

A cultural algorithm for data clustering

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Abstract

Clustering is a widespread data analysis and data mining technique in many fields of study such as engineering, medicine, biology and the like. The aim of clustering is to collect data points. In this paper, a Cultural Algorithm (CA) is presented to optimize partition with N objects into K clusters. The CA is one of the effective methods for searching into the problem space in order to find a near optimal solution. This algorithm has been tested on different scale datasets and has been compared with other well-known algorithms in clustering, such as K-means, Genetic Algorithm (GA), Simulated Annealing (SA), Ant Colony Optimization (ACO) and Particle Swarm Optimization (PSO) algorithm. The results illustrate that the proposed algorithm has a good proficiency in obtaining the desired results.

Keywords : Data Clustering; Genetic Algorithm; Cultural Algorithm; Particle Swarm Optimization.

1 Introduction

Clustering is an important problem that must often be solved as a part of the more complicated tasks in image processing, anomaly detection, medicine, construction management, marketing, data retrieval, reliability, portfolio optimization, selecting supplier and data envelopment analysis. Clustering is partitioning a set of objects into clusters, where the objects in the same cluster are more similar to each other. Hence, the clustering method is also known as hierarchy clustering method, mixture-model clustering, learning network clustering, and objective function based clustering and partition-based clustering. (Po et al. 2009) [15]. K-means clustering algorithm is one of the most popular and classic clustering algorithms (Duda and Hart 1973[2]). This method is simple, efficient and

fast with linear time complexity. However, the results of k-means highly depend on the initial state in order for them to reach the local optimal solution. There are a large number of researchers who have applied different optimization techniques to eliminate this problem. For example, Genetic Algorithm-based method to solve the clustering problem was proposed by Cowgill et al. 1999 [7]; Maulik and Bandyopadhyay 2000 [13]. Tabu search-based heuristic for clustering was developed by Sung and Jin 2000) [20]. Shelokar et al. (2004) [19] have proposed an ant colony optimization based approach for optimal clustering N objects into K clusters. An HBMO algorithm, which was inspired by the process of marriage in real honey-bee world, was used to solve the clustering problem by Fathian et al. (2007) [9]. Kao et al. (2008) [12] have introduced a hybrid technique that combines the PSO algorithm, NelderMead simplex search, and K-means algorithm. Cao and Cios (2008) [3] have proposed a hybrid algorithm (GAKREM) based on

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the genetic algorithm, K-means and logarithmic regression expectation maximization. GAKREM has three main advantages, namely, there is no need to specify the number of clusters a priori, it avoids being trapped into a local optimum, and it requires not lengthy computations. Niknam and Amiri (2010) [14] have presented a hybrid evolutionary optimization algorithm according to a fuzzy adaptive PSO, ACO and K-means, called FAPSO-ACOK, to solve the clustering problem. Using the advantages of the K-means algorithm and also the output of hybrid FAPSOACO algorithm is considered as an initial state of k-means. Chuang et al. (2011) [5] showed the outstanding application of PSO in multi-dimensional space clustering performance. However, the rate of convergence when searching for global optima is still not sufficient (Kao et al. 2008) [12]. For this reason, they combined Chaotic-map Particle Swarm Optimization (CPSO) with an accelerated convergence rate strategy. This technique allows the ACP SO algorithm to cluster arbitrary data better than previous algorithms. Results of the conducted experimental trials on a variety of data sets taken from several real-life situations demonstrate that ACP SO was superior to the K-means, PSO, NM-PSO, CPSO, K-PSO and K-NM-PSO algorithms (Kao et al. 2008) [12]. Cura (2012) [8] have proposed a particle swarm optimization approach to clustering. Apart from many of the previously-proposed approaches, PSO algorithm is applicable when the number of clusters is either known or unknown.

Cultural algorithm (CA) is a branch of evolutionary computation introduced by Reynolds in 1995 (Reynolds 1995) which has three major components as population space, belief space and communication protocol where the belief space is a knowledge component. In CA there is a dual evolution- dual improvement evolutionary based algorithms. This improvement can lead to a faster convergence and also a robust performance (Xie et al. 2009; Ma and Wang 2009). In this study, CA algorithm is extended for solving clustering problems. The performance of the algorithm has been tested on different scale datasets and compared with several other proposed clustering algorithms.

