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Research Article

Dirichlet series and approximate analytical solutions of MHD flow over a linearly stretching sheet

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Abstract

The paper presents the semi-numerical solution for the magnetohydrodynamic (MHD) viscous flow due to a stretching sheet caused by boundary layer of an incompressible viscous flow. The governing partial differential equations of momentum equations are reduced into a nonlinear ordinary differential equation (NODE) by using a classical similarity transformation along with appropriate boundary conditions. Both nonlinearity and infinite interval demand novel the mathematical tools for their analysis. The solution of the resulting third order nonlinear boundary value problem with an infinite interval is obtained using fast converging Dirichlet series method and approximate analytical method viz. method of stretching of variables. These methods have the advantages over pure numerical methods for obtaining the derived quantities accurately for various values of the parameters involved at a stretch and they are valid in much larger parameter domain as compared with HAM, HPM, ADM and the classical numerical schemes. Also, these methods require less computer memory space as compared with pure numerical methods.

Keywords : Magnetohydrodynamics (MHD); Boundary layer flow; Shrinking sheet; Dirichlet series; Powell's method; Method of stretching variables.

1 Introduction

The boundary layer flow of conducting fluid fast stretching surface has numerous important engineering applications. The flow situations encountered in many industrial applications are extrusion of polymer sheet from a die or in the drawing of plastic films and heat-treated materials travelling between a feed roll and a wind-up roll or materials manufactured by extrusion process, glass-fiber and paper production, cooling of metallic sheets, crystal growing etc. In all these cases, the mechanical properties of the final product strictly depend on the stretching and cooling rates during the process. In the manufacture of these sheets the melt issues from a slit and is subsequently stretched to achieve the desired thickness. The pioneering work of Sakiadis [1, 2] gives various aspects of boundary layer flow on a continuously stretching surface with constant speed. Crane [3, 4] analysed this configuration for the stretching surfaces. Specifically Crane's problem for flow of an incompressible viscous fluid fast stretching surface has become classic in the literature. It admits an exact analytical solution and many researchers have analysed various aspects of this problem. Among those Mcleod and Rajgopal [5] discussed the uniqueness of exact analytical solution. Gupta and Gupta [6] examined the

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heat and mass transfer on a stretching flow subject to suction or injection. Brady and Acrivos [7] analysed the flow inside a stretching channel or tube with accelerating velocity. Wang [8] examined the fluid flow due to a stretching cylinder. In another paper, Wang [9], analysed the three dimensional axi-symmetric flow due to stretching surfaces. Wang [10] and Usha and Sridharan [11] have discussed the unsteady flows induced by stretching film. The extended work of Crane's problem for second grade, micropolar and power-law fluid models have been studied by Rajagopal et al[12], Sankara and Watson[13] and Andersson et al. [14] respectively. The contributions due to the boundary layer flow over a stretching surface have been made by Pavlov[15], Sachdev et al. [16], Mahapatra et al. [17], Ahmed and Asghar[18], Makinde and Charles [19], Makinde et al. [20], Recently, Vajravelu and Prasad [21] have analysed various aspects of MHD stretching/shrinking sheet problems involving boundary layer theory. Vajravelu et al.[22] have analysed the fluid flow and heat transfer over permeable stretching cylinder. Hayat et al. [23] have analysed MHD flow of nanofluids over an exponentially stretching sheet in a porous medium with convective boundary conditions. Nadeem et al. [24] have examined MHD flow of a Casson fluid over an exponentially shrinking sheet. Zaimi et al. [25] have discussed boundary layer flow and heat transfer over a nonlinearly permeable stretching/shrinking sheet in a nanofluid.

The present investigation is to analyse the MHD viscous flow caused by a stretching sheet. The solution of the resulting third order nonlinear boundary value problem with an infinite interval is obtained using Dirichlet series method and method of stretching of variables. The third order nonlinear differential equation with infinite boundary admits a Dirichlet series solution; necessary conditions for the existence and uniqueness of these solutions may also be found in [26, 27]. The specific type of the boundary condition i.e. $f'(\infty)=0$, the Dirichlet series solution is particularly useful for obtaining solution and the derived quantities exactly. A general discussion of the convergence of the Dirichlet series may also be found in Riesz [28]. The accuracy as well as uniqueness of the solution can be confirmed using other powerful semi-numerical schemes. Sachdev et al. [16] have analysed various problems from fluid dynamics of stretching sheet using this approach and found more accurate solution compared with earlier numerical findings. Vishwanath et al. [29, 30, 31] and Kudenatti et al. [32] have analysed the problems from MHD boundary layer flow with nonlinear stretching sheet using these methods and found more accurate results compared with the classical numerical methods. In this paper, we present Dirichlet series solution and an approximate analytical method-method of stretching of variables. This method is quite easy to use especially for nonlinear ordinary differential equations and requires less computer time compared with pure numerical methods and easy to solve, compared with other approximate methods (for example, Homotopy perturbation method (HPM) Pade' technique, Adomain decomposition methods (ADM)) etc.

