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Research Article

A Multi-supplier Inventory Model with Permissible Delay in Payment and Discount

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Abstract

This paper proposes a multi-supplier multi-product inventory model in which the suppliers have unlimited production capacity, allow delayed payment, and offer either an all-unit or incremental discount. The retailer can delay payment until after they have sold all the units of the purchased product. The retailers warehouse is limited, but the surplus can be stored in a rented warehouse at a higher holding cost. The demand over a finite planning horizon is known. This model aims to choose the best set of suppliers and also seeks to determine the economic order quantity allocated to each supplier. The model will be formulated as a mixed integer and nonlinear programming model which is NP-hard and will be solved by using genetic algorithm (GA), simulated annealing (SA) algorithm, and vibration damping optimization (VDO) algorithm. Finally, the performance of the algorithms will be compared.

Keywords : Economic order quantity; Genetic algorithm; Simulated annealing; Vibration damping optimization

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1 Introduction

Inventory control plays the main role in decreasing a companys costs. It helps a retailer reduce its procurement costs, holding cost, shortage cost and etc. One of the most common methods used in inventory control is economic order quantity (EOQ). By considering that suppliers adopt various policies to attract buyers, selecting and determining an appropriate EOQ will not be easy.

Many suppliers allow their customers to pay after a predetermined interval without charging any interest, but once the payment term expires, customers should pay daily fines for their delay. Goyal [7] proposed the first EOQ model considering permissible delay in payment. The two conditions which he considered were (1) the payment period is longer than the order cycle and (2) the order cycle is longer than the payment period. Shinn et al. studied an EOQ model in which the ordering cost included a fixed ordering and a freight costs. By assuming the freight cost having a quantity discount, they solved the problem under permissible delay in payments. Chang (2004) [5] presented a similar model considering inflation rate and deterioration rate as well as delay in payment. An assumption in this model is earning daily interest for selling all units of a product before the grace period. Huang (2007) [10] studied an EOQ model under permissible delay in payment. The main difference of his work with the previously research was to consider a partial delay in payments when the order quantity is less

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than the amount of quantity that leads to fully delayed payments. Sana and Chaudhuri (2008) [34] considered an inventory model under permissible delay time and discount to maximize the profit, where the amount of discount depended on the length of the grace time. Liang and Zhou (2008) [12] developed a model in which permissible delay in payment is a key consideration and the items are assumed to be of a deteriorating type. The retailer has storage space limitation, but they can rent a warehouse with a less deterioration rate and more holding costs. Ouyang et al. (2009) [28] presented an EOQ model under deterioration rate and partial permissible delay time. In their research, when the order quantity is less than a predetermined quantity for a fully delayed payment, the retailer must pay the partial payment by taking a loan with an interest charged per dollar per year. Roy and Samanta extended Goyals (1985) [7] model to include unequal unit selling and purchasing prices (Roy and Samanta (2011) [32]). Abad and Jaggi (2003) [7] considered an inventory model under credit period in which the end demand was price- sensitive. Moreover, both the credit period and the price were considered sellers decision variables. Pasandideh et al. (2014) [29] considered a multiproduct EOQ problem where delay in payment is permissible and the retailer can benefit cash discounts. Also, the amount of discount and the length of the grace period depend on the order quantity and all the costs increase by an inflation rate. Moreover, the shortage is backlogged and the limited warehouse space leads to a constraint for storage. They first formulate the problem and then proposed a hybrid genetic algorithm and simulated annealing (GA+SA) to solve it.

Another factor which should be considered in an EOQ model is the fact that retailers often fulfill their requirements with the help of more than one supplier which have capacity constraints. In the model proposed by Yang et al. [38], the costs are inflated during each order cycle, and the products are of a perishable kind. The retailer owns a warehouse with limited storage capacity, but they can rent a warehouse with unlimited capacity. Basnet and Leung (2005) [2] proposed a multi-product, multi-supplier, and multi-period model in which costs of transactions, holding, and purchasing determine order size and supplier selection. Chang et al. (2006) [4] considered a single-item multi-supplier system with different discount policies, limited warehouse space, and variable lead time. Burke et al. [3] introduced a model in which a retailer demands to buy a known quantity of a single item for a single period from the suppliers who have limited production and offer either an incremental, all-unit, or linear discount policy. Sadeghi-Moghadam et al. [33] considered a model in which the demand rate is not constant and transaction, purchasing, and holding costs are the only considerations. Mohammad Ebrahim et al. [25] presented a single-item multi-supplier model in which each supplier only offers one kind of discount policy (e.g., all-unit, incremental, and total volume discount) and has capacitated production. Mendoza and Ventura [24] proposed two models for selecting suppliers with capacity constraints for a single item. In the first model, the size of the order placed with a supplier is independent of the order placed with the other suppliers. In the second model, the order placed by the retailer with all the selected suppliers should be the same size. Rezaei and Davoodi [31] studied a multi-item, multi-period, and multi-supplier scenario where the suppliers have capacitated production rates. They studied order size and supplier selection under the assumptions of defective items and limited storage space. Zhang and Zhang [39] presented a model for supplier selection under stochastic demand. In their model, the suppliers have limited production capacities with maximum and minimum bounds. Mafakheri et al. [14] developed a decision making model for supplier selection and order allocation within a multi-criterion framework. Huang et al. [9] investigates an inventory control system for an online retailer with discrete demand. The retailer normally replenishes its inventory according to a continuous review (nQ, R) policy in which lead time is constant, shortages are permitted and a fraction of them will be lost. Zhang et al. [40] proposed a two-item inventory model in which the demand for a minor item is correlated to that of a major item since cross-selling and partial backordering for both products is assumed. A comprehensive survey of this research may be found in Pentico and Drake [30]. Mansini et al. [15] presented a model for supplier selection and order size specification. In their model, suppliers offer all-unit discounts, and transportation cost is based on the

