (u - v, \theta + \eta, \lambda + \phi)_{LR}$.

Definition 2.6 *[12, 13] Let $\tilde{u}$, $\tilde{v}$ be two fuzzy numbers as in definition 1; then*

$$(u, \theta, \lambda)_{LR} \otimes (v, \phi, \eta)_{LR} \cong (uv, u\phi + v\theta, u\eta + v\lambda)_{LR}$$

for $\tilde{u}, \tilde{v}$ positive;

$$(u, \theta, \lambda)_{LR} \otimes (v, \phi, \eta)_{LR} \cong (uv, v\theta - u\eta, v\lambda - u\phi)_{LR}$$

for $\tilde{v}$ positive, $\tilde{u}$ negative.

Definition 2.7 *Let $\tilde{A} = (\tilde{a}_{ij})$ and $\tilde{B} = (\tilde{b}_{ij})$ be two $m \times n$ and $n \times p$ fuzzy matrices. We define $\tilde{A} \otimes \tilde{B} = \tilde{C} = (\tilde{c}_{ij})$ which is the $m \times p$ matrix where*

$$\tilde{c}_{ij} = \sum_{k=1,\dots,n}^{\oplus} \tilde{a}_{ik} \otimes \tilde{b}_{kj}.$$

Definition 2.8 *We say that $(u, \theta, \lambda)_{LR} \preceq (v, \phi, \eta)_{LR}$ if $u \le v$, $u - \theta \le v - \phi$ and $u + \lambda \le v + \eta$.*

Definition 2.9 *[23] A ranking function is a function $\Re : F(\mathbb{R}) \longrightarrow \mathbb{R}$, where $F(\mathbb{R})$ is a set of fuzzy numbers defined on set of real numbers, which maps each fuzzy number into the real line, where a natural order exists. Let $(u, \theta, \lambda)_{LR}$ be a fuzzy number then $\Re(\tilde{u}) = \frac{(u-\theta)+2u+(u+\lambda)}{4}$.*

2.1 Fully fuzzy linear programming problem

Linear programming is concerned with the optimization (minimization or maximization) of a linear function while satisfying a set of linear equality and/ or inequality constrains or restrictions. In the real life problems there may exists uncertainty about the parameters. In such a situation the parameters of linear programming problems may be represented as fuzzy numbers.

Consider the following fully fuzzy linear programming problem.

$$Min\ (or\ Max)\ (\tilde{c}_1 \otimes \tilde{x}_1) \oplus \dots \oplus (\tilde{c}_n \otimes \tilde{x}_n)$$

$$st \quad \begin{array}{l} (\tilde{a}_{11} \otimes \tilde{x}_1) \oplus \dots \oplus (\tilde{a}_{1n} \otimes \tilde{x}_n) \preceq \tilde{b}_1 \\ (\tilde{a}_{21} \otimes \tilde{x}_1) \oplus \dots \oplus (\tilde{a}_{2n} \otimes \tilde{x}_n) \preceq \tilde{b}_2 \\ \vdots \\ (\tilde{a}_{m1} \otimes \tilde{x}_1) \oplus \dots \oplus (\tilde{a}_{mn} \otimes \tilde{x}_n) \preceq \tilde{b}_m \end{array}$$

$$\tilde{x}_1,\ \tilde{x}_2,\ \dots,\ \tilde{x}_n \ge 0.$$

Using matrix notation we get

$$\begin{array}{l} Min\ (or\ Max)\ \tilde{c}^T \otimes \tilde{x} \\ \quad \tilde{A} \otimes \tilde{x} \preceq \tilde{b}, \\ \tilde{x}\ is\ a\ nonnegative\ fuzzy\ number. \end{array} \tag{2.1}$$

Here $\tilde{A} = (A, M, N)$ and $\tilde{c} = (c, p, r)$ have positive and negative fuzzy elements. This linear programming problem is called a fully fuzzy linear programming problem.

