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Research Article

Evaluating the efficiency and classifying the fuzzy data: A DEA based approach

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Abstract

Data envelopment analysis (DEA) has been proven as an efficient technique to evaluate the performance of homogeneous decision making units (DMUs) where multiple inputs and outputs exist. In the conventional applications of DEA, the data are considered as specific numerical values with explicit designation of being an input or output. However, the observed values of the data are sometimes imprecise (i.e. input and output variables cannot be measured precisely) and data are sometimes flexible (measures with unknown status of being input or output are referred to as flexible measures in the literature). In the current paper a number of methods are proposed to evaluate the relative efficiency and to identify the status of fuzzy flexible measures. Indeed, the modified fuzzy DEA models are suggested to accommodate flexible measures. In order to obtain correct results, alternative optimal solutions are considered to deal with the fuzzy flexible measures. Numerical examples are used to illustrate the procedure.

Keywords : Data envelopment analysis; Fuzzy numbers; Flexible measures; Inputs, Outputs.

1 Literature review

Data envelopment analysis (DEA) is a non-parametric technique to measure the relative efficiency of a set of similar units referred to as decision making units (DMUs). In the standard DEA, it is assumed that the input versus output status of each performance measure related to the DMUs has been known. However, in some situations the role of a variable may be flexible. There are some papers investigating flexible measures, among them are Beasley [3] who presented a formulation to evaluate universities in the UK where the variable "research funding" is assumed as both an input and an output. The alternative and corrected version of Beasley's model was suggested by Cook et al. [5]. Then, Cook and Zhu [6] proposed a different method to classify inputs and outputs by considering flexible measures in the CCR ratio model. A revised model was introduced by Toloo in 2009 [10]. Afterwards, Toloo [11] suggested another method by considering alternative solutions. Amirteimoori and Emrouznejad [1] also stated Cook and Zhu's model overestimating the efficiency and suggested an approach to determine the status of flexible measures. Moreover, Amirteimoori and Emrouznejad [2] indicated that the proposed model by Toloo [10] is infeasible in some situations.

Furthermore, there are many contexts on fuzzy data envelopment analysis; for instance, see Wang and Chin [12], Hatami-marbini et al. [7]. Nabahat and Esmaeeli [9] suggested a technique to

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evaluate the efficiency in the presence of flexible and fuzzy measures in order to classify flexible measures. In fact, they applied the α-level based approach but fuzzy DEA models built on the basis of α-level sets require the situation of a series of LP models and thus considerable computational efforts. Therefore, in the current paper the fuzzy expected value approach [12] is modified for containing the flexible measures in the DEA analysis. Indeed, the modified fuzzy DEA models are proposed to identify the status of flexible measures.

The current paper is organized as follows: In Section 2, the proposed methods by Cook and Zhu [6] and Toloo [11] to determine the status of flexible measures are mentioned briefly. Moreover, a summary of fuzzy DEA problem utilized and extended in this paper will be stated. Section 3 provides the suggested models. Numerical examples are presented in Section 4 to illustrate the procedure. Conclusions are reached in Section 5.

2 Preliminaries

In this section, we review the previous studies on flexible measures and the fuzzy data envelopment analysis in brief.

2.1 Flexible measures

Assume the purpose is to evaluate the efficiency of n decision making units; $(DMU_j\,;\, j = 1, ..., n)$ in which there are m inputs $x_{ij}\,, i = 1, ..., m$, s outputs y_{rj}, $r = 1, ..., s$ and l flexible measures w_{lj}, $l = 1, ..., L$. Cook and Zhu [6] proposed the following mathematical programming model to determine the status of flexible measures.

$$\begin{array}{ll} Max & \frac{\sum_{r=1}^{s} u_r y_{ro} + \sum_{l=1}^{L} d_l \gamma_l w_{lo}}{\sum_{i=1}^{m} v_i x_{io} + \sum_{l=1}^{L} (1-d_l) \gamma_l w_{lo}} \\ s.t. & \frac{\sum_{r=1}^{s} u_r y_{rj} + \sum_{l=1}^{L} d_l \gamma_l w_{lj}}{\sum_{i=1}^{m} v_i x_{ij} + \sum_{l=1}^{L} (1-d_l) \gamma_l w_{lj}} \leq 1, \quad j = 1, 2, ..., n \\ & d_l \in \{0, 1\}, \forall l, \; u_r, v_i, \gamma_l \geq 0, \; \forall r, i, l. \end{array} \tag{2.1}$$

Then, they transformed the mentioned model into the mixed integer linear programming by using the Charnes- Cooper transformation [4] and changing of variable. Readers are referred to Cook and Zhu [6] for more information regarding this.