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number of truck loads required for shipment.

This paper proposes a multi-supplier multi-product inventory model in which the suppliers have unlimited production capacity, allow delayed payment, and offer either an all-unit or incremental discount. The retailer can delay payment until after they have sold all the units of the purchased product. The retailers warehouse is limited, but the surplus can be stored in a rented warehouse at a higher holding cost. The demand over a finite planning horizon is known.

The model is a combination of supplier selection models and EOQ models and considers many applicable assumptions and is closer to the real-world problems and will be formulated as a mixed integer and nonlinear programming model and will be solved by three metaheuristic algorithms named GA, simulated annealing (SA) algorithm, and vibration damping optimization (VDO) algorithm.

2 Model description

In this section a mathematical model which considers a multi-supplier multi-product inventory system, is presented. In this model, the retailer purchase from a set of suppliers. Each supplier offer either all-unit or incremental discount and has uncapacitated production. Delayed payment is allowed depending on the quantity of purchase. If the retailer sells their stocked products before the permitted delay in payments, they would earn daily interest until the payment deadline. However, if the retailer does not sell the whole amount of a stock before the payment deadline, they should pay daily fines for the delay and would also lose the price discount for their procurement. The presented model is illustrated in Fig. 1.

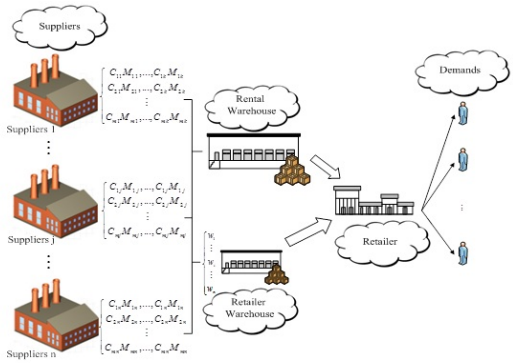

Figure 1: Schematic representation of the model.

2.1 Assumptions

- Lead time is zero; inventory replenishment happens exactly after an order is placed.
- Shortage is allowed and backlogged.
- The retailer cannot pay for a purchased product before selling the whole of the procured quantity.
- Supplier production is uncapacitated.
- Each supplier offers a single kind of price discount.
- Demand rate is constant and known.
- Delivered items are thoroughly inspected, and the defective items are rejected.
- Warehouse space is limited, but the retailer can rent an unlimited warehouse space.
- The holding cost of the rented warehouse is more than that of the retailers warehouse.
- All the products are sold at a constant interest rate.

2.2 Parameters and variables

D˙i	Demand for product i
r	Daily interest rate for selling all units ofproduct i before payment deadline
l	Cost of holding product i in the retailers warehouse
lˆ’	Cost of holding product i in the rented warehouse
Q˙i	Size of order for product i placed with supplier j
C˙ij	Price per unit offered by supplier j at the kth discount level for product i
h˙ij	Cost of holding product i purchased from supplier j in the retailers warehouse
h˙iˆ’	Cost of holding product i purchased from supplier j in the rented warehouse
A˙ij	Transaction cost for product i purchased from supplier j
Π_i	Back ordering cost per unit for product i

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γ_{ij}	Delay penalty rate for product i to be paid to supplier j
T˙i	Order cycle of product i
N˙i	Number of order cycle of product i on the planning horizon
T˙iˆ’	Part of the order cycle where the inventory of product i is not zero
Tˆ’	The time when the inventory level of product i in the rented warehouse has not reachedzero
E	Number of suppliers which offer all-unit discount ($m - E$: number of suppliers which offer
M˙ij	Permissible delay in paying supplier j for product i
b˙i	The amount of shortage of product i
W˙i	Storage space for product i in retailers warehouse
P˙ij	Average percentage of the defective items in batch of product i delivered by supplier j
m	Number of available suppliers
k	Number of discount levels offered by suppliers
n	Number of different products required
TS˙i	Total transaction cost of product i
TB˙i	Total shortage cost of product i
TH˙i	Total holding cost of product i
TM˙i	Total delay cost of product i
TP˙iˆ’	Total cost of purchasing product i without price discount
TP˙iˆ”	Total cost of purchasing product i with price discount
TP˙i	Total cost of purchasing product i
TIn˙i	Total income from purchasing product i
In	Interest rate for selling product i
u	Selling interest for all the products
TI	Total annual interest
gi	Selling price of product i
O˙ij	1 if the retailer purchases product i from supplier j, zero otherwise
S˙ij	1 if the retailer pays on time for product i purchased from supplier j, zero otherwise
F˙i	Size of order for product i is greater than the available space in the retailers warehouse
X˙ijk	Quantity of non-defective product i purchased from supplier j at the kth discount level
Y˙ijk	Quantity of product i purchased from supplier j at the kth discount level

