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Research Article

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Dynamical system and Semi-Hereditarily hypercyclic property

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Abstract

In this paper we give conditions for a tuple of commutative bounded linear operators which holds in the property of the Hypercyclicity Criterion. We characterize topological transitivity and semi-hereiditarily hypercyclicity of a dynamical system given by an n-tuple of operators acting on a separable infinite dimensional Banach space X.

Keywords : Tuple; hypercyclic vector; Hypercylicity Criterion; hereditarily hypercyclicity; dynamical systems; periodic point.

1 Introduction

Topological transivity was first introduced by G. D. Birkhoff in 1920, for the study of flows and characterized a dynamical system. See [3, 4, 8] for more information.
Dynamical systems have different behavior, for example, some of them have dense periodic points and some others may be minimal and so without any periodic point. We will consider a dynamical system $(X, \mathcal{T})$ given by an n-tuple $\mathcal{T} = (T_1, T_2, ..., T_n)$ acting on an infinite dimensional Banach space X. We give some conditions on X (or $\mathcal{T}$) to be hold in semi-hereditarily hypercyclic and topological transitive properties.

The symbol $(X, \mathcal{T})$ can be used in real physical system, where a state is never given or it measured with a certain error. Two current definitions of topological transitivity on an elementary dynamical system (X, f), where X is a metric space and $f : X \to X$ is continuous as follows:

a) For every pair of nonempty open subsets U and V of X, there is a positive integer n such that $f^n(U) \bigcap V \neq \emptyset$.

b) There is a point $x_0 \in X$ such that the orbit of x_0, $(\{x_0, f(x_0), ..., f^n(x_0), ...\})$, is dense in X.
The above definitions do not imply the others. For example, take $X = \{0\} \bigcup \{1/n : n \in \mathbb{N}\}$ with standard metric and $f : X \to X$ defined by $f(0) = 0$ and $f(1/n) = 1/(n+1), n = 1, 2, ...$. Clearly the point $x_0 = 1$ is the only point which its orbits is dense in X, but by taking $U = \{1/2\}$ and $V = \{1\}$, one can see that there is no positive integer n such that $f^n(U) \bigcap V \neq \emptyset$. So (b) does not imply (a). Let $I = [0, 1]$, $g(x) = 1 - |2x - 1|$ be the self map on I, X be the set of all periodic points of g and $f = g|_X$. Then the system (X, f) does not satisfy the definition (b) but it holds in (a).This follows from the fact that for any nondegenerate subinterval J of I there is a positive integer k with $g^k(J) = I$. Hence, whenever J_1 and J_2 are nonempty open subintervals of I , there is a periodic orbit of g which intersects both J_1 and J_2. This gives (a) for (X, f).

Under some conditions on space X (or on the map f), the two definitions are equivalent.

Theorem 1.1 *([10]) If X has no isolated point then (b) implies (a). If X is separable and second*

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category then (a) implies (b).

By an n-tuple of operators we mean a finite sequence of length n of commuting continuous linear operators on a Banach space X.

Definition 1.1 *Let $\mathcal{T} = (T_1, T_2, ..., T_n)$ be an n-tuple of operators acting on an infinite dimensional Banach space X. Let*

$$\mathcal{F} = \{{T_1}^{k_1}{T_2}^{k_2}...{T_n}^{k_n} : k_i \in \mathbb{Z}^+, \ \ i = 1, ..., n\}$$

be the semigroup generated by $\mathcal{T}$. For $x \in X$, the orbit of x under the tuple $\mathcal{T}$ is the set $Orb(\mathcal{T}, x) = \{Sx : S \in \mathcal{F}\}$. A vector x is called a hypercyclic vector for $\mathcal{T}$ if $Orb(\mathcal{T}, x)$ is dense in X and in this case the tuple $\mathcal{T}$ is called hypercyclic.