In the following, the data clustering analysis is discussed in Section 2. In Section 3, Cultural algorithm is presented. The implementation of

the CA algorithm in clustering is illustrated in Section 4. In Section 5 the performance of the proposed algorithm has been demonstrated and compared with that of the original GA, SA, PSO and K-means for different datasets, and finally, conclusions are presented in Section 6.

2 Data Clustering Analysis

The K-means algorithm (Duda and Hart 1973[2]) searches to find the cluster centers, C_1, C_2, \dots, C_K , in such a way that the sum of the squared distances (i.e., objective function) of each data point (X_i) to its nearest cluster center (C_k) is minimized, as shown in Equation (2.1), where d is a distance function. Typically, d is chosen as the Euclidean distance which is derived from Minkowski metric and can be defined as Equation (2.2).

$$f(X, C) = \sum_{i=1}^N \left(\min_{k=1,2,\dots,K} d(X_i - C_k) \right)^2. \quad (2.1)$$

$$d(x, y) = \left(\sum_{i=1}^m |x_i - y_i|^r \right)^{\frac{1}{r}} \quad (2.2)$$

$$\Rightarrow d(x, y) = \sqrt{\sum_{i=1}^m (x_i - y_i)^2}$$

The main procedures of the K-means algorithm are as follows.

- (1) To randomly select k points as initial centroids.
- (2) To assign each point to the nearest centroids.
- (3) To update the locations of each centroid by calculating the mean value of the objects assigned to it.
- (4) To stop, if the termination criterion is met, or to go to step 2, if otherwise. This means that either the iterations reach the maximum number or no change in the location of the centroids occurs.

3 Cultural algorithm (CA)

A cultural algorithm (CA) is a dual-inheritance system (Reynolds 1994 [17]), which maintains two search spaces, that is, the population space

to represent a genetic component based on Darwinian principles, and a belief space to represent a cultural component. It is the latter that distinguishes CAs from other evolutionary algorithms (EA). The belief space models the cultural information about the population, while the population space represents the individuals on a genotypic and/or phenotypic level. Both the population and belief spaces evolve in parallel, with both influencing one another. A communication protocol therefore forms an integral part of a CA. Such a protocol defines two communication channels; one for a select group of individuals to adapt the set of beliefs, and another defining the way that the beliefs influence all of the individuals in the population space. The pseudo code and general framework of the cultural algorithm are presented in Algorithm 1 and Figure 1, respectively. In every iteration, individuals are first

Algorithm 1 Cultural Algorithm

Set the generation counter, $t = 0$;
 Create and initialize the population space, $C(0)$;
 Create and initialize the belief space, $B(0)$;
while stopping condition(s) not true **do**
 Evaluate the fitness of each $x_i(t) \in C(t)$;
 Adjust ($B(t)$, Accept ($C(t)$));
 Variate ($C(t)$, Influence ($B(t)$));
 $t = t + 1$;
 Select the new population;
End

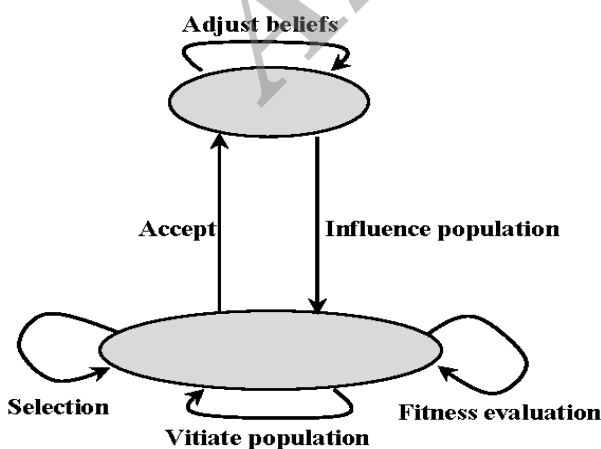


Figure 1: Framework of cultural algorithm.

evaluated using the fitness function specified for

the EA on the population level. An acceptance function is then used to determine which individuals from the current population have an influence on the current beliefs. The experience of the accepted individuals is then used to adjust the beliefs. The adjusted beliefs are then used to influence the evolution of the population. The variation operators use the beliefs to control the changes in individuals. This is usually achieved through self-adapting control parameters, as functions of the beliefs. The population space is searched using any of the standard EAs, for example an EP (Reynolds and Chung 1997 [18]) or GA (Reynolds 1991 [16]). Recently, Particle Swarm Optimization (PSO) has been used on the population space level (Coelho and Mariani 2006 [6]).

