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New characterization of some linear groups

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Abstract

There are a few finite groups that are determined up to isomorphism solely by their order, such as $\mathbb{Z}_2$ or $\mathbb{Z}_{15}$. Still other finite groups are determined by their order together with other data, such as the number of elements of each order, the structure of the prime graph, the number of order components, the number of Sylow p-subgroups for each prime p, etc. In this paper, we investigate the possibility of characterizing the projective special linear groups $L_n(2)$ by simple conditions when $2^n - 1$ is a prime number. Our result states that: $G \cong L_n(2)$ if and only if $|G| = |L_n(2)|$ and G has one conjugacy class length $\frac{|L_n(2)|}{2^n-1}$, where $2^n - 1 = p$ is a prime number. Furthermore, we will show that Thompson's conjecture holds for the simple groups $L_n(2)$, where $2^n - 1$ prime is a prime number. By Thompson's conjecture if L is a finite non-Abelian simple group, G is a finite group with a trivial center, and the set of the conjugacy classes size of L is equal to G, then $L \cong G$.

Keywords : Projective special linear groups; conjugacy class size; Thompson's conjecture.

1 Introduction

Throughout this paper, groups under consideration are finite, and by a simple group, we always mean a non-Abelian simple. In this paper, we investigate the possibility of characterizing $L_n(2)$ by simple conditions when $2^n - 1$ is a prime number.

For related results, Chen et al. in [5] shows that the projective special linear groups $L_2(p)$ recognizable by their order and one conjugacy class length, where p is a prime number. As a consequence of their result, they showed that Thompson's conjecture is valid for $L_2(p)$.

Put $N(G) = \{n : G$ has a conjugacy class of size $n\}$. By Thompson's conjecture if L is a finite non-Abelian simple group, G is a finite group with a trivial center, and $N(G) = N(L)$, then $L \cong G$.

Similar characterizations have been found in [11] for the groups: sporadic simple groups, and simple K_3-groups (a finite simple group is called a simple K_n-group if its order is divisible by exactly n distinct primes).

The *prime graph* of a finite group G that denoted by $\Gamma(G)$ is the graph whose vertices are the prime divisors of G and where prime p is defined to be adjacent to prime q ($\neq p$) if and only if G contains an element of order pq.

We denote by $\pi(G)$ the set of prime divisors of $|G|$. Let $t(G)$ be the number of connected components of $\Gamma(G)$ and let $\pi_1, \pi_2, \ldots, \pi_{t(G)}$ be the connected components of $\Gamma(G)$. If $2 \in \pi(G)$, then we always suppose $2 \in \pi_1$.

We can express $|G|$ as a product of integers m_1,

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$m_2, \dots, m_{t(G)}$, where $\pi(m_i) = \pi_i$ for each i. The numbers m_i are called the order components of G. In particular, if m_i is odd, then we call it an odd component of G. Write $OC(G)$ for the set $\{m_1, m_2, \dots, m_{t(G)}\}$ of order components of G and $T(G)$ for the set of connected components of G. According to the classification theorem of finite simple groups and [10, 12, 9], we can list the order components of finite simple groups with disconnected prime graphs as in Tables 1-4 in [4].

If n is an integer, then denote the r-part of n by $n_r = r^a$ or by $r^a \parallel n$, namely, $r^a \mid n$ but $r^{a+1} \nmid n$. The other notation and terminology in this paper are standard, and the reader is referred to [6] if necessary.

2 Preliminary Results

For the proof of the main theorem we need to the following Lemmas:

Lemma 2.1 *[7, Lemma 8.1] Let $q > 1$ be an integer, m be a nature number, and p be an odd prime. If p divides $q-1$, then $(q^m - 1)_p = m_p.(q-1)_p$.*

Lemma 2.2 *[13] Let a, b and n be positive integers such that $(a, b) = 1$. Then there exists a prime p with the following properties:*

p divides $a^n - b^n$,

p does not divide $a^k - b^k$ for all $k < n$, with the following exceptions: $a = 2$, $b = 1$, $n = 6$ and $a + b = 2^k$, $n = 2$.