The present paper is structured as follows. In Section 2 the mathematical formulation of the proposed problem with relevant boundary conditions is given. Section 3 is devoted to the solution of the problem using Dirichlet series. Section 4 gives the solution by means of method of stretching of variables. In Section 5 detailed results obtained are compared with the corresponding numerical schemes and Section 6 is about the conclusion.

2 Mathematical Formulation

Consider a steady two-dimensional incompressible boundary layer flow of an electrically conducting isothermal Newtonian liquid over a linearly stretching sheet as shown in Fig. 1. The

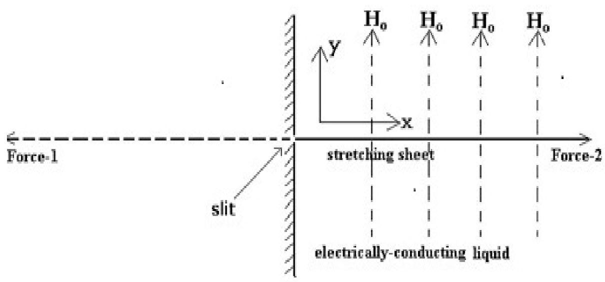

Figure 1: Schematic of the stretching sheet problem.

uniform transverse magnetic field H_0 acts parallel to the y-axis and the conducting liquid in the space $y>0$ is considered. Two equal and opposite forces are applied along the x-axis, so that the wall is stretched and keep the origin fixed. A Hartmann formulation is done for the MHD problem. The conservation of mass and momentum boundary layer equation for the quadratic stretching sheet problem are as (Makinde et al.[20])

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{2.1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=v\frac{\partial^2 u}{\partial y^2}-\frac{\sigma H_0^2}{\rho}u. \tag{2.2}$$

The relevant boundary conditions for the present flow are

$$u(x,y)=cx, v=v_w \qquad at \qquad y=0 \tag{2.3}$$

$$u(x,y)=0 \qquad as \qquad y\to\infty \tag{2.4}$$

where u and v are the liquid velocity components in the x and y directions respectively, $c>0$ is the streching rate, ν is the kinematic viscosity, H_0 is the applied magnetic field and σ is the electrical conductivity of the fluid. The constant v_w is the suction/ blowing parameter, where $v_w<0$ corresponds to the suction and $v_w>0$ to the blowing of the fluid and v_w=0 , characterize the impermeable. Introducing the dimensionless variables f and η as

$$\psi=\sqrt{c\nu}xf(\eta) \qquad and \qquad \eta=y\sqrt{\frac{c}{\nu}} \tag{2.5}$$

The velocity components u and ν are related to the physical stream function ψ defined by

$$\psi=\frac{\partial\psi}{\partial y}, \qquad v=-\frac{\partial\psi}{\partial x}. \tag{2.6}$$

is introduced such that the continuity equation is automatically satisfied. Introducing the stream function $\psi(x,y)$ and the non-dimensional form of Eqs. (2.2-2.4) becomes

$$v\frac{\partial^3\psi}{\partial y^3}+\frac{\partial(\psi,\frac{\partial\psi}{\partial y})}{\partial(x,y)}-\frac{\sigma H_0^2}{\rho}\frac{\partial\psi}{\partial y}=0, \tag{2.7}$$

$$\frac{\partial\psi}{\partial y}=cx, \frac{\partial\psi}{\partial x}=v_w \qquad at \qquad y=0, \tag{2.8}$$

$$\frac{\partial\psi}{\partial y}=0, \qquad as \qquad y\to\infty, \tag{2.9}$$

Substituting the similarity transformation for stream function in equation 2.5 into equations 2.7 - 2.9, we obtain

$$f'''+ff''-f'^2-Mf'=0,'=\frac{d}{d\eta}. \tag{2.10}$$

$$f(0)=f_w, f'(0)=1, f'(\infty)=0. \tag{2.11}$$

where $M=\frac{H_0^2\sigma}{c\rho}$ is the Magnetic parameter and $f_w=\frac{v_w}{\sqrt{c\rho}}$ suction /blowing parameter. In this case, we assume $f(0)$=f_w and $f_w>0$ correspond to suction, $f_w<0$ correspond to blowing and $f_w=0$ is the case when the surface is impermeable. The functions $f(\eta)$ allow us to determine the skin friction coefficient given as $C_f=-f''(0)$.

