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Research Article

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The embedding method to obtain the solution of fuzzy linear systems

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Abstract

In this paper,we study (investigates) general fuzzy linear system.The main aim of this paper is based on embedding approach.we find that it is necessary and sufficient conditions for existence of fuzzy solution.Finally,Numerical examples are presented to illustrate the proposed model.

Keywords : Fuzzy linear system; Embedding method; Fuzzy numbers.

1 Literature review

System of linear equation play a major role in various areas of science.In many problems at various areas of science, can be solved by solving a system of linear equation. since some of the systems are parametric and measurements are vague or imprecise, are represented by fuzzy numbers.Development of mathematical models and numerical procedures that would appropriately treat general fuzzy linear systems and solve them, is important. The system of linear equations, $AX = b$, is called fuzzy system of linear equations(FSLE), in which coefficients matrix $(n \times n)$ A is crisp and b is a column matrix and a fuzzy number vector. The fuzzy linear equations has been studied by many authors.Friedman [17] proposed a general model for solving such fuzzy linear systems by using the embedding approach where replace the original system by $(2n) \times (2n)$ representation. In following Friedman et al. [17], Allahviranloo et al.in [3, 4, 5, 6, 7, 8] and other authors such as Abbasbandy et al. [1, 2], Asady et al. [9], Dehghan et al. [15], Wang et al. [20], Zheng et al. [21], Ezzati et al. [16] designed some numerical methods for calculating the solution of fuzzy linear system. In this paper we propose a general model for solving an (FSLE). We use the embedding method and replace original $n \times n$ (FSLE) by two $n \times n$ crisp linear systems. In the Section 2, we introduce preliminary and (FSLE). In Section 3rd the proposed model for solving (FSLE) is discussed. The proposed model is illustrated by solving some example in section 4th.

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2 Preliminaries

Definition 2.1 *[13] For Arbitrary fuzzy number $\widetilde{u}$ in parametric form, is represented by an ordered pair of functions $(\underline{u}(r), \overline{u}(r)), 0 \leq r \leq 1$ which satisfy the following requirements:*

(i) $\underline{u}(r)$ is a bounded left-continuous non-decreasing function over $[0, 1]$

(ii) $\overline{u}(r)$ is a bounded left-continuous non-increasing function over$[0, 1]$

(iii) $\underline{u}(r) \leq \overline{u}(r), 0 \leq r \leq 1$.

The set of all these fuzzy numbers is denoted by E.

Definition 2.2 *[17]For arbitrary fuzzy numbers* $\widetilde{x} = (\underline{x}(r), \overline{x}(r))$, $\widetilde{y} = (\underline{y}(r), \overline{y}(r))$ *and real number* k *we define equality* $\widetilde{x} = \widetilde{y}$, *addition* $\widetilde{x} + \widetilde{y}$ *and multiplication as follows:*
(i) $\widetilde{x} = \widetilde{y}$ *if and only if* $\underline{x}(r) = \underline{y}(r)$ *and* $\overline{x}(r) = \overline{y}(r)$.
(ii) $\widetilde{x} + \widetilde{y} = (\underline{x}(r) + \underline{y}(r), \overline{x}(r) + \overline{y}(r))$.
(iii) $k\widetilde{x} = \begin{cases} (k\underline{x}, k\overline{x}), & k \geq 0 \\ (k\overline{x}, k\underline{x}), & k \leq 0 \end{cases}$

Definition 2.3 *[17] The* $n \times n$ *linear system of equations*

$$\begin{cases} a_{11}\,\widetilde{x}_1 + a_{12}\,\widetilde{x}_2 + \cdots + a_{1n}\,\widetilde{x}_n = \widetilde{b}_1, \\ a_{21}\,\widetilde{x}_1 + a_{22}\,\widetilde{x}_2 + \cdots + a_{2n}\,\widetilde{x}_n = \widetilde{b}_2, \\ \vdots \\ a_{n1}\,\widetilde{x}_1 + a_{n2}\,\widetilde{x}_2 + \cdots + a_{nn}\,\widetilde{x}_n = \widetilde{b}_n, \end{cases} \tag{2.1}$$

where the coefficients matrix, $A = [a_{ij}]_{i,j=1}^{n}$ *is a crisp* $n \times n$ *matrix and* $\widetilde{b}_i$ *are fuzzy numbers, is called a fuzzy linear system. The matrix form of system (2.1) is as follows:*

$$AX = b, \tag{2.2}$$

where $X = (\widetilde{x}_1, \widetilde{x}_2, \ldots, \widetilde{x}_n)^T$, $b = (\widetilde{b}_1, \widetilde{b}_2, \ldots, \widetilde{b}_n)^T$ are the fuzzy number vectors.