3.1 Belief space

In general, the belief space contains at least two knowledge components (Reynolds and Chung 1997 [18]):

- A situational knowledge component, which keeps track of the best solutions found at each generation
- A normative knowledge component, which provides standards for individual behaviors, used as guidelines for mutational adjustments to individuals. In the case of function optimization, the normative knowledge component maintains a set of intervals, one for each dimension of the problem being solved. These intervals characterize the range of what is believed to be good areas to search in each dimension.

If only these two components are used, the belief space is formed as,

$$B(t) = (S(t), N(t)) \quad (3.3)$$

Where $S(t)$ represents the situational knowledge component, and $N(t)$ represents the normative knowledge component. The situational component is the set of best solutions is formed as,

$$S(t) = \{\hat{y}_l(t) : l = 1, \dots, n_S\} \quad (3.4)$$

and the normative component is represented as

$$N(t) = (X_1(t), X_2(t), \dots, X_{n_x}(t)) \quad (3.5)$$

where, for each dimension, the following information is stored:

$$X_j(t) = (I_j(t), L_j(t), U_j(t)) \quad (3.6)$$

I_j denotes the closed interval, $I_j(t) = [x_{\min,j}(t), x_{\max,j}(t)] = \{x : x_{\min,j} \leq x \leq x_{\max,j}\}$, $L_j(t)$ is the score for the lower bound, and $U_j(t)$ is the score for the upper bound.

3.2 Acceptance Functions

The acceptance function determines which individuals from the current population will be used to shape the beliefs for the entire population. Static methods use absolute ranking, based on fitness values, to select the top $n\%$ individuals. Providing that the number of individuals remains the same, the acceptance function chooses individuals who impact the creation of the current belief space. A number of different classes of acceptance functions can be employed. One class of functions uses only the information of top individuals in the population to renew the belief space. For example, if the top 20% rule is applied, then just the top 20% individuals impact the belief space in each generation. A second class of functions, called modified-top 20% is also static in nature but uses temporal information about the number of generations used so far to determine the number of individuals accepted at time (generation number) t .

3.3 Adjusting the Belief Space

For the purpose of this part, it is assumed that the belief space maintains a situational and normative knowledge component, with the number of accepted individuals, $nB(t)$, known, the two knowledge components can be updated as follows (Coelho and Mariani, 2006 [6]):

- Situational knowledge: Assuming that only one element is kept in the situational knowledge component,

$$S(t+1) = \{\hat{y}(t+1)\} \quad (3.7)$$

Where

$$\hat{y}(t+1) = \begin{cases} \min_{l=1,2,\dots,n_B(t)} \{x_l(t)\} & \text{if} \\ \min_{l=1,2,\dots,n_B(t)} \{x_l(t)\} \prec \hat{y}(t) & \\ \hat{y}(t) & \text{otherwise} \end{cases} \quad (3.8)$$

- Normative knowledge: In adjusting the normative knowledge component, a conservative approach is followed when narrowing intervals, thereby delaying too early exploration. Widening of intervals is applied more progressively. The interval update rule is as follows:

$$x_{\min,j}(t+1) = \begin{cases} x_{l,j}(t)\{x_l(t)\} & \text{if} \\ x_{l,j}(t) \leq x_{\min,j} & \\ \text{or } f(x_l(t)) \prec L_j(t) & \\ x_{\min,j}(t) & \text{otherwise} \end{cases} \quad (3.9)$$

$$x_{\max,j}(t+1) = \begin{cases} x_{l,j}(t)\{x_l(t)\} & \text{if} \\ x_{l,j}(t) \geq x_{\max,j} & \\ \text{or } f(x_l(t)) \prec U_j(t) & \\ x_{\max,j}(t) & \text{otherwise} \end{cases} \quad (3.10)$$

$$L_j(t+1) = \begin{cases} f(x_l(t)) & \text{if} \\ x_{l,j}(t) \leq x_{\min,j} & \\ \text{or } f(x_l(t)) \prec L_j(t) & \\ L_j(t) & \text{otherwise} \end{cases} \quad (3.11)$$