The exact solution of MHD flow of Newtonian fluid is obtained by Chakrabarti and Gupta [33] and non-Newtonian fluid by Bhattacharyya et al [34]. We assume the solution in more general form as

$$f(\eta)=a+b\exp(-\lambda\eta). \tag{2.12}$$

where a, b and λ are constants with $\lambda>0$. Substituting Eq. 2.12 into Eq. 2.10 and Eq. 2.11, we get

$$b=-\frac{1}{\lambda}, \qquad a=f_w+\frac{1}{\lambda}\text{and} \qquad \lambda=\frac{1}{2}(f_w+\sqrt{4+f_w^2+4M}) \tag{2.13}$$

So, the analytical solution reduces to

$$f=f_w-\frac{2}{f_w\pm\sqrt{4+f_w^2+4M}} \frac{2\exp((\frac{1}{2})(-f_w\mp\sqrt{4+f_w^2+4M})\eta)}{f_w\pm\sqrt{4+f_w^2+4M}} \tag{2.14}$$

3 Dirichlet Series Solution

We seek an elegant semi-numerical method i.e. Dirichlet series solution of Eq. 2.10 satisfying last boundary condition $f'(\infty)=0$ automatically in the form (Kravchenko and Yablonskii [26, 27])

$$f \quad = \quad \gamma_1+6\gamma\sum_{i=1}^{\infty}b_i a^i\exp(-i\gamma\eta) \tag{3.15}$$

where γ and a are parameters which are to be determined. Substituting Eq. 3.15 into Eq. 2.10, we get

$$\sum_{i=1}^{\infty}(-\gamma^2 i^3+\gamma\gamma_1 i^2+Mi)b_i a^i \exp(-i\gamma\eta)$$
$$+6\gamma^2\sum_{i=2}^{\infty}\sum_{k=1}^{i-1}[k^2-k(i-k)]b_k b_{i-k}a^i\exp(-i\gamma\eta)$$
$$=0 \tag{3.16}$$

For $i=1$ we have

$$\gamma_1=\gamma^2-M. \tag{3.17}$$

Substituting Eq. 3.17 into Eq. 3.16 the recurrence relation for obtaining coefficients is given by

$$b_i=\frac{6\gamma^2}{i(i-1)[\gamma^2 i+M]}\sum_{k=1}^{i-1}[k^2-k(i-k)]b_k b_{i-k} \tag{3.18}$$

for $i=2,3,4,\dots$. If the Eq. 3.15 converges absolutely when $\gamma=0$ for some η_0 , this series converges absolutely and uniformly in the half plane Reη $\geq$Reη_0 and represents an analytic $\frac{2\pi}{\gamma}$ periodic function $f=f(\eta_0)$ such that $f'(\infty)=0$ ([kra26]). The Eq. 3.15 contains two free unknown parameters namely a and γ which are to be determined from the remaining boundary conditions of Eq. 2.11 at $\eta=0$

$$f(0)=\frac{\gamma^2-M}{\gamma}+6\gamma\sum_{i=1}^{\infty}b_i a^i=\alpha_1 \tag{3.19}$$

and

$$f'(0)=6\gamma^2\sum_{i=1}^{\infty}(-i)b_i a^i=\beta_1 \tag{3.20}$$

The solution of the above transcendental Eq. 3.19 and Eq. 3.20 yield constants a and γ . The solution of the above transcendental equations is equivalent to the unconstrained minimization of the functional

$$[\frac{\gamma^2-M}{\gamma}+6\gamma\sum_{i=1}^{\infty}b_i a^i-\alpha_1]^2+[6\gamma^2\sum_{i=1}^{\infty}(-i)b_i a^i-\beta_1]^2 \tag{3.21}$$

We use Powell's method of conjugate directions (Press et. al. [35]) which is one of the most efficient techniques for solving unconstrained optimization problems. This helps in finding the unknown parameters a and γ uniquely for different values of the parameters α_1 ,β_1 and M. Alternatively, Newton's method is also used to determine the unknown parameters a and γ accurately. The physical quantity of the interest for the problem is shear stress at the surface and velocity profiles. The shear stress at the surface of the problem is given by

$$f''(0)=6\gamma\sum_{i=1}^{\infty}b_i a^i(i\gamma)^2 \tag{3.22}$$

The velocity profiles of the problem is given by

$$f'(\eta)=6\gamma^2\sum_{i=1}^{\infty}(-i)b_i a^i\exp(-i\gamma\eta) \tag{3.23}$$