Remark 2.1 *If $b = 1$, the prime p is called the Zsigmondy prime. If p is a Zsigmnody of $a^n - 1$, then Fermat's little theorem shows that $n \mid p - 1$. Put $Z_n(a) = \{p : p$ is a Zsigmondy prime of $a^n - 1\}$. If $r \in Z_n(a)$ and $r \mid a^m - 1$, then $n \mid m$.*

3 Main results

By [1, Corollary 2.11], $L_n(2)$ has one conjugacy class length $\frac{|L_n(2)|}{2^n-1}$.

Theorem 3.1 *Let G be a group. Then $G \cong L_n(2)$ if and only if $|G| = |L_n(2)|$ and G has one conjugacy class length $\frac{|L_n(2)|}{2^n-1}$, where $2^n - 1 = p$ is a prime number.*

Proof. The necessity of the theorem can be checked easily. We only need to prove the sufficiency. Since $2^n - 1$ is a prime, by [8], n is a prime. If $n = 2$ or 3, then since $L_2(2) \cong S_3$ and $L_3(2) \cong L_2(7)$, by [5] $G \cong L_n(2)$. Thus we only consider that $n \geq 5$.

By hypothesis, there exists an element x of order p in G such that $C_G(x) =< x >$ and $C_G(x)$ is a Sylow p-subgroup of G. By the Sylow theorem, we have that $C_G(y) =< y >$ for any element y in G of order p . So, $\{p\}$ is a prime graph component of G and $t(G) \geq 2$. In addition, p is the maximal prime divisor of $|G|$ and an odd order component of G. We are going to prove the theorem 3.1 in the following steps.

Step 1. G has a normal series $1 \trianglelefteq H \trianglelefteq K \trianglelefteq G$ such that H and G/K are π_1-groups, K/H is a non-Abelian simple group and H is a nilpotent group.

Let $g \in G$ be an element of order p, then $C_G(g) =< g >$. Set $H = O_{p'}(G)$ (the largest normal p'-subgroup of G). Then H is a nilpotent group since g acts on H fixed point freely. Let K be a normal subgroup of G such that K/H is a minimal normal subgroup of G/H. Then K/H is a direct product of copies of some simple group. Since $p \mid |K/H|$ and $p^2 \nmid |K/H|$, K/H is a simple group. Since $< g >$ is a Sylow p-subgroup of K, $G = N_G(< g >)K$ by the Frattini argument and so $|G/K|$ divides $p - 1$.

If $|K/H| = p$, then by Lemma 2.2, there is a prime $r \in Z_{n-1}(2) \cap \pi(G)$ and so $|L_n(2)|_r = |2^{n-1} - 1|_r \leq |G|_r$. Since $\pi(G) = \pi(K) \cup \pi(H) = \pi_1(G) \cup \pi_2(G)$, then $r \in \pi(H)$. Since H is nilpotent, a Sylow r-subgroup is normal in G. It follows that the Sylow p-subgroup of G acts fixed point freely on the set of elements of order r and so $p \mid |L_n(2)|_r - 1$. Thus $p \leq |L_n(2)|_r \leq ((2^{\frac{n-1}{2}} - 1)^2)_r < 2^{n-1} - 1 < p$; a contradiction.

If K/H has an element of order rq where r and q are primes, then G has also such element. Hence by definition of order components, an odd order component of G must be an odd order component of K/H. Note that $t(K/H) \geq 2$.

Step 2. K/H is isomorphic to $L_n(2)$.

According to the classification theorem of finite simple groups and the results in Tables 1–4 in [4], K/H is an alternating group, sporadic group or simple group of Lie type.

First we prove that K/H can not be an alternating group A_m.