Consider the FFLP problem (2.1) where $\tilde{x} = (x, y, z)$ is unknown positive fuzzy vector and $\tilde{b} = (b, g, l)$ is known arbitrary fuzzy vector. We define $\tilde{A} = \tilde{D} \oplus \tilde{H}$ and $\tilde{c} = \tilde{u} \oplus \tilde{w}$ where $\tilde{D} = (\tilde{d}_{ij})$, $\tilde{H} = (\tilde{h}_{ij})$, $\tilde{u} = (\tilde{u}_i)$ and $\tilde{w} = (\tilde{w}_i)$ with new notation $\tilde{D} = (D, \alpha, \beta)$, $\tilde{H} = (H, \gamma, \delta)$, $\tilde{u} = (u, \psi, \nu)$ and $\tilde{w} = (w, \omega, \kappa)$ where

$$\tilde{d}_{ij} = \begin{cases} \tilde{a}_{ij} = (a_{ij}, m_{ij}, n_{ij}), & a_{ij} - m_{ij} \ge 0, \\ \tilde{0} = (0, 0, 0), & otherwise, \end{cases}$$

$$\tilde{h}_{ij} = \begin{cases} \tilde{a}_{ij} = (a_{ij}, m_{ij}, n_{ij}), & a_{ij} + n_{ij} \le 0, \\ \tilde{0} = (0, 0, 0), & otherwise, \end{cases}$$

$$\tilde{u}_i = \begin{cases} \tilde{c}_i = (c_i, p_i, r_i), & c_i - p_i \geq 0, \\ \tilde{0} = (0,0,0), & otherwise \end{cases}$$

and

$$\tilde{w}_i = \begin{cases} \tilde{c}_i = (c_i, p_i, r_i), & c_i + r_i \leq 0, \\ \tilde{0} = (0,0,0), & otherwise. \end{cases}$$

For solving FFLP problem (2.1), we replace (2.1) by

$$\begin{aligned} & Min \ (or \ Max) \ (\tilde{u}^T \oplus \tilde{w}^T) \otimes \tilde{x} \\ & \quad (\tilde{D} \oplus \tilde{H}) \otimes \tilde{x} \preceq \tilde{b}, \\ & \tilde{x} \ is \ a \ nonnegative \ fuzzy \ number, \end{aligned}$$

then [12]

$$\begin{aligned} & Min \ (or \ Max) \ (\tilde{u}^T \otimes \tilde{x}) \oplus (\tilde{w}^T \otimes \tilde{x}) \\ & \quad (\tilde{D} \otimes \tilde{x}) \oplus (\tilde{H} \otimes \tilde{x}) \preceq \tilde{b}, \\ & \tilde{x} \ is \ a \ nonnegative \ fuzzy \ number. \end{aligned} \tag{2.2}$$

2.2 Application of ranking function for solving FFLP problems

The fuzzy optimal solution of FFLP problem (2.1) will be a fuzzy number $\tilde{x}$ if it satisfies the following characteristics:

(i) $\tilde{x}$ is a non-negative fuzzy number vector,

(ii) $\tilde{A} \otimes \tilde{x} \preceq \tilde{b}$,

(iii) If there exist any non-negative fuzzy number vector $\tilde{x}'$ such that $\tilde{A} \otimes \tilde{x}' \preceq \tilde{b}$, then $\Re(\tilde{c}^T \otimes \tilde{x}) < \Re(\tilde{c}^T \otimes \tilde{x}')$ (in case of minimization problem) and $\Re(\tilde{c}^T \otimes \tilde{x}) > \Re(\tilde{c}^T \otimes \tilde{x}')$ (in case of maximization problem).

The final target of this paper is to find the optimal solution of FFLP problem (2.2). Therefore, we use definitions 5 and 6, we have

$$Min \ \frac{(4(u+w) - \psi - \omega + \nu + \kappa)x + (u+w)z - (u+w)y}{4}$$

$$st \quad \begin{aligned} & Dx + Hx \leq b \\ & Dy + \alpha x + \gamma x - Hz \leq g, \\ & Dz + \beta x + \delta x - Hy \leq l \\ & y_i, x_i, x_i - y_i, z_i \geq 0, \\ & for \ i = 1, \ldots, n. \end{aligned} \tag{2.3}$$