4 Approximate Analytical solution

Most of the nonlinear ODE arising in MHD problems are not amenable for obtaining analytical solutions. In such situations, attempts have been made to develop an approximate analytical method for the solution of these problems. The numerical approach is always based on the idea of stretching of variables of the flow problems. Method of stretching of variables is used here for the solution of such problems. In this method, we have to choose suitable derivative function H' such that the derivative boundary conditions are satisfied automatically and integration of H' will satisfy the remaining boundary condition. Substitution of this resulting function into the given equation gives the residual of the form $R(\xi,\alpha)$ which is called defect function. Using Least squares method, the residual of the defect function can be minimized. For details see Ariel [36]. Using the transformation $f=f_w+F$ into Eq. 2.10, we get

$$F'''+(f_w+F)F''-F'^2-MF'=0,'=\frac{d}{d\eta} \tag{4.24}$$

and the relevant boundary conditions 2.11 become

$$F(0)=0,F'(0)=1,F'(\infty)=0 \tag{4.25}$$

We introduce two variables ξ and G in the form

$$G(\xi)=\alpha F(\eta) \qquad and \qquad \xi=\alpha\eta \tag{4.26}$$

where $\alpha > 0$, is an amplification factor. In view of Eq. 4.26, the system 4.24-4.25 are transformed to the form

$$\alpha^2 G''' + (f_w\alpha + G)G'' - G'^2 - MG' = 0, \quad (4.27)$$

where $' = \frac{d}{d\xi}$ and the boundary conditions in Eq. 4.25 become

$$G(0) = 0, G'(0) = 1, G'(\infty) = 0 \quad (4.28)$$

We choose a trail velocity profile

$$G' = \exp(-\xi) \quad (4.29)$$

which satisfies the derivative conditions in Eq. 4.28. Integrating Eq. 4.29 with respect to ξ from 0 to ξ using conditions 4.28, we get

$$G = 1 - \exp(-\xi). \quad (4.30)$$

Substituting Eq. 4.30 into Eq. 4.27, we get the residual of defect function

$$R(\xi, \alpha) = (-\alpha^2 + f_w\alpha - 1 + M)\exp(-\xi). \quad (4.31)$$

Using the Least squares approximation method as discussed in Ariel [?], the equation 4.31 can be minimized for which

$$\frac{\partial}{\partial\alpha}\left(\int_0^\infty R^2(\xi, \alpha)d\xi\right) = 0. \quad (4.32)$$

Substituting 4.31 into equation 4.32 and solving cubic equation in α for a positive root, we get

$$\alpha = \frac{1}{2}\left(f_w \pm \sqrt{-4 + 4M + f_w^2}\right) \quad (4.33)$$

Once the amplification factor is calculated, then using Eq. 4.24, original function f can be written as

$$f = f_w + \frac{1}{\alpha}(1 - \exp(-\alpha\eta)). \quad (4.34)$$

With α defined in Eq. 4.33. Thus Eq. 4.34 gives the solution of Eq. 2.10 for all values of f_w and M .

5 Result and discussion

The megnetohydrodynamic (MHD) viscous flow due to a stretching sheet caused by boundary layer of an incompressible viscous flow is analysed by the more suggestive ways by using the Fortran programming and Mathematica. The third order nonlinear ordinary differential Eq. 2.11 subject to the infinite boundary conditions 2.11 (has been solved semi-numerically using Dirichlet series method and method of stretching of variables for different values of the parameters f_w and M . Table 1. shows that the results of skin friction for different parameters f_w and M which agree accurately with available pure numerical method. Fig. 2 presents the effect of increasing magnetic field strength on the momentum boundary layer thickness. The effect of increasing magnetic strength on the boundary layer creating the drag force that opposes the fluid motion causing the velocity to decrease. Increase the Magnetic parameter it slow down the motion of the fluid and decreases the boundary layer thickness. Fig. 3. shows a similar trend with increasing suction $f_w > 0$ while an injection $f_w < 0$ causes the boundary layer to thicken by increasing the fluid velocity. Fig. 4. shows that the skin friction increases generally with increase in the intensity of magnetic field. As the intensity of fluid suction increases, a skin friction increases and an injection causes a decrease in the local skin friction at the surface.

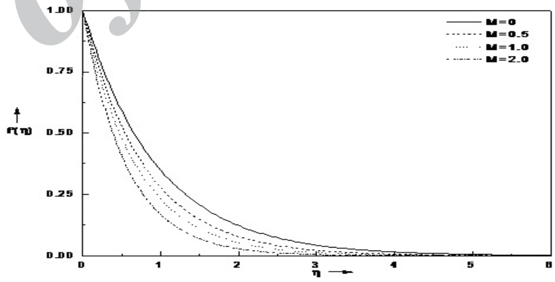

Figure 2: Effect of increasing magnetic field intensity on velocity profile for $f_w = 0.1$.