Definition 2.4 *[17]A fuzzy number vector* $X = (\widetilde{x}_1, \widetilde{x}_2, \ldots, \widetilde{x}_n)^T$ *that has been given by* $\widetilde{x}_j = (\underline{x}_j(r), \overline{x}_j(r))$, $1 \leq j \leq n$, $0 \leq r \leq 1$ *is called a solution of (2.1) if*

$$\underline{\sum_{j=1}^{n} a_{ij}\widetilde{x}_j} = \sum_{j=1}^{n} \underline{a_{ij}\widetilde{x}_j} = \underline{b}_i$$
$$\overline{\sum_{j=1}^{n} a_{ij}\widetilde{x}_j} = \sum_{j=1}^{n} \overline{a_{ij}\widetilde{x}_j} = \overline{b}_i \tag{2.3}$$
$$i = 1, 2, \ldots, n.$$

3 The proposed model

In this section at first we are going to define an embedding map to form a new crisp system.

Definition 3.1 *For an arbitrary fuzzy number* $\widetilde{x}$ *in parametric form the embedding* $\pi : \mathbb{R}^2 \to \mathbb{R}^2$ *is defined as follows*

$$\pi(\underline{x}(r), \overline{x}(r)) = (\overline{x}(r) - \underline{x}(r), \overline{x}(r) + \underline{x}(r)). \tag{3.4}$$

Lemma 3.1 *Let* $\widetilde{x} = (\underline{x}(r), \overline{x}(r))$, $\widetilde{y} = (\underline{y}(r), \overline{y}(r))$ *are arbitrary fuzzy numbers and let* k *is real number. Then*
(i) $\widetilde{x} = \widetilde{y}$ *if and only if* $\pi(\widetilde{x}) = \pi(\widetilde{y})$
(ii) $\pi(\widetilde{x} + \widetilde{y}) = \pi(\widetilde{x}) + \pi(\widetilde{x})$
(iii) $\pi(k\ \widetilde{x}) = \pi(k(\underline{x}(r), \overline{x}(r))) = (|k|(\overline{x}(r) - \underline{x}(r)), k(\overline{x}(r) + \underline{x}(r)))$

Proof. *The properties (i) and (ii) are clear. We just prove the item (iii): Let* $k \geq 0$, *since* $k\widetilde{x} = (k\underline{x}, k\overline{x})$. *Then* $\pi(k\ \widetilde{x}) = (k(\overline{x}(r) - \underline{x}(r)), k(\overline{x}(r) + \underline{x}(r))) = (|k|(\overline{x}(r) - \underline{x}(r)), k(\overline{x}(r) + \underline{x}(r)))$.

Let $k \leq 0$ *since* $k\widetilde{x} = (k\overline{x}, k\underline{x})$. *Then* $\pi(k\ \widetilde{x}) = (-k(\overline{x}(r) - \underline{x}(r)), k(\overline{x}(r) + \underline{x}(r))) = (|k|(\overline{x}(r) - \underline{x}(r)), k(\overline{x}(r) + \underline{x}(r)))$.

By the previous lemma (3.1) , Eq. (2.1) can be replaced by the following parametric system:

$$\pi\left(\sum_{j=1}^{n}(a_{ij}(\underline{x}_j(r), \overline{x}_j(r))\right) = \pi\left(\underline{b}_i(r), \overline{b}_i(r)\right), \tag{3.5}$$
$$i = 1, 2, \ldots, n.$$

$$\sum_{j=1}^{n}\left(\pi\left(a_{ij}(\underline{x}_j(r), \overline{x}_j(r))\right)\right) =$$
$$\left(\overline{b}_i(r) - \underline{b}_i(r), \overline{b}_i(r) + \underline{b}_i(r)\right), i = 1, 2, \ldots, n. \tag{3.6}$$

$$|a_{ij}|(\overline{x}_j(r) - \underline{x}_j(r)), a_{ij}(\overline{x}_j(r) + \underline{x}_j(r)) =$$
$$\sum_{j=1}^{n}\left((\overline{b}_i(r) - \underline{b}_i(r), \overline{b}_i(r) + \underline{b}_i(r)\right), \tag{3.7}$$
$$i = 1, 2, \ldots, n.$$

$$|a_{ij}|(\overline{x}_j(r) - \underline{x}_j(r)), \sum_{j=1}^{n} a_{ij}(\overline{x}_j(r) + \underline{x}_j(r)) =$$
$$\left(\sum_{j=1}^{n}\left(\overline{b}_i(r) - \underline{b}_i(r), \overline{b}_i(r) + \underline{b}_i(r)\right), \tag{3.8}\right.$$
$$i = 1, 2, \ldots, n.$$