Furthermore, Toloo [11] introduced a new mixed integer linear programming model to overcome the problem of not considering alternative optimal solutions in the previous models. Toloo's model is as follows:

$$\begin{array}{ll} Min & \theta_o \\ s.t & \sum_{j=1}^{n} \lambda_j x_{ij} \leq \theta_o x_{io} \quad \forall i \\ & \sum_{j=1}^{n} \lambda_j y_{kj} \geq y_{ko} \quad \forall k \\ & \sum_{j=1}^{n} \lambda_j w_{lj} \leq \theta_o w_{lo} + M \bar{d}_l \quad \forall l \\ & \sum_{j=1}^{n} \lambda_j w_{lj} \geq w_{lo} - M(1 - \bar{d}_l) \quad \forall l \\ & \bar{d}_l \in \{0, 1\} \; \forall l \quad \lambda_j \geq 0 \; \forall j \end{array} \tag{2.2}$$

where M is a large positive number.

2.2 The fuzzy Data Envelopment Analysis model

In this subsection, the issue of evaluating the efficiency of DMUs in the presence of fuzzy inputs and outputs is mentioned.

Consider there are n DMUs that consume m inputs to produce s outputs. Assume $\tilde{x}_{ij}$, $(i = 1, ..., m)$ and $\tilde{y}_{rj}$, $(r = 1, ..., s)$indicate inputs and outputs of the$j$th DMU$(j = 1, ..., n)$, respectively characterized by trapezoidal fuzzy numbers $\tilde{x}_{ij} = (x^l_{ij}, x^M_{ij}, x^N_{ij}, x^u_{ij})$ and $\tilde{y}_{rj} = (y^l_{rj}, y^M_{rj}, y^N_{rj}, y^u_{rj})$with $x^l_{ij} \geq 0$ and $y^l_{rj} \geq 0$ for $i = 1, ..., m$, $r = 1, ..., s$ and $j = 1, ..., n$.

Wang and Chin [12] showed the efficiency of DMU_jas:

$$\theta_j = \frac{\sum_{r=1}^{s} u_r (y^L_{rj} + y^M_{rj} + y^N_{rj} + y^U_{rj})}{\sum_{i=1}^{m} v_i (x^L_{ij} + x^M_{ij} + x^N_{ij} + x^U_{ij})}, \; j = 1, ..., n. \tag{2.3}$$

Indeed, they used the fuzzy expected value of a fuzzy variable that is defined by Liu and Liu [8]. Readers can refer to Wang and Chin [12], Liu and Liu [8] for more information. Wang and Chin [12] proposed the following LP for determining the efficiency from an optimistic point of view.

$$\begin{array}{ll} Max & \frac{\sum_{r=1}^{s} u_r (y^l_{ro} + y^M_{ro} + y^N_{ro} + y^u_{ro})}{\sum_{i=1}^{m} v_i (x^l_{io} + x^M_{io} + x^N_{io} + x^u_{io})} \\ s.t. & \frac{\sum_{r=1}^{s} u_r (y^l_{rj} + y^M_{rj} + y^N_{rj} + y^u_{rj})}{\sum_{i=1}^{m} v_i (x^l_{ij} + x^M_{ij} + x^N_{ij} + x^u_{ij})} \leq 1\,, j = 1, ..., n. \\ & u_r, v_i \geq 0 \quad i = 1, ..., m\;, \quad r = 1, ..., s. \end{array} \tag{2.4}$$

Then they transformed the model into the linear programming by using the Charnes-Cooper transformation [4] as follows:

$$\begin{array}{ll} Max & \sum_{r=1}^{s} u_r(y_{ro}^l + y_{ro}^M + y_{ro}^N + y_{ro}^u) \\ s.t. & \sum_{i=1}^{m} v_i(x_{io}^l + x_{io}^M + x_{io}^N + x_{io}^u) = 1 \\ & \sum_{r=1}^{s} u_r(y_{rj}^l + y_{rj}^M + y_{rj}^N + y_{rj}^u) - \\ & \sum_{i=1}^{m} v_i(x_{ij}^l + x_{ij}^M + x_{ij}^N + x_{ij}^u) \leq 0, j = 1, ..., n \\ & u_r, v_i \geq 0 \quad i = 1, ..., m \ , \quad r = 1, ..., s. \end{array} \tag{2.5}$$

In the next Section we will extend the fuzzy DEA model by considering flexible measures.