Definition 1.2 *By $\mathcal{T}_d^{(k)}$ we will refer to the set of all k copies of an element of $\mathcal{F}$, i.e.*

$$\mathcal{T}_d^{(k)} = \{S_1 \oplus ... \oplus S_k : S_1 = ... = S_k \in \mathcal{F}\}.$$

We say that $\mathcal{T}_d^{(k)}$ is hypercyclic provided there exist $x_1, ..., x_k \in X$ such that

$$\{W(x_1 \oplus ... \oplus x_k) : W \in \mathcal{T}_d^{(k)}\}$$

is dense in the k copies of X, $X \oplus ... \oplus X$.

Note that if $T_1, T_2, ..., T_n$ are commutative bounded linear operators on a Banach space X, and $\{m_j(i)\}_j$, is a sequence of natural numbers for $i = 1, ..., n$, then we say

$$\{{T_1}^{m_j(1)}{T_2}^{m_j(2)}...{T_n}^{m_j(n)} : j \geq 0\}$$

is hypercyclic if there exists $x \in X$ such that

$$\{{T_1}^{m_j(1)}{T_2}^{m_j(2)}...{T_n}^{m_j(n)}x : j \geq 0\}$$

is dense in X.

We say that a tuple $\mathcal{T} = (T_1, T_2, ..., T_n)$ is topologically transitive with respect to a tuple of nonnegative integer sequences

$$(\{k_{j(1)}\}_j, \{k_{j(2)}\}_j, ..., \{k_{j(n)}\}_j),$$

if for every nonempty open subsets U, V of X there exists $j_0 \in \mathbb{N}$ such that $T_1^{k_{j_0(1)}}T_2^{k_{j_0(2)}}...T_n^{k_{j_0(n)}}(U) \cap V \neq \emptyset$. Also, we say that an n-tuple $\mathcal{T}$ is topologically transitive if it is topologically transitive with respect an n-tuple of nonnegative integer sequences.

Definition 1.3 *A tuple $\mathcal{T} = (T_1, T_2, ..., T_n)$ is called topologically mixing if for any given open sets U and V, there exist positive integers $M(1)$, ..., $M(n)$ such that*

$$T_1^{m(1)}...T_n^{m(n)}(U) \cap V \neq \emptyset,$$
$$\forall m(i) \geq M(i), \quad i = 1, ..., n.$$

Definition 1.4 *We say that a tuple $\mathcal{T} = (T_1, T_2, ...T_n)$ is hereditarily hypercyclic with respect to a tuple of nonnegative increasing sequences*

$$(\{k_{j(1)}\}_j, \{k_{j(2)}\}_j, ..., \{k_{j(n)}\}_j)$$

of integers provided for all tuples of subsequences

$$(\{k_{j_i(1)}\}_i, \{k_{j_i(2)}\}_i, ..., \{k_{j_i(n)}\}_i)$$

of $(\{k_{j(1)}\}_j, \{k_{j(2)}\}_j, ..., \{k_{j(n)}\}_j)$, the sequence

$$\{T_1^{k_{j_i(1)}}T_2^{k_{j_i(2)}}...T_n^{k_{j_i(n)}} : i \geq 1\}$$

is hypercyclic. We say that an n-tuple $\mathcal{T}$ is hereditarily hypercyclic, if it is hereditarily hypercyclic with respect to an n-tuple of nonnegative increasing sequences of integers.