2.3 Objective function

The objective function is the difference between total income and total costs, which can be calculated as follows:

$$\begin{aligned} TI = &\sum_{j=1}^{n}\sum_{i=1}^{m} TIn_i N_i \\ &- \sum_{j=1}^{n}\sum_{i=1}^{m}(N_i[TS_i + TB_i + TM_i \\ &+ TH_i + TP_i]) \end{aligned} \tag{2.1}$$

2.3.1 Total costs

- Transaction cost:

$$TS_i = \sum_i \sum_j A_{ij} o_{ij}$$

- Total shortage (backlogged) cost:

$$TB_i = \sum_i b_i \pi_i$$

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Table 1: Discount price and permitted delay in payment at different levels

Order size	Discount	Permitted delay in payment
$0 < Q_i \leq q_{i,j,1}$	$C_{i,j,1}$	$M_{i,j,1}$
$q_{i,j,1} < Q_i \leq q_{i,j,2}$	$C_{i,j,2}$	$M_{i,j,2}$
$\vdots$	$\vdots$	$\vdots$
$q_{i,j,k-1} < Q_i \leq q_{i,j,k}$	$C_{i,j,k}$	$M_{i,j,k}$
$q_{i,j,k} < Q_i \leq U$	$C_{i,j,k+1}$	$M_{i,j,k+1}$

- Purchasing cost:

Table 1 presents the discounted price and permitted delay period for payments in different amount of ordered quantities. The purchasing cost depends on the time of payment. If the retailer is able to pay on time, they can use the promised discounts. Otherwise, they should pay without any discounts. Thus, if $M_{ij} < T_i^{'}$, the purchasing cost is calculated as follows:

$$TP_i^{'} = \sum_{j=1}^{n} P_{ij} Q_i C_{i,j,1}$$

when the discount policy is all-unit, the purchasing cost can be computed as:

$$TP_i^{''} = \sum_{j=1}^{n} P_{ij} Q_i X_{ijk} C_{ijk}$$

and when the discount policy is incremental, it can be computed as follows:

$$TP_i^{''} = \sum_{j=E}^{n} \Bigg(\sum_{k=2}^{a+1} \Bigg((P_{ij}Q_i - q_{i,j,k-1}) C_{i,j,k} + \sum_{f=1}^{k-1} (q_{i,j,k-f} - q_{i,j,k-f-1}) C_{i,j,k-f} \Bigg) x_{i,j,k} + P_{i,j} Q_i C_{i,j,1} x_{i,j,1} \Bigg)$$

Hence, the total annual purchasing cost is as follows:

$$TP_i = TP_i^{''} * S_i + TP_i^{'}(1 - S_i)$$

- Delayed payment

When the retailer cannot pay by the payment deadline, they should pay an additional cost for the delay. Thus, when $M_i < T_i$:

$$TM_i^{'} = \left(\frac{(P_{ij}Q_i - b_i)}{D_i - M_{ijk}} \right)_{ij}$$

When the retailer can pay on time, no additional cost will be incurred. Thus, for $M_i \geq T_i^{'}$:

$$TM_i^{''} = 0$$

These equations lead to:

$$TM_i = \max \left\{ 0, \left(\frac{(P_{ij}Q_i - b_i)}{D_i - M_{ijk}} \right)_i j \right\}$$

- Holding cost

For calculating the holding cost of each product, the model should be studied in two following conditions ($C1$ and $C2$):

$C1$: Maximum inventory level of product i is smaller than or equal to the retailers warehouse space ($P_{ij}Q_i - b_i \geq w_i$), as shown in Fig. 2.

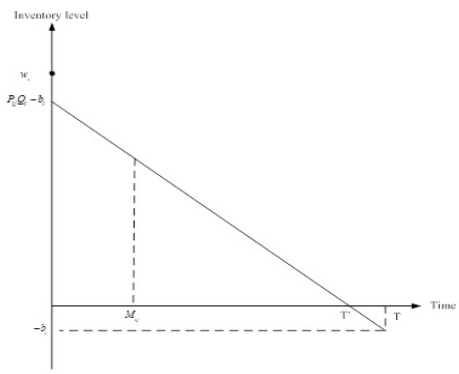

Figure 2: Graphical representation of Condition 1.