Now we have the following equations :

$$\sum_{j=1}^{n}|a_{ij}|(\overline{x}_j(r) - \underline{x}_j(r)) = \overline{b}_i(r) - \underline{b}_i(r), \tag{3.9}$$
$$i = 1, 2, \ldots, n.$$

$$\sum_{j=1}^{n} a_{ij}(\overline{x}_j(r) + \underline{x}_j(r)) = \overline{b}_i(r) + \underline{b}_i(r), \tag{3.10}$$

$$i = 1, 2, \ldots, n.$$

Consequently in order to solve the system given by (2.1) we must solve two $(n \times n)$ crisp linear system of Eq. (3.9) and (3.10).
The matrix form of systems (3.9) and (3.10) is as follows:

$$BU = Z, AY = W \tag{3.11}$$

where the coefficients matrix $B = [\ |a_{ij}|\ ]_{i,j=1}^{n}$ and $A = [a_{ij}]_{i,j=1}^{n}$ are crisp $n \times n$ matrices and the right hand side columns are the vectors $Z = (\bar{b}_1(r) - \underline{b}_1(r), \ldots, \bar{b}_n(r) - \underline{b}_n(r))^T$, $W = (\bar{b}_1(r) + \underline{b}_1(r), \ldots, \bar{b}_n(r) + \underline{b}_n(r))^T$.
$U = (\overline{x}_1(r) - \underline{x}_1(r), \ldots, \overline{x}_n(r) - \underline{x}_n(r))^T$ and $Y = (\overline{x}_1(r) + \underline{x}_1(r), \ldots, \overline{x}_n(r) + \underline{x}_n(r))$ are the solutions of the crisp linear systems of Eq. (3.11) .

Theorem 3.1 *The fuzzy linear system (2.1) has a unique solution if and only if the matrices A and B are both nonsingular.*
Proof. *It is obvious.*

So the solution vector is unique but it is not still an appropriate fuzzy number vector.
The following theorems explain guarantied conditions for recieving fuzzy number vector solution.
In order to have an appropriate solution we use following theorems.

Theorem 3.2 *[17] The unique solution X of Eq. (3.9) is nonnegative for arbitrary Z if and only if B^{-1} is nonnegative.*

Proof. Let $B^{-1} = (t_{ij})$, $\quad 1 \leq i, j \leq n$. Then $U = B^{-1}Z$ and $Z = (\bar{b}_1(r) - \underline{b}_1(r), \ldots, \bar{b}_n(r) - \underline{b}_n(r))^T$

$$u_i = \sum_{j=1}^{n} t_{ij} z_j \tag{3.12}$$

Let $u_i \geq 0$, $1 \leq i \leq n$, since $z_j \geq 0$, $1 \leq j \leq n$, then $t_{ij} \geq 0$, $1 \leq i, j \leq n$.
Because, if we consider $\exists i \exists j$, $t_{ij} < 0$ and can choose $z_j = e_j$ consequently $u_i < 0$ and this is contradiction. Converse case is obvious .

Theorem 3.3 *[19] The inverse of a nonnegative matrix A is nonnegative if and only if A is a generalized permutation matrix.*

Theorem 3.4 *The fuzzy linear system (2.1) has a fuzzy solution*
If B^{-1} , $B^{-1} - A^{-1}$, $B^{-1} + A^{-1}$ are nonnegative matrices.