Definition 1.5 *We say that a tuple $\mathcal{T} = (T_1, T_2, ...T_n)$ is semi-hereditarily hypercyclic with respect to a tuple of nonnegative increasing sequences*

$$(\{k_{j(1)}\}_j, \{k_{j(2)}\}_j, ..., \{k_{j(n)}\}_j)$$

of integers provided for all tuples of subsequences

$$(\{k_{j_{i_1}(1)}\}_{i_1}, \{k_{j_{i_2}(2)}\}_{i_2}, ..., \{k_{j_{i_n}(n)}\}_{i_n})$$

of $(\{k_{j(1)}\}_j, \{k_{j(2)}\}_j, ..., \{k_{j(n)}\}_j)$, the sequence

$$\{T_1^{k_{j_i(1)}}T_2^{k_{j_i(2)}}...T_n^{k_{j_i(n)}} : i_j \geq 1, j = 1, ..., n\}$$

is hypercyclic. We say that a pair $\mathcal{T}$ is semi-hereditarily hypercyclic, if it is semi-hereditarily hypercyclic with respect to a pair of nonnegative increasing sequences.

Clearly, if a tuple $\mathcal{T} = (T_1, T_2, ...T_n)$ is hereditarily hypercyclic, then it is semi-hereditarily hypercyclic.

Definition 1.6 *An strictly increasing sequence of positive integers $\{n_k\}$ is said to be syndetic if $sup_n\{n_{k+1} - n_k\} < \infty$.*

The formulation of the Hypercyclicty Criterion in the next section was given by N. S. Feldman [6] for the case of $n = 2$. Here, we want to extend some properties of hypercyclic operators to a tuple of commuting operators. For some other topics we refer to [1, 2, 5, 6, 7, 9, 11, 12, 13, 14, 15, 16, 17, 18, 19].

2 Main Results

In this Section we characterize the equivalent conditions for a tuple of operators, satisfying the Hypercyclicity Criterion.

Theorem 2.1 *(The Hypercyclicity Criterion) Suppose that X is a separable infinite dimensional Banach space and $\mathcal{T} = (T_1, T_2, ..., T_n)$ be the n-tuple of operators T_1, T_2, ..., T_n acting on X. If there exist two dense subsets Y and Z in X, and strictly increasing sequences $\{m_j(i)\}_j$ for $i = 1, ..., n$ such that :*
1. $T_1^{m_j(1)} ... T_n^{m_j(n)} y \to 0$ for all $y \in Y$,
2. There exist a sequence of functions $\{S_j : Z \to X\}$ such that for every $z \in Z$, $S_j z \to 0$, and $T_1^{m_j(1)} ... T_n^{m_j(n)} S_j z \to z$ as $j \to \infty$,
then $\mathcal{T}$ is a hypercyclic tuple.

Theorem 2.2 *Let $\mathcal{T}$ be a tuple of operators $\{T_1, T_2, ..., T_n\}$ on a separable infinite dimensional Banach space X. Also, let T_i^* has no eigenvalues for $i = 1, ..., n$. Then the followings are equivalent:*
(i) $\mathcal{T}_d^{(2)}$ is hypercyclic.
(ii) for every nonempty open subsets U, V of X and every neighborhood W of 0, there exist integers $m_1, ..., m_n$ such that

$$T_1^{m_1} T_2^{m_2} ... T_n^{m_n}(U) \cap W \neq \emptyset$$

and

$$T_1^{m_1} T_2^{m_2} ... T_n^{m_n}(W) \cap V \neq \emptyset.$$

Proof. (i) implies (ii): Let (U, V) be a pair of nonempty open subsets of X and W be a neighborhood of 0. Put $U_1 = U$, $V_1 = U_2 = W$ and $V_2 = V$. Since $\mathcal{T}_d^{(2)}$ is hypercyclic, there exists a tuple of nonnegative integers $(m_1, ..., m_n)$ such that

$$T_1^{m_1} T_2^{m_2} ... T_n^{m_n}(U_1) \cap V_1 \neq \emptyset$$

and

$$T_1^{m_1} T_2^{m_2} ... T_n^{m_n}(U_2) \cap V_2 \neq \emptyset.$$

This proves (ii).
(ii) implies (i): In order to show that $\mathcal{T}_d^{(2)}$ is hypercyclic, we will show that for every nonempty open subsets U, V of X, there exists a tuple of integers $(m_1, ..., m_n)$ such that