3 Fuzzy DEA models with classifying Inputs and outputs

As the previous section, consider n DMUs consume varying amounts of m different inputs to produce s different outputs. Assume $\tilde{x}_{ij}$ $(i = 1, 2, ..., m)$, $\tilde{y}_{rj}$ $(r = 1, 2, ..., s)$ and $\tilde{w}_{lj}$ $(l = 1, 2, ..., L)$represent the fuzzy input , output and flexible measure of the jth DMU_j $(j = 1, 2, ..., n)$, respectively. All input, output and flexible data are assumed to be characterized by trapezoidal fuzzy numbers $\tilde{x}_{ij} = (x_{ij}^l, x_{ij}^M, x_{ij}^N, x_{ij}^u)$,$\tilde{y}_{rj} = (y_{rj}^l, y_{rj}^M, y_{rj}^N, y_{rj}^u)$ and $\tilde{w}_{lj} = (w_{lj}^l, w_{lj}^M, w_{lj}^N, w_{lj}^u)$,respectively with $x_{ij}^l \geq 0$, $y_{rj}^l \geq 0$ and $w_{lj}^l \geq 0$, for $i = 1, ..., m$, $r = 1, ..., s$ and $l = 1, ..., L$.

3.1 A method for determining the status of fuzzy flexible measures

The following model is suggested to evaluate the performance of DMU_o and to classify inputs and outputs:

Max

$$\frac{\sum_{r=1}^{s}}{\sum_{i=1}^{m}} \frac{u_r(y_{ro}^l+y_{ro}^M+y_{ro}^N+y_{ro}^u)+\sum_{l=1}^{L}\gamma_l d_l(w_{lo}^l+w_{lo}^M+w_{lo}^N+w_{lo}^u)}{v_i(x_{io}^l+x_{io}^M+x_{io}^N+x_{io}^u)+\sum_{l=1}^{L}\gamma_l(1-d_l)(w_{lo}^l+w_{lo}^M+w_{lo}^N+w_{lo}^u)}$$

s.t.

$$\frac{\sum_{r=1}^{s}}{\sum_{i=1}^{m}} \frac{u_r(y_{rj}^l+y_{rj}^M+y_{rj}^N+y_{rj}^u)+\sum_{l=1}^{L}\gamma_l d_l(w_{lj}^l+w_{lj}^M+w_{lj}^N+w_{lj}^u)}{v_i(x_{ij}^l+x_{ij}^M+x_{ij}^N+x_{ij}^u)+\sum_{l=1}^{L}\gamma_l(1-d_l)(w_{lj}^l+w_{lj}^M+w_{lj}^N+w_{lj}^u)}$$

$$\leq 1,$$
$$j = 1, ..., n\, , \; d_l \in \{0, 1\}, \; u_r, v_i, \gamma_l \geq 0$$
$$i = 1, ..., m\ , \quad r = 1, ..., s, \; l = 1, ..., L.$$
(3.6)

It is clear that model (3.6) is nonlinear. Thus, it can be linearized, the same as Cook and Zhu [6], by using the Charnes-Cooper transformation [4] and the changing of variable $\delta_l = d_l\gamma_l$ as follows:

$$\begin{array}{l} Max \ \sum_{r=1}^{s} u_r(y_{ro}^l + y_{ro}^M + y_{ro}^N + y_{ro}^u) \\ + \sum_{l=1}^{L} \delta_l(w_{lo}^l + w_{lo}^M + w_{lo}^N + w_{lo}^u) \\ s.t \ \ \sum_{i=1}^{m} v_i(x_{io}^l + x_{io}^M + x_{io}^N + x_{io}^u) \\ + \sum_{l=1}^{L} \gamma_l(w_{lo}^l + w_{lo}^M + w_{lo}^N + w_{lo}^u) - \\ \quad \sum_{l=1}^{L} \delta_l(w_{lo}^l + w_{lo}^M + w_{lo}^N + w_{lo}^u) = 1, \\ \quad \sum_{r=1}^{s} u_r(y_{rj}^l + y_{rj}^M + y_{rj}^N + y_{rj}^u) \\ + \sum_{l=1}^{L} 2\delta_l(w_{lj}^l + w_{lj}^M + w_{lj}^N + w_{lj}^u) - \\ \quad \sum_{i=1}^{m} v_i(x_{ij}^l + x_{ij}^M + x_{ij}^N + x_{ij}^u) \\ - \sum_{l=1}^{L} \gamma_l(w_{lj}^l + w_{lj}^M + w_{lj}^N + w_{lj}^u) \leq 0 \\ \quad 0 \leq \delta_l \leq M d_l \\ \quad \delta_l \leq \gamma_l \leq \delta_l + M(1 - d_l) \\ \quad d_l \in \{0, 1\}, \quad v_i, u_r \geq 0, \quad \forall r, i; \gamma_l, \delta_l \geq 0, \ \forall l. \end{array} \tag{3.7}$$

M is a large positive number. Where $d_l = 1$, factor l designates as an output and it is an input where $d_l = 0$.

3.2 An alternative method to determine the status of fuzzy flexible measures

In this subsection an alternative method is proposed to classify inputs and outputs and to identify the efficiency. Actually, model (2.2) will extend to evaluate the efficiency when fuzzy data are presented.

$$\begin{array}{ll} Min & \theta \\ s.t & \sum_{j=1}^{n} \lambda_j(x_{ij}^l + x_{ij}^M + x_{ij}^N + x_{ij}^u) \leq \\ & \theta(x_{io}^l + x_{io}^M + x_{io}^N + x_{io}^u) \qquad i = 1, ..., m \\ & \sum_{j=1}^{n} \lambda_j(y_{rj}^l + y_{rj}^M + y_{rj}^N + y_{rj}^u) \geq \\ & (y_{ro}^l + y_{ro}^M + y_{ro}^N + y_{ro}^u) \qquad r = 1, ..., s \\ & \sum_{j=1}^{n} \lambda_j(w_{lj}^l + w_{lj}^M + w_{lj}^N + w_{lj}^u) \leq \\ & \theta(w_{lo}^l + w_{lo}^M + w_{lo}^N + w_{lo}^u) + M\tilde{d}_l \quad l = 1, ..., L \\ & \sum_{j=1}^{n} \lambda_j(w_{lj}^l + w_{lj}^M + w_{lj}^N + w_{lj}^u) \geq \\ & (w_{lo}^l + w_{lo}^M + w_{lo}^N + w_{lo}^u) - M(1 - \tilde{d}_l), \\ & l = 1, ..., L \quad \tilde{d}_l \in \{0, 1\} \ \forall l \ , \lambda_j \geq 0 \ \forall j. \end{array} \tag{3.8}$$

If $\tilde{d}_l = 1$, factor l is an output and when $\tilde{d}_l = 0$, it is an input.

After solving model (3.7) or model (3.8), the status of flexible measures will be determined by applying the majority role among DMUs that do not have similar results of selecting flexible measure as an input or output. Since triangular fuzzy numbers and crisp numbers are special cases of

Table 1: Data for numerical Example 4.1.

DMU	Input-1	Input-2	output-1	flexible-1
1	(12,14,16,19)	(1.1,2.3,3.6,4.3)	(6,8,10,11)	(2,3,4,5)
2	(3,6,7,9)	(1.9,2.6,3.7,4.8)	(12,15,16,18)	(1,4,6,10)
3	(5,6,7,8)	(1.6,3.4,4.1,5.5)	(2,3,6,9)	(1,3,4,8)
4	(1,3,4,6)	(3.5,4.1,5.2,6.7)	(7,10,12,14)	(1,2,3,4)
5	(5,7,8,10)	(1,4,5,6)	(4,6,8,10)	(1,2,4,5)

Table 2: The results of models (3.7) and (3.8).

DMU	Flexible as Input	Flexible as output	Efficiency of model (3.7)	Efficiency of model (3.8)	d^*
1	0.8059	0.767	0.8059	0.767	0
2	1	1	1	1	0 or 1
3	0.3891	0.7326	0.7326	0.3891	1
4	1	1	1	1	0 or 1
5	0.6386	0.4762	0.6386	0.4762	0

Table 3: Inputs and the flexible measure of higher education institutions.