3.4 Influence Functions

Beliefs are used to adjust individuals in the population space to conform closer to the global beliefs. The adjustments are realized via influence functions. The belief space is used to determine the mutational step sizes, and the direction of changes. Reynolds and Chung proposed four ways in which the knowledge components can be used within the influence function. In this paper we use only one of them as follow:

The normative component is used to determine step sizes and the situational component is used to determine change directions:

$$x'_{ij}(t) = \begin{cases} x_{ij}(t) + |\sigma_{ij}(t) \cdot \beta \cdot N_{ij}(0,1)| & \text{if} \\ x_{ij}(t) \prec \hat{y}_j(t) \in S(t) & \\ x_{ij}(t) - |\sigma_{ij}(t) \cdot \beta \cdot N_{ij}(0,1)| & \text{if} \\ x_{ij}(t) \succ \hat{y}_j(t) \in S(t) & \\ x_{ij}(t) + \sigma_{ij}(t) \cdot \beta \cdot N_{ij}(0,1) & \text{otherwise} \end{cases} \quad (3.12)$$

In Equation (3.12), β is control parameter and $N_{ij}(0,1)$ shows a random number from a normal distribution where the mean value and the standard deviation are zero and one respectively. Finally, σ_{ij} is the strategy parameter associated with component j of individual i which is obtained as follow:

$$\sigma_{ij}(t) = \text{size}(I(t)) \quad (3.13)$$

4 Application of CA in clustering

To implement the CA algorithm in the clustering problem, the following steps should be taken and repeated:

- Step 1: Initialize belief space, population space and parameters of algorithm.
- Step 2: Evaluate the objective function value for each individual in the initial population and the individuals with less fitness value are chosen as the third step.
- Step 3: Choosing top individuals (% n) as acceptance function.
- Step 4: Forming and updating the belief space with the individuals accepted (Adjusting).
- Step 5: Update the population using belief space (Influence).
- Step 6: Check the termination criteria. If the current iteration number reaches the predetermined maximum iteration number, stop the search procedure; otherwise, go back to Step 2.

5 Results and discussion

We test the CA on five different scale datasets and compared with other well-known algorithms. All algorithms are implemented in MATLAB software and executed on a 2GHz laptop with 6GB of RAM. Two of datasets are artificial taken from Kao et al. (2008) [12] and the three of them are well-known iris, thyroid, and wine datasets taken from Machine Learning Laboratory (Blake and Merz [1]). They have been considered by many authors to study and evaluate the performance of their algorithms, and can be described as follows:

Dataset 1: Artificial data set one ($n = 600$, $d = 2$, $k = 4$). This is a two featured problem with four unique classes. A total of 600 patterns were drawn from four independent bivariate normal distributions, where classes were distributed according to

$$M_2 = \left(\mu = \begin{pmatrix} \omega_i \\ \omega_i \end{pmatrix}, \Sigma \begin{bmatrix} 0.5 & 0.05 \\ 0.05 & 0.5 \end{bmatrix} \right)$$

$$i = 1, 2, \dots, 4, \omega_1 = -3, \omega_1 = 0, \omega_1 = 3, \omega_1 = 6$$

μ and Σ being mean vector and covariance matrix, respectively. The data set is illustrated in Figure 2.

Dataset 2: Artificial data set two ($n = 250$, $d = 3$, $k = 5$). This is a three featured problem with five classes, where every feature of the classes was distributed according to Class 1-Uniform (85, 100), Class 2-Uniform(70, 85), Class 3-Uniform(55, 70) Class 4-Uniform(40, 55), Class 5-Uniform(25, 40). The data set is illustrated in Figure 2.

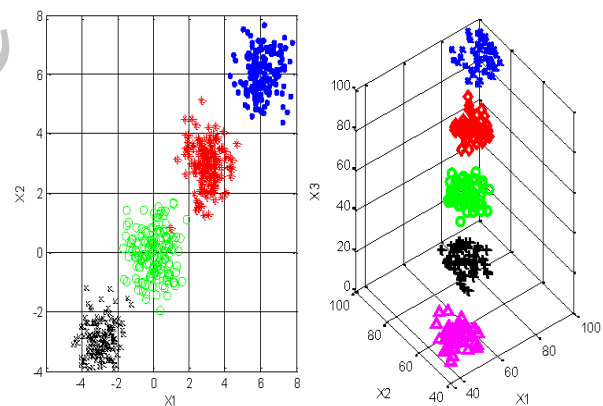


Figure 2: Two artificial data sets.