$$T_1^{m_1} T_2^{m_2} ... T_n^{m_n}(U) \cap V \neq \emptyset$$

and

$$T_1^{m_1+1} T_2^{m_2+1} ... T_n^{m_n+1}(U) \cap V \neq \emptyset.$$

For this let (U, V) be any pair of nonempty open subsets of X. Also, let W be any neighborhood of 0. In assertion (ii) of the theorem substitute W by $W \cap T_1^{-1} T_2^{-1} ... T_n^{-1}(W)$. Then there exists a tuple of integers $(m_1, ..., m_n)$ such that

$$T_1^{m_1} T_2^{m_2} ... T_n^{m_n}(U) \cap (W \cap T_1^{-1} T_2^{-1} ... T_n^{-1}(W)) \neq \emptyset$$

and

$$T_1^{m_1} T_2^{m_2} ... T_n^{m_n}(W \cap T_1^{-1} T_2^{-1} ... T_n^{-1}(W)) \cap V \neq \emptyset.$$

Hence

$$T_1^{j_1} T_2^{j_2} ... T_n^{j_n}(U) \cap W \neq \emptyset$$

and

$$T_1^{j_1} T_2^{j_2} ... T_n^{j_n}(W) \cap V \neq \emptyset$$

for $m_i \leq m_i + 1$, $i = 1, ..., n$. Now, let u, v be arbitrary elements in U and V, respectively. Choose $k_0 \in \mathbb{N}$ such that $B(u, \frac{1}{k_0}) \subset U$ and $B(v, \frac{1}{k_0}) \subset V$. For $k \geq k_0$ put $U_k = B(u, \frac{1}{k})$, $V_k = B(v, \frac{1}{k})$ and $W_k = B(0, \frac{1}{k})$. Then there exists a tuple of positive integers $(m_k(1), m_k(2), ..., m_k(n))$ such that

$$T_1^{j_1} T_2^{j_2} ... T_n^{j_n}(U_k) \cap W_k \neq \emptyset$$

and

$$T_1^{i} T_2^{j}(W_k) \cap V_k \neq \emptyset$$

for $m_k(i) \leq m_k(i) + 1$, $i = 1, ..., n$. So there exist $u_k, u'_k \in U_k$, $v_k, v'_k \in V_k$ and $w_k, w'_k \in W_k$ such that

$$u_k, u'_k \longrightarrow u, \ w_k, w'_k \longrightarrow 0,$$

$$T_1^{m_k(1)} T_2^{m_k(2)} ... T_n^{m_k(n)} u_k \longrightarrow 0,$$

$$T_1^{m_k(1)+1} T_2^{m_k(2)+1} ... T_n^{m_k(n)+1} w_k \longrightarrow v,$$

$$T_1^{m_k(1)+1} T_2^{m_k(2)+1} ... T_n^{m_k(n)+1} u'_k \longrightarrow 0,$$

and

$$T_1^{m_k(1)+1} T_2^{m_k(2)+1} ... T_n^{m_k(n)+1} w'_k \longrightarrow v.$$

Hence, $u_k + w_k \longrightarrow u$ and so

$$T_1^{m_k(1)} T_2^{m_k(2)} ... T_n^{m_k(n)} (u_k + v_k) \longrightarrow v.$$

Also, $u'_k + w'_k \longrightarrow u$ and so

$$T_1^{m_k(1)+1} T_2^{m_k(2)+1} ... T_n^{m_k(n)+1} (u'_k + v'_k) \longrightarrow v.$$