$$T' = \frac{P_{ij}Q_i - b_i}{D_i}$$

$$THO_i = \frac{(P_i jQ_i - b_i)^2}{2D_i} O_{ij} h_i$$

$C2$: Maximum inventory level of product i is greater than the retailers warehouse space, and the retailer needs a rented warehouse ($P_{ij}Q_i - b_i > w_i$), as shown in Fig. 3. $T_i^{''}$ is time that

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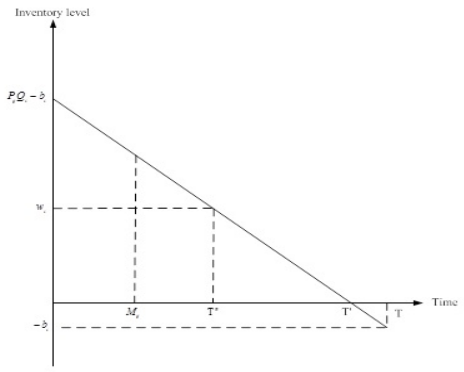

Figure 3: Graphical representation of Condition 2.

rented warehouses inventory level of product i hasnt reach zero and rental warehouse inventory level for product i is more than zero, the retailer incurs the holding cost of both warehouses.

Holding cost of the rented warehouse until T'', can be calculated as follows:

$$T'' = \frac{p_i j Q_i - b_i - w_i}{D_i}$$
$$\frac{(p_{ij}Q_i - b_i - w_i)^2}{2D_i} O_{ij} h_i'$$

The inventory level of product i in the retailers warehouse which does not change during T'' is as follows

$$\frac{w_i(2p_{ij}Q_i - 2b_i - w_i)}{2D_i} O_{ij} h_i$$

Hence:

$$THR_i = \frac{(p_{ij}Q_i - b_i - w_i)^2}{2D_i} O_{ij} h_i' + \frac{w_i(2p_{ij}Q_i - 2b_i - w_i)}{2D_i} O_{ij} h_i$$

Total annual holding cost can be computed as follows:

$$THi = [(THO_i(1 - f_i) + THR_i f_i)]$$

2.3.2 Total income

Calculating total annual income demands that the selling price and the rate of daily interest earned from early sales be calculated.

Selling price of product i can be computed as follows:

$$g_i = In \times C_{i,j,1}$$

Rate of daily interest earned from early sales is as follows:

$$z_i' = \max\left\{1, (1+r)^{M_{ij} - T_i'}\right\}$$

So, total annual income can be calculated as follows:

$$TIn_i = \sum_i \sum_j P_{ij} Q_i g_i Z_i'$$

2.4 Model formulation

$$\max TI_i \tag{2.2}$$
$$s.t.$$
$$TP_i'' = \sum_{j=1}^{E} P_{ij} Q_i X_{ijk} C_{ijk} O_{ij} \tag{2.3}$$
$$TP_i'' = \sum_{j=E}^{n} \left(\sum_{k=2}^{a+1} \left((P_{ij}Q_i - q_{i,j,k-1}) C_{i,j,k} + \sum_{f=1}^{k-1} (q_{i,j,k-f} - q_{i,j,k-f-1}) C_{i,j,k-f} \right) x_{i,j,k} + P_{i,j} Q_i C_{i,j,1} x_{i,j,1} \right) O_{ij} \tag{2.4}$$
$$TP_i' = \sum_{j=1}^{n} P_{ij} Q_i C_{i,j,1} O_{ij} \tag{2.5}$$
$$h_i = \sum_{j=1}^{n} \left(\left(l \frac{TP_i''}{P_{ij} Q_i} \right) s_{ij} + \left(l c_{i,1} \times (1 - s_{ij}) \right) \right) O_{ij} \tag{2.6}$$

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$$h_i^{'} = \sum_{j=1}^{n} \left(l^{'} \frac{TP_i^{''}}{P_{ij}Q_i} \right) s_{ij} O_{ij}$$
$$+ (l^{'} c_{i,j,1} \times (1 - s_{ij}))O_{ij} \tag{2.7}$$
$$\sum_{j=1}^{n} \left(\sum_{k=0}^{a} q_{i,j,k} y_{i.j,k+1} \right) \geq \sum_{j=1}^{n} Q_i O_{ij}$$
$$\leq \sum_{j=1}^{n} (\sum_{k=1}^{a} q_{i,j,k} y_{i,j,k} + U y_{i,j,a+1}) \tag{2.8}$$
$$\sum_{k=0}^{a} q_{i,j,k} x_{i.j,k+1} \leq p_{i,j} Q_i O_{ij}$$
$$\leq \sum_{k=1}^{a} q_{i,j,k} x_{i,j,k} + u x_{i,j,a+1} \tag{2.9}$$
$$\sum_{j=1}^{n} \sum_{k=1}^{a+1} y_{i,j,k} = 1 \tag{2.10}$$
$$\sum_{j=1}^{n} \sum_{k=1}^{a+1} x_{i,j,k} = 1 \tag{2.11}$$
$$\sum_{i} \geq \left(\sum_{j=1}^{n} \left(\frac{P_{ij}Q_i - b_i}{D_i} \right) \right.$$
$$\left. - \sum_{j=1}^{n} \sum_{k=1}^{a+1} y_{ijk} M_{ijk} \right)_{ij} \tag{2.12}$$
$$Z_i s_{ij} = 0 \tag{2.13}$$
$$Z_i + s_{ij} > 0 \tag{2.14}$$
$$z_i^{'} \leq (1 + r)^{\sum_{j=1}^{n} M_{ij}.o_{ij} - T_i^{'}} \tag{2.15}$$
$$z_i^{'} \geq 1 \tag{2.16}$$
$$P_{ij}Q_i - b_i \geq w_i F_i \tag{2.17}$$
$$P_{ij}Q_i - b_i \leq V F_i + w_i \tag{2.18}$$
$$\sum_{j=1}^{n} o_{ij} = 1 \tag{2.19}$$
$$x_{i,j} = 0, 1,\ y_{ij} = 0, 1,\ s_{ij} = 0, 1,$$
$$z_i \geq 0,\ Q_i \geq 0,\ b_i \geq 0,\ F_i = 0, 1 \tag{2.20}$$