DMU	$\tilde{x}_1$	$\tilde{x}_2$	$\tilde{w}_1$
1	(432,530,600)	(30,50,100)	(70,250,400)
2	(800,2600,4760)	(220,300,340)	(1090,1480,1890)
3	(118,310,490)	(15,25,30)	(12,36,80)
4	(300,1640,2900)	(60,400,800)	(780,900,1120)
5	(100,490,924)	(50,80,120)	(50,100,170)
6	(800,2600,4550)	(270,460,1250)	(2000,2900,3910)
7	(330,430,500)	(0,0,0)	(40,280,560)
8	(200,800,1600)	(101,120,150)	(301,770,1190)
9	(200,400,900)	(8,30,60)	(17,40,60)
10	(30,500,1100)	(60,86,110)	(112,340,570)

Table 4: Outputs of higher education institutions.

DMU	$\tilde{y}_1$	$\tilde{y}_2$	$\tilde{y}_3$
1	(70,140,220)	(18,28,32)	(0,0,0)
2	(130,370,635)	(24,65,124)	(8,17,24)
3	(36,46,54)	(4,8,14)	(2,4,6)
4	(200,280,375)	(6,50,94)	(0,0,0)
5	(43,92,140)	(14,32,49)	(1,10,12)
6	(56,350,646)	(41,160,310)	(2,3,5)
7	(38,73,98)	(20,40,60)	(10,11,12)
8	(40,194,375)	(12,34,50)	(0,0,0)
9	(26,70,80)	(2,15,30)	(0,0,0)
10	(44,87,100)	(32,40,45)	(4,14,30)

trapezoidal fuzzy numbers, the above models are also applicable to crisp and triangular measures.

Now, suppose inputs, outputs and flexible measures of DMUs are triangular fuzzy numbers. We indicate inputs, outputs and flexible measures as $\tilde{x}_{ij} = (x^l_{ij}, x^M_{ij}, x^u_{ij})$, $\tilde{y}_{rj} = (y^l_{rj}, y^M_{rj}, y^u_{rj})$ and $\tilde{w}_{lj} = (w^l_{lj}, w^M_{lj}, w^u_{lj})$, respectively with $x^l_{ij} \geq 0$, $y^l_{rj} \geq 0$and $w^l_{lj} \geq 0$, for $i = 1, ..., m$, $r = 1, ..., s$ and $l = 1, ..., L$. Therefore, models (3.7) and (3.8) can be substituted with models (3.9) and (3.10)

Table 5: The results of models (3.9) and (3.10)

DMU	Flexible as Input	Flexible as output	Efficiency of model (3.9)	Efficiency of model (3.10)	d^*
1	1	1	1	1	0 or 1
2	0.6047	0.6689	0.6689	0.6047	1
3	1	0.7379	1	0.7379	0
4	0.6428	0.723	0.723	0.6428	1
5	1	0.9173	1	0.9173	0
6	0.7246	1	1	0.7246	1
7	1	1	1	1	0 or 1
8	0.8668	1	1	0.8668	1
9	1	0.5591	1	0.5591	0
10	1	1	1	1	0 or 1

respectively as shown below:

$$\begin{aligned}
&Max\ \textstyle\sum_{r=1}^{s} u_r(y_{ro}^l + 2y_{ro}^M + y_{ro}^u) + \textstyle\sum_{l=1}^{L} \delta_l(w_{lo}^l + 2w_{lo}^M + w_{lo}^u) \\
&s.t\ \ \textstyle\sum_{i=1}^{m} v_i(x_{io}^l + 2x_{io}^M + x_{io}^u) + \textstyle\sum_{l=1}^{L} \gamma_l(w_{lo}^l + 2w_{lo}^M + w_{lo}^u) - \textstyle\sum_{l=1}^{L} \delta_l(w_{lo}^l + 2w_{lo}^M + w_{lo}^u) = 1 \\
&\quad \textstyle\sum_{r=1}^{s} u_r(y_{rj}^l + 2y_{rj}^M + y_{rj}^u) + \textstyle\sum_{l=1}^{L} 2\delta_l(w_{lj}^l + 2w_{lj}^M + w_{lj}^u) - \textstyle\sum_{i=1}^{m} v_i(x_{ij}^l + 2x_{ij}^M + x_{ij}^u) - \textstyle\sum_{l=1}^{L} \gamma_l(w_{lj}^l + 2w_{lj}^M + w_{lj}^u) \le 0 \\
&\quad 0 \le \delta_l \le M d_l \\
&\quad \delta_l \le \gamma_l \le \delta_l + M(1 - d_l) \\
&\quad d_l \in \{0,1\}, \quad v_i, u_r \ge 0, \ \ \forall r, i; \gamma_l, \delta_l \ge 0, \ \forall l.
\end{aligned} \tag{3.9}$$