2.3 Fully fuzzy linear systems

In this subsection, we propose FFLP problem for solving fully fuzzy linear systems.
A linear system such as

$$\begin{aligned} & (\tilde{a}_{11} \otimes \tilde{x}_1) \oplus ... \oplus (\tilde{a}_{1n} \otimes \tilde{x}_n) = \tilde{b}_1 \\ & (\tilde{a}_{21} \otimes \tilde{x}_1) \oplus ... \oplus (\tilde{a}_{2n} \otimes \tilde{x}_n) = \tilde{b}_2 \\ & \vdots \\ & (\tilde{a}_{n1} \otimes \tilde{x}_1) \oplus ... \oplus (\tilde{a}_{nn} \otimes \tilde{x}_n) = \tilde{b}_n \end{aligned}$$

where $\tilde{a}_{ij}$, $1 \leq i,j \leq n$ are positive or negative LR fuzzy numbers, the elements $\tilde{b}_i$ in the right-hand vector are LR fuzzy numbers and the unknown elements $\tilde{x}_j$ are non-negative ones, is called a fully fuzzy linear system.

Using matrix notation, we have

$$\tilde{A} \otimes \tilde{x} = \tilde{b}. \tag{2.4}$$

Definition 2.10 *Consider a fully fuzzy linear system (2.4). We say that $\tilde{x}$ is a non-negative fuzzy solution if*

$$\begin{aligned} & Dx + Hx = b \\ & Dy + \alpha x + \gamma x - Hz = g, \\ & Dz + \beta x + \delta x - Hy = l \\ & y_i, x_i - y_i, z_i \geq 0, \ for \ i = 1, 2, \ldots, n. \end{aligned} \tag{2.5}$$

Assuming that $D, D+H, D - HD^{-1}H$ are non-singular crisp matrices. Thus we easily get

$$x = (D+H)^{-1}b, \tag{2.6}$$

and then by equation (2.6), we have

$$y = D^{-1}(g + Hz - (\alpha + \gamma)(D+H)^{-1}b), \tag{2.7}$$

by equations (2.6-2.7)

$$\begin{aligned} z = & (D - HD^{-1}H)^{-1}[l + HD^{-1}g - HD^{-1} \\ & (\alpha + \gamma)(D+H)^{-1}b - (\beta + \delta)(D+H)^{-1}b]. \end{aligned} \tag{2.8}$$

Theorem 2.1 *Let $\tilde{A} = (\tilde{a}_{ij})$ for $1 \leq i,j \leq n$ have positive and negative fuzzy elements, $\tilde{b}$ is arbitrary fuzzy number vector, D^{-1} and $(D - HD^{-1}H)^{-1}$ are two non-negative crisp matrices and $(D+H)^{-1}b \geq 0$. Also let*

$g + Hz \geq (\alpha+\gamma)(D+H)^{-1}b$, $l + HD^{-1}g \geq HD^{-1}(\alpha+\gamma)(D+H)^{-1}b + (\beta+\delta)(D+H)^{-1}b$ and $(D^{-1}(\alpha+\gamma)+I)(D+H)^{-1}b \geq D^{-1}(g+Hz)$. Then the system $\tilde{A} \otimes \tilde{x} = \tilde{b}$ has a nonnegative fuzzy solution.

Proof. *By hypotheses $x = (D+H)^{-1}b \geq 0$. On the other hand, $g + Hz \geq (\alpha+\gamma)(D+H)^{-1}b$ and $l + HD^{-1}g \geq HD^{-1}(\alpha+\gamma)(D+H)^{-1}b + (\beta+\delta)(D+H)^{-1}b$. Thus, with $y = D^{-1}(g + Hz - (\alpha+\gamma)(D+H)^{-1}b)$, $z = (D - HD^{-1}H)^{-1}[l + HD^{-1}g - HD^{-1}(\alpha+\gamma)(D+H)^{-1}b - (\beta+\delta)(D+H)^{-1}b]$, we have $y \geq 0$ and $z \geq 0$.*
So, $\tilde{x} = (x, y, z)$ is a fuzzy vector which satisfies $\tilde{A} \otimes \tilde{x} = \tilde{b}$. Since $x - y = (D+H)^{-1}b - D^{-1}(g+Hz-(\alpha+\gamma)(D+H)^{-1}b)$, the positivity property of $\tilde{x}$ can be obtained from $(D^{-1}(\alpha+\gamma)+I)(D+H)^{-1}b \geq D^{-1}(g+Hz)$. □

Now, we propose linear programming for solving fully fuzzy linear systems with the inequality $y_i, x_i - y_i, z_i \geq 0,\ for\ i = 1, 2, \ldots, n$ denotes the constraint.