Let $K/H \cong A_m$ with $m \geq 5$, then since $2^n - 1 = p$ is a prime and $p \in \pi(K/H)$, $m \geq 2^n - 1$. Thus there is a prime $u \in \pi(A_m) \cap \pi(G)$ such that $\frac{p+1}{2} < u < p$. Since $|G|= |L_n(2)|$, there exists $t \in \{2i, i : 1 < i < n-1\} \cup \{n\}$ such that $u \in Z_t(2)$. Obviously $u > \frac{2^n-1+1}{2} = 2^{n-1}$ and so $u = 2^{n-1}+1$. Since n, u are primes, then $n-1 = 2^k+1$ and so $n = 2$; a contradiction.

Let K/H be sporadic simple groups, we can rule out this case by considering their odd order component since the odd components of K/H is $p = 2^n - 1$. Therefore, K/H is isomorphic to a simple group of Lie type. We consider the following cases.

Case 1: Let $t(K/H) = 2$. Then we have that $OC_2(K/H) = p = 2^n - 1$.

1.1. Let $K/H = C_m(q)$, where $m = 2^u > 2$, then $\frac{q^m+1}{(2,q-1)} = 2^n - 1$. If q is odd, then $q^m - 1 = 2^{n+1} - 4$. On the other hand, $q^m - 1 = 2^2(2^{n-1} - 1)$. Since $2 \mid (q-1)$ and $m = 2^u$, then by Lemma 2.1, $(q^m-1)_2 = (q-1)_2.m_2 = 2^2$. But $m \geq 3$, $(q^m-1)_2 .m_2 \geq 2^3$; a contradiction. If q is even, then $q^m - 1 = 2^{n-1} - 1$ and hence, $m = 1$, $n = 1$; a contradiction.

Similarly, we can rule out the cases $K/H = B_m(q)$ or $C_m(q)$ with $m = 2^u \geq 4$.

1.2. Let $K/H = C_r(3)$ or $B_r(3)$, then $\frac{3^r-1}{2} = 2^n - 1$. Thus $3^r = 2^{n+1} - 1$, which contradicts Lemma 2.2.

Similarly, we can rule out these cases $K/H = D_r(3)$ or $D_{r+1}(3)$.

1.3. Let $K/H = C_r(2)$, then $2^r - 1 = 2^n - 1$ and so $n = r$. Therefore, $2^n + 1 \mid |G|= |L_n(2)|$; a contradiction. Similarly we can rule out the cases $K/H = D_r(2)$ or $D_{r+1}(2)$.

1.4. Let $K/H = D_r(5)$ where $r \geq 5$, then $\frac{5^r-1}{4} = 2^n - 1$. Thus $5^r = 2^{n+2} - 3$ and so $r = 1 = n$ or $r = 3$, $n = 5$; a contradiction.

1.5. Let $K/H = {}^2D_m(3)$, where $9 \leq m = 2^r + 1$ and m is not a prime, then $\frac{3^{m-1}+1}{2} = 2^n - 1$ and hence, $3(3^{m-2} - 1) = 2^n$; a contradiction. Also we can rule out $K/H = {}^2D_{m+1}(2)$.

1.6. Let $K/H = {}^2D_m(2)$, where $m = 2^r + 1 \geq 5$, then $2^{m-1} + 1 = 2^n - 1$ and hence, $2^{m-2} = 2^{n-1} - 1$; a contradiction.

1.7. Let $K/H = {}^2D_r(3)$, where $r \neq 2^m + 1 \geq 5$, then $\frac{3^r+1}{4} = 2^n - 1$ and hence, $3^r = 2^{n+2} - 5$. Thus $n = 3$ and $r = 3$; a contradiction.

1.8. Let $K/H = G_2(q)$, where $2 < q \equiv \varepsilon \mod 3$ and $\varepsilon = \pm 1$, then $q^2 - \varepsilon q + 1 = 2^n - 1$ and hence, $(q-2)(q+1) = 2^n$ or $(q+2)(q-1) = 2^n$ and hence, $q = 2$ or 3 and $n = 2$; a contradiction.