Thus the sets

$$T_1^{m_k(1)} T_2^{m_k(2)} ... T_n^{m_k(n)} (U) \cap V$$

and

$$T_1^{m_k(1)+1} T_2^{m_k(2)+1} ... T_n^{m_k(n)+1} (U) \cap V$$

are both nonempty for k large enough and so the proof is complete.□

Proposition 2.1 *Let $\mathcal{T}$ be a tuple of operators $T_1, T_2, ..., T_n$ on a separable infinite dimensional Banach space X. Then $\mathcal{T} = (T_1, T_2, ..., T_n)$ is semi-hereditarily hypercyclic with respect to a tuple of increasing sequences of nonnegative integers*

$$(\{m^{(1)}_{k(1)}\}_k, \{m^{(2)}_{k(2)}\}_k, ..., \{m^{(n)}_{k(n)}\}_k)$$

if and only if for all given any two open sets U, V, there exists a pair of positive integers $(M_1, M_2, ..., M_n)$ such that

$$T_1^{m^{(1)}_{k(1)}} T_2^{m^{(2)}_{k(2)}} ... T_2^{m^{(n)}_{k(n)}} (U) \cap V \neq \emptyset$$

for any $k(i) > M_i$ for $i = 1, ..., n$.

Proof. Let $\mathcal{T} = (T_1, T_2, ..., T_n)$ be semi-hereditarily hypercyclic with respect to a tuple of increasing sequences of nonnegative integers

$$(\{m^{(1)}_{k(1)}\}_k, \{m^{(2)}_{k(2)}\}_k, ..., \{m^{(n)}_{k(n)}\}_k).$$

Suppose that there exist some open sets U, V such that for all positive integers $i_1, ..., i_n$,

$$T_1^{m^{(1)}_{k_{i_1}(1)}} T_2^{m^{(2)}_{k_{i_2}(2)}} ... T_n^{m^{(n)}_{k_{i_n}(n)}} (U) \cap V = \emptyset$$

for some tuple of subsequences

$$(\{m^{(1)}_{k_{i_1}(1)}\}_{i_1}, \{m^{(2)}_{k_{i_2}(2)}\}_{i_2}, ..., \{m^{(n)}_{k_{i_n}(n)}\}_{i_n})$$

of

$$(\{m^{(1)}_{k(1)}\}_k, \{m^{(2)}_{k(2)}\}_k, ..., \{m^{(n)}_{k(n)}\}_k).$$

Since $\mathcal{T}$ is semi-hereditarily hypercyclic with respect to

$$(\{m^{(1)}_{k(1)}\}_k, \{m^{(2)}_{k(2)}\}_k, ..., \{m^{(n)}_{k(n)}\}_k),$$

thus

$$\{T_1^{m^{(1)}_{k_{i_1}(1)}} T_2^{m^{(2)}_{k_{i_2}(2)}} ... T_n^{m^{(n)}_{k_{i_n}(n)}} : i_j \geq 0 \; , \; j = 1, ..., n\}$$

is hypercyclic and so we get a contradiction. Conversely, suppose that

$$(\{m^{(1)}_{k_{i_1}(1)}\}_{i_1}, \{m^{(2)}_{k_{i_2}(2)}\}_{i_2}, ..., \{m^{(n)}_{k_{i_n}(n)}\}_{i_n})$$

be an arbitrary subsequence of

$$(\{m^{(1)}_{k(1)}\}_k, \{m^{(2)}_{k(2)}\}_k, ..., \{m^{(n)}_{k(n)}\}_k).$$

Let U, V be open sets in X, so there exists positive integers M_i, $i = 1, ..., n$ such that

$$T_1^{m^{(1)}_{k(1)}} T_2^{m^{(2)}_{k(2)}} ... T_2^{m^{(n)}_{k(n)}} (U) \cap V \neq \emptyset$$

for any $k(i) > M_i$ for $i = 1, ..., n$. This implies clearly that

$$T_1^{m^{(1)}_{k_{i_1}(1)}} T_2^{m^{(2)}_{k_{i_2}(2)}} ... T_n^{m^{(n)}_{k_{i_n}(n)}} (U) \cap V \neq \emptyset$$

for any $i_j > M_j$ and all $j = 1, ..., n$. Now the proof is complete.□

Theorem 2.3 *Let $\mathcal{T}$ be a tuple of operators $T_1, T_2, ..., T_n$ on a separable infinite dimensional Banach space X. Then $\mathcal{T} = (T_1, T_2, ..., T_n)$ is semi-hereditarily hypercyclic with respect to a tuple of increasing syndetic sequences of nonnegative integers if and only if $\mathcal{T}$ is topologically mixing.*