Dataset 3: iris dataset. The Iris flower dataset or Fisher's Iris dataset is a multivariate dataset introduced by Sir Ronald Fisher (1936) as an example of discriminate analysis. This dataset contains three categories of 50 objects each, where each category refers to a type of iris plant. In the iris dataset, there are 150 instances with four attributes, which are sepal length in cm, sepal width in cm, petal length in cm and petal width in cm.

Dataset 4: wine dataset. These data are

the results of a chemical analysis of wines grown in the same region in Italy extracted from three different cultivars. This dataset contains 178 instances with 13 continuous numeric attributes. The attributes are alcohol, malic acid, ash, alkalinity of ash, magnesium, total phenols, flavanoids, nonflavanoid phenols, proanthocyanins, color intensity, hue, OD280/OD315 of diluted wines and proline. All attributes are continuous. There is no missing attribute value.

Dataset 5: Contraceptive Method Choice (CMC) dataset. This dataset is a subset of the 1987 National Indonesia Contraceptive Prevalence Survey. The samples were married women who either were not pregnant or did not know if they were at the time of interview. The problem is to predict the current contraceptive method choice (no use of long-term methods, or short-term methods) of a woman based on her demographic and socio economic characteristics. The attributes are wife age, wife education, husband's education, number of children ever born, wife's religion, wife's now working, husband's occupation, standard-of-living index, media exposure, and contraceptive method used. To evaluate the performance of the CA, we have compared it with the following clustering algorithms: K-means, GA, TS, SA, ACO and PSO which are taken from Niknam and Amiri (2010) [14]. There are four control parameters in CA, the Population Size PS, the Acceptance Function AF, the Maximum Iteration MI and control parameter μ . They are set as follows: PS = 100, AF = %30, MI = 100 and $\mu = 0.3$. The comparison of results for each dataset is based on the best solution, obtained after more than 20 different simulations, for each algorithm. The sum of the intra-cluster distances, i.e. the distances between data vectors within a cluster and the centroid of this cluster, as defined in Equation 2.1 is used to measure the quality of a clustering. Clearly, the smaller the sum of the distances is, the higher the quality of clustering. MATLAB software is used in a 2GHz laptop with 6GB of RAM, to encode this algorithm. The results of K-means, GA, SA, ACO and PSO can be found in Niknam and Amiri (2010) [14]. Highest values are indicated in bold type. The results of the simulations for comparing the proposed algorithm with different algorithms

Data set	Criteria	Algorithms					
		K-means	GA	SA	ACO	PSO	CA
Artset1	Best	516.04	518.09	518.95	517.87	515.93	515.87
	(Std)	295.84	189.86	195.15	2.01	180.24	0.00
	Average	721.57	638.094	684.68	519.88	627.74	515.87
Artset2	Best	1746.9	1743.20	1743.20	1743.20	1743.20	1743.20
	(Std)	720.66	437.05	429.02	134.06	415.02	2.01
	Average	2762.00	2667.30	2686.84	1948.97	2517.20	1745.40
Iris	Best	97.333	113.98	97.45	97.10	96.89	96.54
	(Std)	14.631	14.56	2.01	0.36	0.34	0.01
	Average	106.05	125.19	99.95	97.17	97.23	96.58
Wine	Best	16555.68	16530.53	16473.48	16530.53	16345.96	16292.29
	(Std)	793.21	0.00	753.08	0.00	85.49	0.01
	Average	18061.00	16,530.53	17521.09	16530.53	16417.47	16292.31
CMC	Best	5842.20	5705.63	5849.03	5701.92	5700.98	5693.82
	(Std)	47.16	50.36	50.86	45.63	46.95	0.00
	Average	5893.60	5756.59	5893.48	5819.13	5820.96	5693.82

Table 1: Comparison of different clustering algorithms.

on Artset1, Artset2, Iris, and Wine and Liver disorder datasets are represented in table1. The results obtained by CA algorithm for Artset1 dataset are equal to 515.87, which significantly is better than other algorithms. For the Artset2 dataset, CA such as GA, SA, ACO and PSO algorithm is converged to the global optimum of 1743.20. Note that the standard deviation of solution obtained by CA is lower than other algorithm. For the Iris datasets, the proposed algorithm is converged to the global optimum of 96.54, while other algorithms cannot reach this value even after more than 20 runs. For the Wine dataset, the CA algorithm has the best result. Finally, For the Liver disorder dataset, the CA algorithm achieved the optimum value of 9851.72, which is significantly better than the other algorithms.