Objective function (2.2) maximizes total annual interest. Constraints (2.3) and (2.4) consider the product purchased under all-unit and incremental discount policy, respectively. Constraints (2.5) consider the purchased material without any discount policy. Constraints (2.6) and (2.7) calculate per unit cost of holding product i in the retailers warehouse and rented warehouse, respectively. Constraints (2.8) and (2.9) consider the delay in the payment for product i purchased from supplier j at the kth discount level. Constraints (2.10) and (2.11) ensure the correct price of the quantity of product i purchased from supplier j at the kth discount level. Constraints (2.12) consider the cost of delay in the payment for all products. Constraints (2.13) and (2.14) ensure that the retailer benefits from price discount for product i only if payment is made on time. Constraints (2.15) and (2.16) considered that the retailer earns daily interest only if payment is made before the deadline. Constraints (2.17) and (2.18) assure the amount of inventory stock in the retailer's warehouse and the rented warehouse. Constraint (2.19) ensures that each product is purchased only from one supplier. Constraints (2.20) consider the range of the decision variables.

3 The meta-heuristic algorithms

The proposed model is a mixed integer nonlinear programming model. The solution will be hard and time-consuming if exact methods are used. The presented model in subsection *2.4* is an MINLP problem; solving the MINLP problems are hard with exact methods because the MINLP is an NP-hard problem (Garey and Johnson, [6]; Murty and Kabadi, [27]; Vavasis, [37]).Thus, meta-heuristic algorithms were employed to solve and compare the numerical examples. The algorithms were GA, SA, and VDO. A description of these methods is considered in the following subsections.

3.1 Parameter calibration

The appropriate design of parameters has significant impact on efficiency of meta-heuristics. In this paper, the Taguchi [36] method applied to calibrate the parameters of the proposed algorithms, namely SA, VDO and GA. This method is based on maximizing performance measures called signal-to-noise (S/N) ratios in order to find the optimized levels of the effective factors in the experiments. This ratio refers to the mean-square deviation of the objective function that minimizes the mean and variance of quality characteristics to make them closer to the expected values. For the factors that have significant impact on S/N ratio, the highest S/N ratio provides the optimum level for that factor. As mentioned before,

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the purpose of Taguchi method is to maximize the S/N ratio. In this subsection, the parameters for experimental analysis are determined.

Table 2 lists different levels of the factors for SA, VDO and GA. In this paper according to the levels and the number of the factors, respectively the Taguchi method L9 is used for the adjustment of the parameters for the SA and L27 are used for the VDO and GA. Figures 5, 6 and 7 show S/N ratios. According to these figures 1500, 60, 0.99, 10, 60, 0.1, 1200, 0.5, 450, 0.3, 0.1, 0.95, 200 are the optimal level of the factors T_0, L, α, A_0, $L_{\max}$, γ, t, σ, Npop, P_m, P_c, S_m and iteration.

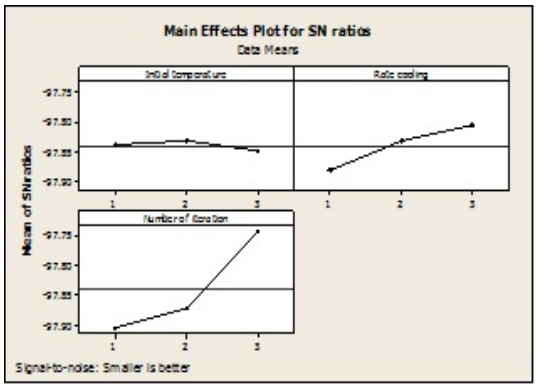

Figure 4: SN ratios for the SA algorithm.