1.9. Let $K/H = {}^2F_4(2)$. Since $|{}^2F_4(2)|= 2^{11}.3^3.5^2.13$, $2^n - 1 = 13$; a contradiction. Also we can rule out $K/H = {}^2A_3(2)$.

1.10. Let $K/H = L_r(q)$, where $(r, q) \neq (3, 2)$, $(3, 4)$. Since $\frac{q^r-1}{(q-1)(r,q-1)} = 2^n - 1$, then $r = n$ and $q = 2$, as desired.

1.11. Let $K/H = U_r(q)$, then $\frac{q^r+1}{(q+1)(r,q+1)} = 2^n - 1$.

(a) Let q is odd. If $(r, q+1) = 1$, then $q^{r-1} - q^{r-2} + ... - q + 1 = 2^n - 1$ and hence, $q^{r-1} - q^{r-2} + ... - q = 2(2^{n-1} - 1)$. It follows that $q = 2$; a contradiction. If $(r, q+1) = r$, then $r = 2$ or $3 \leq r$ is a prime.

If $r = 2$, then $(q+1) \mid (q^2+1)$; a contradiction. Thus $r \geq 3$ and so $r \mid (q+1) \mid (q^r+1)$. It follows that $r \mid q^{2r} - 1$. Then by Fermat's little theorem, $r \mid \varphi(2r) = r - 1$, and so $r = 1$; a contradiction.

(b) Let q is even. If $(r, q+1) = 1$, then $q^{r-1} - q^{r-2} + ... - q + 1 = 2^n - 1$ and hence, $q^{r-1} - q^{r-2} + ... - q = 2(2^{n-1} - 1)$. It follows that $q = 2$ and $r = 2$; a contradiction. If $(r, q+1) = r$, then $3 \leq r$ is a prime and so $r \mid (q+1) \mid (q^r+1)$. It follows that $r \mid q^{2r} - 1$. Then by Fermat's little theorem, $r \mid \varphi(2r) = r - 1$, and so $r = 1$; a contradiction.

1.12. Let $K/H = L_{r+1}(q)$, with $(q-1) \mid (r-1)$. Since $\frac{q^r-1}{(r,q-1)} = p$, $p \in Z_r(q)$ and hence $r \mid (p-1) = 2^n - 2 = 2(2^{n-1} - 1)$. It follows that $r = 2$ or $r \in Z_{n-1}(2)$.

(a) Let $r = 2$. Then $q = 4$ or 2. If $q = 4$, then $p = 15$; a contradiction. Hence $q = 2$, $p = 3$ and $n = 2$. It follows that K/H is isomorphic to $L_3(2)$, as desired.

(b) Let $r \in Z_{n-1}(2)$. Then $(q-1) \mid (2^n-1)$ and so q is a Mersenne prime. Order consideration also rules out this case.

Similarly we can rule out the case $K/H = U_{r+1}(q)$.

1.13. Let $K/H = E_6(q)$, where $q = u^a$, then $\frac{q^6+q^3+1}{(3,q-1)} = p = 2^n - 1$. Thus $p \in Z_9(q)$ and hence, $9 \mid (2^n-2) = 2(2^{n-1}-1)$. It follows that $9 \mid (2^{n-1}-1)$ and so $2 \mid n-1$ or $4 \mid n-1$. Thus $n = 2t+1$ or $n = 4t+1$.

(a) Let $n = 2t+1$. Then $p = 2^{2t} - 1 = (2^t - 1)(2^t+1)$ and so $t = 1$, $p = 3$ and $n = 2$. Hence $\frac{q^6+q^3+1}{(3,q-1)} = 3$, but the equation has no solution in $\mathbb{N}$.

(b) Let $n = 4t+1$. Then $p = (2^{2t}-1)(2^{2t}+1)$, the equation has no solution in $\mathbb{N}$.