Proof. Suppose that $\mathcal{T} = (T_1, T_2, ..., T_n)$ is semi-hereditarily hypercyclic with respect to a tuple of increasing syndetic sequences of nonnegative integers

$$(\{m^{(1)}_{k(1)}\}, \{m^{(2)}_{k(2)}\}, ..., \{m^{(n)}_{k(n)}\}).$$

Let U and V be two nonempty open sets in X. We will show that there exist integers M_i, $i = 1, ..., n$ such that

$$T_1^{m_1} T_2^{m_2} ... T_n^{m_n} (U) \cap V \neq \emptyset$$

for any $m_i > M_i$, $i = 1, ..., n$. Put

$$M_i = sup\{m^{(i)}_{k(i)+1} - m^{(i)}_{k(i)} : k(i) \geq 0\}$$

for $i = 1, ..., n$. For all $i_j = 0, ..., M_j$ and $j = 1, ..., n$, define $U_{i_1,i_2,...,i_n} = U$ and

$$V_{i_1,i_2,...,i_n} = T_1^{-i} T_2^{-i} ... T_n^{-i}(V).$$

Thus there exist integers $M^{(j)}_{i_1,i_2,...,i_n}$ such that for any $k(j) > M^{(j)}_{i_1,i_2,...,i_n}$, $j = 1, ..., n$, we have

$$T_1^{m^{(1)}_{k(1)}} T_2^{m^{(2)}_{k(2)}} ... T_2^{m^{(n)}_{k(n)}}(U_{i_1,i_2,...,i_n}) \cap V_{i_1,i_2,...,i_n} \neq \emptyset$$

that is also holds for any $k(j) > m^{(j)}_{M_{i_1,i_2,...,i_n}}$. Put

$$M_o(j) = max\{m^{(j)}_{M_{i_1,i_2,...,i_n}}$$

$$: i_k = 0, 1, 2, ..., M_k,\ k = 1, ..., n\},$$

and let $M_o(j) = m^{(j)}_{k_0(j)}$.

Now if $m_j > m^{(j)}_{k_0(j)}$, there exist $k(j) > k_0(j)$ and $0 \leq i_j \leq M_j$ such that $m_j = m^{(j)}_{k(j)} + i_j$ for $j = 1, ..., n$. Thus we get

$$T_1^{m_1} T_2^{m_2} ... T_n^{m_n}(U) \cap V$$

$$= T_1^{m^{(1)}_{k(1)}+i_1} T_2^{m^{(2)}_{k(2)}+i_2} ... T_n^{m^{(n)}_{k(n)}+i_n}(U_{i_1,i_2,...,i_n}) \cap V$$

$$= T_1^{m^{(1)}_{k(1)}} T_2^{m^{(2)}_{k(2)}} ... T_2^{m^{(n)}_{k(n)}}(U_{i_1,i_2,...,i_n}) \cap T_1^{-i_1} T_2^{-i_2} ... T_n^{-i_n}(V)$$

$$= T_1^{m^{(1)}_{k(1)}} T_2^{m^{(2)}_{k(2)}} ... T_2^{m^{(n)}_{k(n)}}(U_{i_1,i_2,...,i_n}) \cap V_{i_1,i_2,...,i_n} \neq \emptyset.$$

Conversely if $\mathcal{T}$ is topologically mixing, then by Proposition 2.1, $\mathcal{T}$ is semi-hereditarily hypercyclic with respect to the pair of entire sequences and so the proof is complete.□

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