Using arithmetic operations, defined in section 2 and the phase 1 of the two-phase method, we have the following linear programming. In which, we have added the artificial variables $r_1, r_2, \ldots, 3r_n$.

$$Min \quad r_1 + r_2 + \ldots + r_{3n} \tag{2.9}$$

$$st \begin{cases} Dx + Hx + R_1 = b \\ Dy + \alpha x + \gamma x - Hz + R_2 = g, \\ Dz + \beta x + \delta x - Hy + R_3 = l \\ y_i, x_i, x_i - y_i, z_i, r_j \geq 0, \\ for\ i = 1, \ldots, n,\ j = 1, \ldots, 3n, \end{cases}$$

where $R_1^T = (r_1, \ldots, r_n)$, $R_2^T = (r_{n+1}, \ldots, r_{2n})$ and $R_3^T = (r_{2n+1}, \ldots, r_{3n})$. There are various methods for eliminating these artificial variables. One of these methods consists of minimizing their sum, subject to the constraints Eq. (2.5) and $r_i \geq 0,\ i = 1, 2, \ldots, 3n$. If the original FFLS (2.4) has a solution, then the optimal value of this problem is zero, where all the artificial variables drop to zero [7, 26].

2.4 Shortcomings of the existing methods

In this subsection, the shortcomings of the existing methods [1, 3, 11, 24] for solving fuzzy linear systems are pointed out.

(i) Abbasbandy *et al.* [1, 3] investigated the existence of a minimal solution of general dual fuzzy linear equation system of the form $A\tilde{x} + \tilde{f} = B\tilde{x} + \tilde{c}$ and fuzzy linear systems $A\tilde{x} = b$, respectively. The existing methods [1, 3] are applicable only if all the elements of the coefficient matrix are real numbers, eg., they are not possible to find the non-negative fuzzy solution of FFLS, chosen in example 2.

(ii) Dehghan *et al.* [11] considered FFLS of the form $\tilde{A}\tilde{x} = \tilde{b}$ where $\tilde{A}$ is a fuzzy $n \times n$ matrix, the unknown vector $\tilde{x}$ consists of n fuzzy numbers and the constant $\tilde{b}$ is a vector consisting of n fuzzy numbers. The existing method [11] is applicable only if all the elements of the coefficient matrix and those of the right-hand side vector are non-negative fuzzy numbers. But if this is not case, then as the example 2 below shows (in which (-3,1,2), (-5,13,13) and (-2,1,1) are not non-negative fuzzy numbers) the existing method is incapable to find a solution for the FFLS in question.

(iii) Lotfi *et al.* [24] proposed a new method to find the fuzzy optimal solution of FFLP problem with equality constraints. This method can be applied only if the elements of the coefficient matrix are symmetric fuzzy numbers. But if this is not case, then as the example 1 below shows the existing method is incapable to find a solution for the FFLS in question.

3 Numerical examples

To illustrate the technique proposed in this paper, consider the following examples.

Example 3.1 *Consider the following problem*

$$Max\ (2, 1, 1) \otimes \tilde{x}_1 \oplus (-3, 1, 2) \otimes \tilde{x}_2$$

$$st \begin{array}{l}(1,1,1)\otimes\tilde{x}_1\oplus(2,1,2)\otimes\tilde{x}_2\preceq(4,3,4)\\(-2,1,1)\otimes\tilde{x}_1\oplus(3,1,2)\otimes\tilde{x}_2\preceq(5,3,2)\\(3,2,1)\otimes\tilde{x}_1\oplus(-3,2,1)\otimes\tilde{x}_2\preceq(5,4,3)\\\tilde{x}_1,\tilde{x}_2\geq 0.\end{array}$$