Similarly, we can rule out the case $K/H = {}^2E_6(q)$.

Case 2: Let $t(K/H) = 3$. Then $p = 2^n - 1 \in \{OC_2(K/H), OC_3(K/H)\}$.

2.1. Let $K/H = L_2(q)$, where $4 \mid (q+1)$. Then $\frac{q-1}{2} = 2^n - 1$ or $q = 2^n - 1$. If $\frac{q-1}{2} = 2^n - 1$, then $4 \mid (q+1) = 2^n$ and so $n \geq 2$. If $n \geq 3$, then order consideration rules out this case. If $n = 2$, then $q = 7$ and so $|K/H|| \, |L_2(7)|| \, |L_2(2)|$; contradiction. If $q = 2^n - 1 = p$ and so $n \geq 2$, similarly, we can rule out this case.

2.2. Let $K/H = L_2(q)$, where $4 \mid (q-1)$. Then $q = p$ or $\frac{q+1}{2} = p$. If $q = p$, then $4 \mid (2^n - 2)$; a contradiction. If $\frac{q+1}{2} = p$, then $4 \mid (2^{n+1} - 3)$; a contradiction.

2.3. Let $K/H = L_2(q)$, where $q > 2$ and q is even. Then $p \in \{q-1, q+1\}$. If $p = q-1$, then $q = p+1 = 2^n$ and so $(2^n+1) \mid |G|$; a contradiction. If $p = q+1$, then $q = p-1 = 2(2^{n-1}-1)$ and so $n = 2$; a contradiction.

2.4. Let $K/H = U_6(2)$. Then $|K/H| = 2^{15}.3^6.7.11$ and so $2^n - 1 = 11$; a contradiction.

2.5. Let $K/H = L_3(2)$. Then $|K/H| = 2^3.3.7$ and $2^n - 1 = 7$. Thus $n = 3$, which is the desired result.

2.6. Let $K/H = {}^2D_r(3)$, where $r = 2^t + 1 \geq 5$. Then $\frac{3^r+1}{4} = 2^n - 1$ or $\frac{3^{r-1}+1}{2} = 2^n - 1$. If $\frac{3^r+1}{4} = 2^n - 1$, then $3(3^{r-1}-1) = 2^3(2^{n-1}-1)$. Since $2 \mid 3-1$, then by Lemma 2.1, $r - 1 = 4$ and so $t = 2$. It follows that $30 = 2^{n-1} - 1$; a contradiction. If $\frac{3^{r-1}+1}{2} = 2^n - 1$, then $3^{r-1} = 2(2^{n-1}-1)$; a contradiction.

2.7. Let $K/H = G_2(q)$, where $q \equiv 0 \mod 3$. Then $q^2 - q + 1 = 2^n - 1$ or $q^2 + q + 1 = 2^n - 1$. If $q^2 - q + 1 = 2^n - 1$, then $q(q-1) = 2(2^{n-1}-1)$ and so $q = 3$ and $n = 3$. Order consideration rules out this case.

If $q^2+q+1 = 2^n - 1$, then $q(q+1) = 2(2^{n-1}-1)$. But the equation has no solution in $\mathbb{N}$.

Similarly, we can rule out $K/H = {}^2G_2(q)$.

2.8. Let $K/H = F_4(q)$, where q is even. Then $q^4 + 1 = 2^n - 1$ or $q^4 - q^2 + 1 = 2^n - 1$. If $q^4 + 1 = 2^n - 1$, then $q^4 = 2(2^{n-1}-1)$; a contradiction. If $q^4 - q^2 + 1 = 2^n - 1$, then $q^2(q^2-1) = 2(2^{n-1}-1)$, but the equation has no solution in $\mathbb{N}$.