and

$$\begin{aligned}
&Min \quad \theta \\
&s.t\ \ \textstyle\sum_{j=1}^{n} \lambda_j(x_{ij}^l + 2x_{ij}^M + x_{ij}^u) \le \theta(x_{io}^l + 2x_{io}^M + x_{io}^u) \quad i = 1, ..., m \\
&\quad \textstyle\sum_{j=1}^{n} \lambda_j(y_{rj}^l + 2y_{rj}^M + y_{rj}^u) \ge (y_{ro}^l + 2y_{ro}^M + y_{ro}^u) \quad r = 1, ..., s \\
&\quad \textstyle\sum_{j=1}^{n} \lambda_j(w_{lj}^l + 2w_{lj}^M + w_{lj}^u) \le \theta(w_{lo}^l + 2w_{lo}^M + w_{lo}^u) + M\tilde{d}_l \quad l = 1, ..., L \\
&\quad \textstyle\sum_{j=1}^{n} \lambda_j(w_{lj}^l + 2w_{lj}^M + w_{lj}^u) \ge (w_{lo}^l + 2w_{lo}^M + w_{lo}^u) - M(1 - \tilde{d}_l) \quad l = 1, ..., L \\
&\quad \tilde{d}_l \in \{0,1\}\ \forall l \ \ , \lambda_j \ge 0\ \forall j.
\end{aligned} \tag{3.10}$$

Therefore, the status of flexible measures, where triangular fuzzy numbers exist, will be determined by solving model (3.9) and (3.10).

4 Numerical Examples

In this section two numerical examples are stated to illustrate the proposed approach.

Example 4.1 *Assume there are 5 DMUs with two inputs, one output and one flexible measure in which all data are trapezoidal numbers. Data are given in Table 1. Models (3.7) and (3.8) are calculated.*

Results of model (3.7) and model (3.8) are given in the Table 2. As can be seen in Table 2, the results of selecting a flexible measure as input or output are the same for DMU_2 and DMU_4. Thus they should not be considered to classify inputs and outputs. It is clear, 2 DMUs treat flexible measure as input while only one DMU considers it as output. Therefore, the flexible measure is considered as input according to the majority role.

Example 4.2 *Suppose we wish to evaluate institutions of higher education. Two factors, general expenditure ($\tilde{x}_1$) and equipment expenditure ($\tilde{x}_2$) are assumed as inputs; undergraduate students ($\tilde{y}_1$), postgraduate research ($\tilde{y}_2$) and postgraduate teaching ($\tilde{y}_3$) are considered as outputs, and research income ($\tilde{w}_1$) is a flexible measure. All inputs, outputs and flexible measures are triangular fuzzy numbers. Inputs and the flexible measure are shown in Table 3 and outputs can be seen in Table 4.*

The results of models (3.9) and (3.10) are shown in Table 5. As can be seen in Table 5, 3 DMUs consider the flexible measure as input while 4 DMUs assume it as output, and the flexible measure can be deemed as input or output without any effect on efficiency in DMUs 1, 7, and 10. According to the majority role, the flexible measure is considered as an output.

Obviously, models (3.7) and (3.9) are easier and more efficient. Because in models (3.7) and (3.9) when $\theta_j^* = 1$, the least efficiency score is selected in comparing the results of selecting a flexible measure as input or output. Thus, the resulted maximum amount of comparing them is also one. Therefore, DMUs with this condition will be eliminated from consideration to determine the status, conveniently as shown in Toloo [11].

5 Conclusions

In real applications, there are many situations in which measures can play either the role of input or output while measures are vague. For this purpose, the present paper has been proposed two models to evaluate the performance and to determine the status of fuzzy flexible measures where flexible and fuzzy data exist. Actually, fuzzy DEA models have been modified and extended. In the proposed models, similar to the prior models for evaluating the efficiency of DMUs and classifying flexible measures, selecting suitable M is a significant subject for calculating accurate efficiency. Numerical examples have been provided to illustrate the technique.

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