Now using the proposed method, the above FFLP problem is converted into the following crisp problem

$$Max\frac{8x_1-11x_2-2y_1-3z_2+2z_1+3y_2}{4}$$

$$st \begin{array}{l}Dx+Hx\leq b\\Dy+\alpha x+\gamma x-Hz\leq g,\\Dz+\beta x+\delta x-Hy\leq l\\x_1,x_2,y_1,y_2,z_1,z_2,x_1-y_1,x_2-y_2\geq 0,\end{array}$$

where

$$D=\begin{pmatrix}1&2\\0&3\\3&0\end{pmatrix},\ H=\begin{pmatrix}0&0\\-2&0\\0&-3\end{pmatrix},$$

$$\alpha=\begin{pmatrix}1&1\\0&1\\2&0\end{pmatrix},\ \gamma=\begin{pmatrix}0&0\\1&0\\0&2\end{pmatrix},$$

$$\beta=\begin{pmatrix}1&2\\0&2\\1&0\end{pmatrix},\ \delta=\begin{pmatrix}0&0\\1&0\\0&1\end{pmatrix},$$

$$b=\begin{pmatrix}4\\5\\5\end{pmatrix},\ g=\begin{pmatrix}3\\3\\4\end{pmatrix},\ l=\begin{pmatrix}4\\2\\3\end{pmatrix}.$$

Therefore we have $\tilde{x}_1=(\frac{5}{3},0,\frac{4}{9})$ *and* $\tilde{x}_2=(0,0,0)$ *and the optimal value of this problem is* $\frac{32}{9}$.

Example 3.2 *Consider the following fully fuzzy linear system*

$$\begin{array}{l}(2,1,1)\otimes\tilde{x}_1\oplus(-3,1,2)\otimes\tilde{x}_2=(-5,13,13)\\(-2,1,1)\otimes\tilde{x}_1\oplus(5,1,2)\otimes\tilde{x}_2=(11,12,20)\\\tilde{x}_1,\tilde{x}_2\geq 0.\end{array}$$

Now using the proposed method, the above FFLS is converted into the following crisp problem

$$Min\ r_1+r_2+\ldots+r_6$$

$$st \begin{array}{l}Dx+Hx+R_1=b\\Dy+\alpha x+\gamma x-Hz+R_2=g,\\Dz+\beta x+\delta x-Hy+R_3=l\\x_1,x_2,y_1,y_2,z_1,z_2,r_1,r_2,\ldots,r_6\geq 0\\x_1-y_1,x_2-y_2\geq 0,\end{array}$$

where

$$D=\begin{pmatrix}2&0\\0&5\end{pmatrix},\ H=\begin{pmatrix}0&-3\\-2&0\end{pmatrix},$$

$$\alpha=\begin{pmatrix}1&0\\0&1\end{pmatrix},\ \gamma=\begin{pmatrix}0&1\\1&0\end{pmatrix},$$

$$\beta=\begin{pmatrix}1&0\\0&2\end{pmatrix},\ \delta=\begin{pmatrix}0&2\\1&0\end{pmatrix},$$

$$b=\begin{pmatrix}-5\\11\end{pmatrix},\ g=\begin{pmatrix}13\\12\end{pmatrix},\ l=\begin{pmatrix}13\\20\end{pmatrix}.$$

Therefore we have $\tilde{x}_1=(2,1,1)$ *and* $\tilde{x}_2=(3,1,2)$.

4 Conclusion

In this paper, we propose a general model for solving a FFLP problem and system of n fuzzy linear equations with n fuzzy variables. The original problem with fuzzy number matrix $\tilde{A}$ is replaced by $\tilde{D}\oplus\tilde{H}$ where $\tilde{D}$ and $\tilde{H}$ are two fuzzy number matrices. Also, a condition for the existence of a positive fuzzy solution to the FFLS, is presented. A careful comparison between the proposed method and the existing ones shows that this method is more general and more suitable than the other ones. To illustrate the proposed method, we will give a number of solved numerical examples.

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