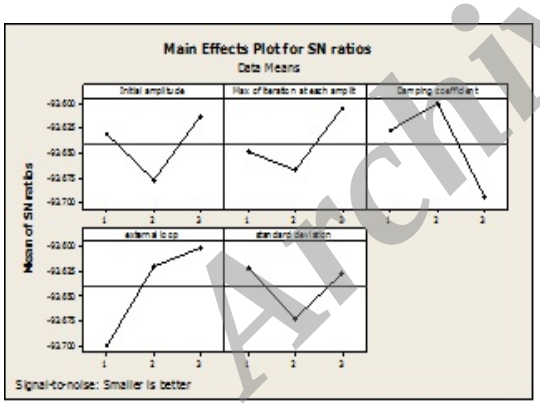

Figure 5: SN ratios for VDO algorithm.

3.2 Genetic algorithm (GA)

This algorithm, initially developed by Holland [8], is based on the mechanics of biological evolution. GA can provide solutions for highly complex search spaces. The first solution set (the first generation) is created randomly, and then as a result of the action of crossover and mutation operators, the solutions improve step by step in the next generations.

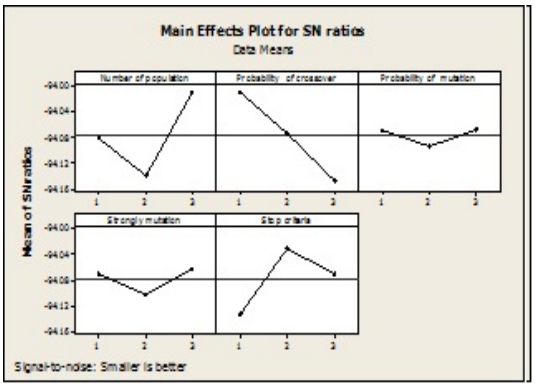

Figure 6: SN ratios for the GA algorithm.

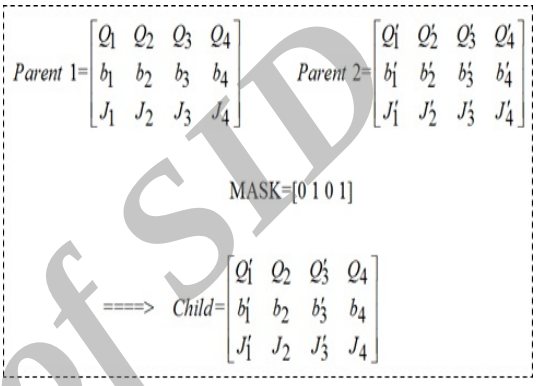

Figure 7: Performance of the mutation operator.

Figures 7 and 8 illustrate the performance of crossover and mutation operators, respectively.

3.3 Simulated annealing (SA) algorithm

This algorithm was developed by Kirkpatrick et al. [11]. It is inspired by annealing in metallurgy, cooling of a material under controlled conditions to reduce its defects. The advantage of this algorithm is that it escapes a local optimum and searches for better solutions. The system moves to the new state if it is better than the previous state. In contrast, if the new state is worse, the system decides about moving by a determined possibility. According to statistical thermodynamics laws, the probability that the energy of the system will increase to ΔE in temperature t is follows:

$$P(\Delta E) = e$$

-$\Delta EK.t$(3.21)Using this probability function, the system decides to stay at a previous better state or accepts the new, yet worse neighbor state.

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Table 2: Factors and their levels

Factor	Algorithm	Notation	Level	Value
Initial temperature		T_0	3	8000, 9000, 10000
Rate cooling	SA	α	3	0.6, 0.65, 0.7
Number of iteration at each temperature		L	3	20, 40, 60
Initial amplitude		A˙0	3	400, 500, 600
Max of iteration at each amplitude		$L_{\max}$	3	90, 12, 150
Damping coefficient	VDO	γ	3	0.05, 0.1, 0.15
external loop		t	3	600, 800, 1200
standard deviation		σ	3	0.5, 1.5, 2
Number of population		Npop	3	220, 240, 260
Probability of mutation		P_m	3	0.2, 0.3, 0.35
Probability of crossover	GA	P_c	3	0.75, 0.8, 0.85
Strongly mutation		Sm	3	0.35, 0.65, 0.95
Stop criteria		Iteration	3	100, 200, 300

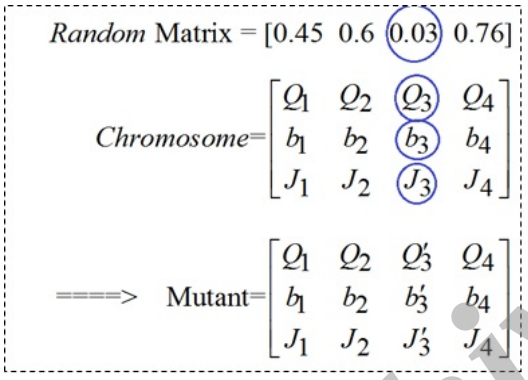

Figure 8: Performance of the crossover operator.

3.4 Vibration damping optimization (VDO) algorithm

Vibration is one of the pivotal topics in dynamics. All elastic objects or systems could have vibrating movement. Mehdizadeh and Tavakoli-moghadam [19] developed a meta-heuristic algorithm based on vibration principle.