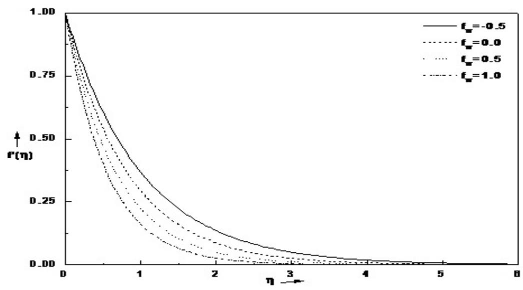

Figure 3: Effect of success/injection on velocity profile for $M = 0.5$.

Table 1: Comparison of result obtained by Dirichlet series, Method of stretching of variables [MSV] and numerical method

M	f_w	Dirichlet series			Numerical [20]	MSV
		a	γ	f"(0)	f"(0)	f"(0)
0	0	-1.000000	0.999999	-0.999999	-1.000000	-1.000000
0	0.5	-0.609612	1.280776	-1.280776	-1.280777	-1.280776
.5	0.5	-0.074074	1.500000	-1.500000	-1.500000	-1.500000
1.0	0.5	-0.058622	1.686141	-1.686141	-1.686140	-1.686140
0.5	-0.5	-0.166666	1.000000	-0.999999	-1.000000	-1.000000
0.5	-1.0	-0.246139	0.822876	-0.822875	-0.822875	-0.822875

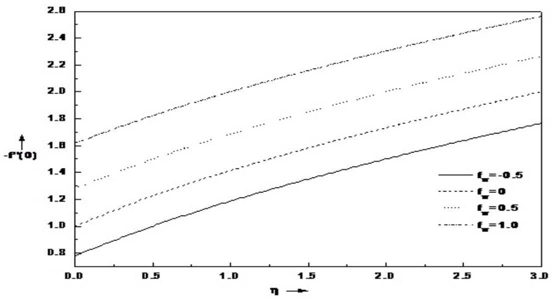

Figure 4: Effect of parameter variation on skin friction coefficients.

Nomenclature:

c constant rate of stretching $[s^{-1}]$

f similarity function

ν kinematic viscosity $[m^2s^{-1}]$

σ electrical conductivity $[mhom^{-1}]$

ρ density $[kgm^{-3}]$

H_0 strength of the magnetic field $[wm^{-2}]$

$\psi(x,y)$ stream function $[m^2s^{-1}]$

$k_l - k_1 = \frac{k_0 c}{\mu}$ viscoelastic parameter

l characteristic length [m]

$H - H_0\sqrt{\frac{\sigma}{c\rho}}$ Hartmann number

$M - H^2$ Magnetic Parameter

u velocity component along the sheet $[ms^{-1}]$

v velocity component normal to the sheet $[ms^{-1}]$

v_w suction / blowing parameter

x coordinate along the sheet $[m]$

y coordinate normal to the sheet $[m]$

η similarity variable

α amplification factor

6 Conclusion

In this article, we describe the analysis of boundary value problem for third order nonlinear ordinary differential equation over an infinite interval arising in MHD boundary layer. The semi-numerical schemes described here offer advantages over solutions obtained by HAM, HPM, ADM and other numerical methods etc. The convergence of the Dirichlet series method is given. The results are presented in Table and graphically, the effects of the emerging parameters are discussed.

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سریهای دیریکله و تقریب جوابهای تحلیلی از جریان MHD بر روی صفحات خطی

چکیده:

با توجه به تبدیل لاپلاس معکوس، از قضیه سوخوتن واندرپل استفاده می کنیم تا یک معادله خود پیچش را برای توابع میتگ-لفلر تعمیم یافته شامل جملاتی از تبدیلات لاپلاس و ملین تشکیل دهیم. هم چنین در حالت های خاص، جواب هایی از معادلات خود پیچش معرفی شده مربوط به توابع μ- ولترا را نتیجه می گیریم. به علاوه چندین معادله خود پیچش جدید را با اعمال تبدیل لاپلاس از توابع میتگ-لفلر تعمیم یافته بیان می کنیم. سرانجام به عنوان یک کاربرد از معادلات خود پیچش در سیستم های ترمودینامیکی، از تبدیل لاپلاس برای حل کردن معادله بولتزمان استفاده می کنیم و جواب این معادله را درجملاتی از توابع میتگ-لفلر تعمیم یافته به دست می آوریم.