Proof. Let $B^{-1} = (t_{ij})$ and $A^{-1} = (s_{ij})$, $1 \leq i, j \leq n$ then

$$U = B^{-1}Z, Y = A^{-1}W \tag{3.13}$$

$u_i = \overline{x}_i(r) - \underline{x}_i(r)$ and $y_i = \overline{x}_i(r) + \underline{x}_i(r)$, $1 \leq i \leq n$, respectively are the solutions of Eq.(3.9) and Eq.(3.10) . Thus we can write : $\overline{x}_i = \frac{1}{2}(y_i + u_i)$

$$\overline{x}_i = \frac{1}{2}\left(\sum_{j=1}^{n} s_{ij} w_j + \sum_{j=1}^{n} t_{ij} z_j\right) \tag{3.14}$$

With replacement $z_j = (\bar{b}_j(r) - \underline{b}_j(r))$ and $w_j = (\bar{b}_j(r) + \underline{b}_j(r))$ in Eq. (3.14) , the result will be

$$\overline{x}_i = \frac{1}{2}\left(\sum_{j=1}^{n} (s_{ij} + t_{ij})\, \bar{b}_j + \sum_{j=1}^{n} (s_{ij} - t_{ij})\, \underline{b}_j\right) \tag{3.15}$$

Since $\bar{b}_j$ is monotonically decreasing and $\underline{b}_j$ is monotonically increasing for all j ,and according to assumptions of theorem, $\overline{x}_i$ to be monotonically decreasing. Similarly $\underline{x}_i = \frac{1}{2}(y_i - u_i)$ is monotonically increasing.

Theorem 3.5 *With notation of theorem (3.4), the fuzzy linear system (2.1) has a fuzzy number solution, if and only if*

$$\begin{cases} u_i \geq 0 \\ |\frac{dy_i}{dr}| \leq -\frac{du_i}{dr} \end{cases} \tag{3.16}$$

where $U = B^{-1}Z$ and $Y = A^{-1}W$.

Proof. Let (FSLE) (2.1) has a fuzzy number solution vector $X = (\widetilde{x}_1, \widetilde{x}_2, \ldots, \widetilde{x}_n)^T$ which $\widetilde{x}_i = (\underline{x}_i(r), \overline{x}_i(r))$. Therefore, $u_i = \overline{x}_i(r) - \underline{x}_i(r) \geq 0$, $i = 1, \ldots, n$. Since $\underline{x}_i = \frac{1}{2}(y_i - u_i)$ is monotonically increasing and $\overline{x}_i = \frac{1}{2}(y_i + u_i)$ is monotonically decreasing, then $\frac{d\underline{x}_i}{dr} \geq 0$ and $\frac{d\overline{x}_i}{dr} \leq 0$. Thus $\frac{d(y_i - u_i)}{dr} \geq 0$, $\frac{d(y_i + u_i)}{dr} \leq 0$ i.e. $-\frac{du_i}{dr} \geq -\frac{dy_i}{dr}$ and $-\frac{du_i}{dr} \geq \frac{dy_i}{dr}$. Consequently, $|\frac{dy_i}{dr}| \leq -\frac{du_i}{dr}$. Conversely is obvious.

4 Numerical example

Example 4.1 *[17] Consider the 2×2 fuzzy system*
$\widetilde{x}_1 - \widetilde{x}_2 = (r, 2 - r)$,
$\widetilde{x}_1 + 3\widetilde{x}_2 = (4 + r, 7 - 2r)$.

$det(A)=4$ and $det(B)=2$, therefore Eq. (3.9) and Eq. (3.10) will have solutions as follow:

$$U=\begin{pmatrix}u_1\\u_2\end{pmatrix}=\begin{pmatrix}\overline{x}_1(r)-\underline{x}_1(r)\\\overline{x}_2(r)-\underline{x}_2(r)\end{pmatrix}=B^{-1}Z=$$

$$\begin{pmatrix}1.5&-0.5\\-0.5&0.5\end{pmatrix}\begin{pmatrix}2-2r\\3-3r\end{pmatrix}=\begin{pmatrix}1.5-1.5r\\0.5-0.5r\end{pmatrix}$$

$$Y=\begin{pmatrix}y_1\\y_2\end{pmatrix}=\begin{pmatrix}\overline{x}_1(r)+\underline{x}_1(r)\\\overline{x}_2(r)+\underline{x}_2(r)\end{pmatrix}=A^{-1}W=$$

$$\begin{pmatrix}0.75&0.25\\-0.25&0.25\end{pmatrix}\begin{pmatrix}2\\11-r\end{pmatrix}=\begin{pmatrix}4.25-0.25r\\2.25-0.25r\end{pmatrix}$$