In summary, the above statements prove that in all datasets the proposed CA algorithm is superior to the other algorithms. It can find high quality solutions and provides small standard deviation. In other words, the CA algorithm converges to global optimum in all the runs while the other algorithms may get trapped in local optimum solutions.

In order to illustrate the significant differences among the results obtained by the proposed CA and the other the clustering algorithms, Statistical testing is used. Specifically, the Friedman test, as well as the ImanDavenport test, is employed to determine whether there are significant differences in the results of the clustering algorithms. If statistically significant differences exist, then we proceed with the Holland as a

post hoc test, in which is used to compare the control method, against the remaining ones. For more details about classification problem, one can refer to (Garcia et al. 2010) [11]. We used $\alpha = 0.05$ as the level of confidence in all cases.

Table 2 depicts the average ranking of clustering algorithms computed through the Friedmans test. The proposed CA algorithm stand alone in the first rank, followed by PSO, ACO, GA, SA and K-means, successively. Table 3 presents the p-value computed by the Friedman test and the ImanDavenport test, which confirm the existence of significant differences among the performance of all the clustering algorithms. Therefore, the

Friedman Ranking	K-means	GA	SA	ACO	PSO	CA
	4.8	4.5	4.6	3.5	2.2	1.4

Table 2: Average ranking of clustering algorithms.

Method	Statistical value	p-Value	Hypothesis
Friedman	14.29	0.014	Rejected
Iman-Davenport	5.3371	0.0028	Rejected

Table 3: Results of Friedmans and ImanDavenport tests.

i	Algorithm	Z	p-Value	1-(1- α) ⁱ	Hypothesis
5	K-means	2.4286	0.015158	0.2262	Rejected
4	SA	2.2857	0.022272	0.1855	Rejected
3	GA	2.2143	0.026808	0.1426	Rejected
2	ACO	1.5	0.133614	0.0975	Not rejected
1	PSO	0.5714	0.567728	0.05	Not rejected

Table 4: Results of Hollands method (CA is the control algorithm).

Hollands method is carried out as a post hoc test to detect effective statistical differences between the control approach, i.e., the one with the lowest Friedmans rank, and the remaining approaches, the results of which are shown in Table 4. The results of the Holms method reveal that the control algorithm (CA) is statistically better than the K-means, GA and SA. In the ACO and PSO cases there is no significant difference based on the Hollands method results. However, the results reported in Table 1 show that the CA algorithm achieved the best results among all the algorithms.

6 Conclusion

Clustering has a various range of application in many fields of study such as engineering, medicine, biology and the like. In this paper, we presented CA algorithm to clustering data vectors for five different scale datasets. CA algorithm could minimize the objective function of the clustering problem in an N-dimensional Euclidean space and also this algorithm can be applied when the number of clusters is known and the clusters are clearly defined. Simulation results clearly presented the efficiency and effectiveness of the proposed CA algorithm which in the future studies this algorithm can be use in a large number of fields. By combining this algorithm with other algorithms, more efficiency and progress is expected.

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کاربرد ماتریس های عملیاتی دقیق برای حل معادلات نوع لین-امدن در فیزیک نجومی

چکیده:

هدف از این مقاله کاربرد ماتریس های عملیاتی دقیق به جای ماتریس های عملیاتی تقریبی شناخته شده برای حل معادلات نوع لین-امدن می باشد. تا به حال، ماتریس های عملیاتی دقیق توسط نویسندگان این مقاله، تنها برای حل معادلات دیفرانسیل با درجه غیر خطی پایین به کار رفته اند که قدرت اصلی روش را به خوبی نشان نمی دهند. لذا، در این مقاله روش را برای حل معادلات لین-امدن به کار می بریم. برای نشان دادن قدرت و دقت روش ارائه شده از ابزار های نرم-1 و نرم باقیمانده استفاده خواهیم کرد و نشان خواهیم داد که ماتریس های عملیاتی دقیق دقت بالاتری نسبت به ماتریس های عملیاتی تقریبی از قبل شناخته شده دارند.