2.9. Let $K/H = {}^2F_4(q)$, where $q = 2^{2t} + 1 > 2$. Then $q^2 + \sqrt{2q^3} + q + \sqrt{2q} + 1 = 2^n - 1$ or $q^2 - \sqrt{2q^3} + q - \sqrt{2q} + 1 = 2^n - 1$. It is easy to get that the equations $q^2 + \sqrt{2q^3} + q + \sqrt{2q} + 1 = 2(2^{n-1}-1)$ and $q^2 - \sqrt{2q^3} + q - \sqrt{2q} + 1 = 2(2^{n-1}-1)$ have no solution in $\mathbb{N}$.

2.10. Let $K/H = E_7(2)$, then $2^n - 1 \in \{73, 127\}$ and so $n = 7$. Order consideration rules out this case.

2.11. Let $K/H = E_7(3)$, then $2^n - 1 \in \{757, 1093\}$, which is impossible.

Case 3: Let $t(K/H) = \{4, 5\}$. Then $p = 2^n - 1 \in \{OC_2(K/H, OC_3(K/H), OC_4(K/H), OC_5(K/H)\}$.

3.1. Let $K/H = L_3(4)$ or ${}^2E_6(2)$. Then $2^n - 1 = 7$ or $2^n - 1 = 19$. If $2^n - 1 = 7$, then $n = 3$, and then order consideration rules out. If $2^n - 1 = 19$, then it is impossible.

3.2. Let $K/H = {}^2B_2(q)$, where $q = 2^{2t} + 1$ and $t \geq 1$. Then $2^n - 1 \in \{q-1, q \pm \sqrt{2q} + 1\}$. If $2^n - 1 = q - 1$, then $n = 2^t + 1$, order consideration rules out. If $2^n - 1 = q \pm \sqrt{2q} + 1$, then $2(2^{n-1}-1) = 2^{t+1}(2^t - 1)$. The equation has no solution in $\mathbb{N}$.

Hence $K/H = L_n(2)$ with $2^n - 1$ prime. Now since $|G| = |L_n(2)|$, $H = 1$ and $K = G \cong L_n(2)$. The Theorem 3.1 is proved.

Corollary 3.1 *Thompson's conjecture holds for the simple groups $L_n(2)$, where $2^n - 1$ prime is a prime number.*

Proof. Let G be a group with a trivial central and $N(G) = N(L_n(2))$. Then it is proved in [2, Lemma 1.4] that $|G| = |L_n(2)|$. Hence; the corollary follows from the Theorem 3.1.

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تشخیص پذیری جدید برای تعدادی از گروههای خطی

چکیده:

تعدادگروههایی که فقط از طریق مرتبه هایشان تشخیص پذیر باشند خیلی کم است مثلا Z_2 و Z_{15} از این دست گروهها می باشند، اما گروههای متناهی زیادی وجود دارند که با استفاده از مرتبه و داده های دیگری تشخیص پذیرند. به عنوان مثال با استفاده از داده های مثل مجموعه مرتبه های عناصر گروه، تعداد سیلو p- زیرگروههای گروه برای هر عدد اول p،مرتبه مولفه های گروه و ... . در این مقاله می خواهیم به امکان تشخیص پذیری گروههای خاص تصویری $L_n(2)$ که در آن 2^n-1 عددی اول است بپردازیم. براساس نتایج بدست آمده در این مقاله گروه G با گروه $L_n(2)$ یکریخت است اگر و فقط اگر مرتبه دو گروه با هم برابر باشند و دوگروه دارای یک کلاس تزویج به طول $\frac{|L_n(2)|}{2^n-1}$ داشته باشند. همچنین در این مقاله نشان خواهیم داد که حدس تامپسون هم برای گروههای $L_n(2)$ که در آن 2^n-1 عددی اول باشد نیز برقرار است. بر اساس حدس تامپسون اگر L یک گروه ساده غیر آبلی و G یک گروه با مرکز بدیهی باشد به طوری مجموعه کلاسهای تزویج دو گروه با هم برابر باشند آنگاه دو گروه باهم یکریخت هستند.