*$\forall r, 0\le r\le 1$, $u_1=1.5-1.5r$ and $u_2=0.5-0.5r$, both are nonnegative .
Also $\forall r, 0\le r\le 1$, $|\frac{dy_i}{dr}|\le -\frac{du_i}{dr}$, $i=1,2$. The result will be*

$$\overline{x}_1=\frac{1}{2}(y_1+u_1)=2.875-0.875r,$$

$$\underline{x}_1=\frac{1}{2}(y_1-u_1)=1.375+0.625r$$

$$\overline{x}_2=\frac{1}{2}(y_2+u_2)=1.375-0.375r,$$

$$\underline{x}_2=\frac{1}{2}(y_2-u_2)=0.875+0.125r$$

Therefore the fuzzy number solutions are $\widetilde{x}_1=(\underline{x}_1(r),\overline{x}_1(r))$, $\widetilde{x}_2=(\underline{x}_2(r),\overline{x}_2(r))$

Example 4.2 *[17] Consider the 3×3 fuzzy system*
$\widetilde{x}_1+\widetilde{x}_2-\widetilde{x}_3=(r,2-r)$,

$\widetilde{x}_1-2\widetilde{x}_2+\widetilde{x}_3=(2+r,3)$,

$2\widetilde{x}_1+\widetilde{x}_2+3\widetilde{x}_3=(-2,-1-r)$.

$det(A)=-13$ and $det(B)=1$, therefore Eq. (3.9) will has solution as follow:

$$U=\begin{pmatrix}u_1\\u_2\\u_3\end{pmatrix}=\begin{pmatrix}\overline{x}_1(r)-\underline{x}_1(r)\\\overline{x}_2(r)-\underline{x}_2(r)\\\overline{x}_3(r)-\underline{x}_3(r)\end{pmatrix}=B^{-1}Z=$$

$$\begin{pmatrix}5&-2&-1\\-1&1&0\\-3&1&1\end{pmatrix}\begin{pmatrix}2-2r\\1-r\\1-r\end{pmatrix}=\begin{pmatrix}7-7r\\-1+r\\-4+4r\end{pmatrix}$$

$\forall r, 0\le r\le 1$, $u_1=7-7r\ge 0$, $u_2=-1+r\le 0$ and $u_3=-4+4r\le 0$, so this (FSLE) will not have fuzzy number solution.

Example 4.3 *Consider the 2×2 fuzzy system*

$\widetilde{x}_1-2\widetilde{x}_2=(r,2-r)$,

$\widetilde{x}_1+3\widetilde{x}_2=(r,2.9-1.9r)$.

$det(A)=5$ and $det(B)=1$, therefore Eq. (3.9) and Eq. (3.10) will have solutions as follow:

$$U=\begin{pmatrix}u_1\\u_2\end{pmatrix}=\begin{pmatrix}\overline{x}_1(r)-\underline{x}_1(r)\\\overline{x}_2(r)-\underline{x}_2(r)\end{pmatrix}=B^{-1}Z=$$

$$\begin{pmatrix}3&-2\\-1&1\end{pmatrix}\begin{pmatrix}2-2r\\2.9-2.9r\end{pmatrix}=\begin{pmatrix}0.2-0.2r\\0.9-0.9r\end{pmatrix}$$

$$Y=\begin{pmatrix}y_1\\y_2\end{pmatrix}=\begin{pmatrix}\overline{x}_1(r)+\underline{x}_1(r)\\\overline{x}_2(r)+\underline{x}_2(r)\end{pmatrix}=A^{-1}W=$$

$$\begin{pmatrix}0.6&0.4\\-0.2&0.2\end{pmatrix}\begin{pmatrix}2\\2.9-0.9r\end{pmatrix}=\begin{pmatrix}2.36-0.36r\\0.18-0.18r\end{pmatrix}$$

$\forall r, 0\le r\le 1$, $u_1=0.2-0.2r$ and $u_2=0.9-0.9r$, both are nonnegative .

Also $\forall r, 0\le r\le 1$, $|\frac{dy_1}{dr}|\ge -\frac{du_1}{dr}$, $|\frac{dy_2}{dr}|\le -\frac{du_2}{dr}$ so this (FSLE) can not have fuzzy number solution.

5 Conclusion

In this paper we propose a general model for solving a system $n\times n$ of (FSLE).The original system with a matrix $\tilde{A}$ is replaced by two crisp linear system with two matrix A and B which matrix $n\times n$, B is formed with absolute matrix element A. Therefore, the matrix B may be singular even if A is nonsingular.

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