The VDO algorithm operates in a similar way to SA. The VDO algorithms probability function for accepting the new, yet worse state is as follows:

$$P(A) = 1 - e$$

-A^22δ^2(3.22)where A is the amplitude of oscillation.

When the energy source of an oscillator is cut, its amplitude reduces and gradually becomes zero. γ is the damping coefficient.

The decrement function of amplitude is as follows:

$$A_t = A_0 e^{-\frac{A^2}{2\delta^2}} \tag{3.23}$$

For more details about the VDO algorithm, one can refer to [19],[23], [22], [26], [20], [17], [21], [18].

4 Result Analysis and comparisons

The model was coded using Lingo 8 [13], and the three meta-heuristic algorithms were coded by MATLAB [16] examples were generated for comparison of meta-heuristics solutions with Lingos solution then, the coded algorithms were run.

The runtime and objective values are shown in Table 3, which shows that the proposed meta-heuristic algorithms were able to provide optimal solutions to very small instances and near-optimal solutions to larger instances within a much reasonable time than did Lingo, perhaps because of the large number of variables and constraints. This software was unable to find optimal solutions to medium-sized examples even after several hours.

After the results were compared with those of Lingos, 30 examples were generated randomly in three size groups: small (Examples 1 to 10), medium (Examples 11 to 20), and large (Examples 21 to 30). For better comparison, the termination factor was fixed at 120 seconds. Each example was run for five times. The average

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Table 3: Comparison of the results obtained from the three meta-heuristic methods and Lingo

Problem	Lingo		GA		SA		VDO	
	Objective	Time	Objective	Time	Objective	Time	Objective	Time
1,2,2	548	00:00:02	548	00:00:01	548	00:00:00:57	548	00:00:01
1,3,4	50069	00:00:09	50096	00:00:02	50096	00:00:01	0096	00:00:01
2,2,2	115456.8	00:00:11	115456.8	00:00:02	15456.8	00:00:01	15456.8	00:00:02
2,3,3	40156	00:01:05	40156	00:00:02	40156	00:00:01	40156	00:00:02
3,3,4	420311.5	00:04:26	4172092	00:00:04	418492.3	00:00:02	420311.5	00:00:03
5,4,4	585552.7	00:59:27	571218.1	00:00:07	581857.3	00:00:03	582971	00:00:05
6,4,5	369095.7	01:49:41	353788	00:00:08	350704.2	00:00:04	359862.5	00:00:06
7,5,5	638751.2 (Local)	03:05:32	724403.7	00:00:10	768875.1	00:00:04	750485.6	00:00:08
10,6,5	NA	05:42:08	674875.4	00:00:17	716537.2	00:00:09	709604.3	00:00:13

Table 4: Comparison of the results obtained from the three meta-heuristic methods and Lingo

Number of example	GA		SA		VDO	
	Average	Best fitness	Average	Best fitness	Average	Best fitness
1	7104184.52	7203749.71	7306262.812	7324505.63	7301697.2	7324245
2	11155985.47	11200053.66	11475346.43	11488437.53	11472457.8	11503986.54
3	9707185.414	9808445.03	9970222.228	9991504.77	9975081.6	9984279.46
4	10273988	10346416.91	10448564.67	10456005.46	10457222.7	10470078.75
5	5691484.43	5720648.86	5692004.074	5712515.65	5695583.0	5707491.31
6	7659329.624	7802707.27	7927664.832	7945809.17	7943393.9	7961664.01
7	5006247.656	5068302.81	5133015.988	5148464.45	5143866.2	5148901.28
8	11682698.46	11810822.35	12222896.96	12264011.57	12253652.3	12286138.79
9	9016836.94	9097927.47	9215361.952	9221491.07	9236933.8	9249977.2
10	12417638.82	12761303.08	13072044.99	13094048.88	13064895.5	13118739.53
11	26340944.85	26576466.28	27437525.91	27484883.21	27373877.1	27469611.02
12	23458863.83	23734975.31	25471327.5	25598790.2	25284720.9	25383246.74
13	29835343.91	30273327.22	31107755.92	31197366.36	31061053.9	31183256.88
14	29570835.35	29685022.7	30323656.3	30418677.19	30355512.9	30398318.3
15	30162115.82	30308728.23	31182206.55	31221416.2	31113822.6	31202104.53
16	23615845.56	23765328.4	25062906.44	25088914.31	24946174.5	25009726.25
17	25450139.62	25861820.15	26876706.64	26960326.28	26685618.9	26762608.04
18	30505700.63	31035170.36	31931871.08	31975001.07	31905639.4	31956110.2
19	28045608.59	28491358.85	29282662.61	29346545.77	29225289.6	29370995.09
20	28102123.31	28373874.87	29305298.51	29373627.03	29187288.1	29305603.41
21	35978953.75	36164121.51	38008333.98	38103386.06	37973696.1	38024564.41
22	34872741.93	35189954.98	37048063.94	37211587.91	36842174.0	37139308.83
23	32016426.58	32206045.23	33176426.58	33226973.51	33204402.0	33376175.35
24	29685461.01	30647209.85	31318007.81	31437864.47	31199602.8	31324213.35
25	36767685.97	36943453.95	38922848.3	39671917.57	38764797.5	39144702.68
26	28155864.12	28508601.02	30484306.59	30957809.55	30439410.1	30536183.75
27	43707606.64	44919491.58	44533242.76	44845223.58	42836860.3	43527094.03
28	24725152.62	25003776.99	24939764.74	25141381	24458567.0	24720524.15
29	28582997.72	28825098.48	29759412.78	29988454.08	28842015.1	29053054.84
30	31305482.81	31411467.36	31563792.5	31657581.92	30894832.8	31054800.51

and best fitness values of the three proposed algorithms for these 150 runs are given in Table 4, and the standard deviations are shown in Fig. 9. The algorithms were compared using 30 randomly generated examples divided into three classes of 10. Each example was run for five times ($30 * 5 = 150$). The obtained values for each algorithm are shown in Table 4.

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The Tukey method was used for comparing the means of the three algorithms in the three classes of examples:

$$H_0 : \mu_{GA} = {}_{SA} = \mu_{RDO} \tag{4.24}$$

$$H_1 : \text{At least one of the means is not equal to the other means.} \tag{4.25}$$

Minitab software was used for comparing these

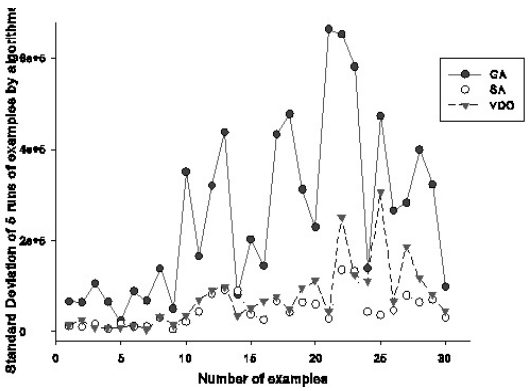

Figure 9: Standard deviation of five runs of each example.

algorithms. The Tukey method showed no difference between GA, SA, and VDO mean sat the 0.05 of confidence level in any 3 size classes.

Table 5 gives us a better understanding of the performance of the proposed algorithms. As for small examples, the VDO proved better in average and best fitness of 50 runs, but the SA had a better standard deviation compared to the other two algorithms. Concerning medium and large examples, the SA had tangible advantages according to all three criterions.

5 Conclusion

In this research, a multi-supplier multi-product system was proposed for choosing a proper set of suppliers and an EOQ. In this model, the suppliers offered price discounts and a permissible delay in payment. The retailer can earn interest if they sell their goods before the payment deadline, but they have to pay fines if they are not able to sell the entire inventory of a product on time. Also, due to warehouse space limitation, the retailer may have to rent another warehouse at a higher holding cost. A mathematical model was developed, and three meta-heuristic algorithms (GA, SA, and VDO) were proposed to solve the problem. Finally, the best algorithm performance for three example sizes was determined.

In a replication of this study, cost of transporting each load to the retailers or the rented warehouse can be factored in. The effect of inflation on different costs and selling prices and the limited production capacity of the suppliers (capacitated production) can also be studied. In addition, purchased items can be regarded perishable.

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Table 5: Comparison of algorithm performances

		GA	SA	VDO
Small	Average	0	3	7
	Best fitness	1	3	6
	Standard deviation	0	6	4
Medium	Average	0	9	1
	Best fitness	0	9	1
	Standard deviation	0	9	1
Large	Average	0	9	1
	Best fitness	1	8	1
	Standard deviation	0	9	1

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A Multi-supplier Inventory Model with Permissible Delay in Payment and Discount

M. Farhangi, E. Mehdizadeh

ارائه یک مدل موجودی چند تامین کننده با مجاز بودن تاخیر در پرداخت و وجود تخفیف

چکیده:

این مقاله یک مدل موجودی چند تامین کننده و چند محصولی ارائه می دهد در شرایطی که در آن تامین کنند ها دارای ظرفیت تولید نامحدود بوده، تاخیر در پرداخت ها مجاز بوده و تخفیف از هر دو نوع کلی و نموی مجاز می باشد. خرده فروش می تواند در پرداخت ها تاخیر داشته باشد تا بعد از آنکه تمام محصولاتی که خریداری کرده است، بفروش برساند. انبار خرده فروش محدود است، اما کالای مازاد می تواند در یک انبار با هزینه نگهداری بالا ذخیره شود. مقدار تقاضا در هر افق برنامه ریزی محدود، معین و معلوم است. هدف مدل انتخاب بهترین مجموعه از تامین کنندگان و همچنین تعیین مقدار سفارش اقتصادی اختصاصی به هر تامین کننده می باشد. مدل ارائه شده از نوع برنامه ریزی عدد صحیح غیرخطی خواهد بود و با توجه به NP-hard بودن مدل، از الگوریتم های ژنتیک، شبیه سازی تبرید و بهینه سازی به کمک میرایی ارتعاش برای حل آن بهره گرفته خواهد شد. در پایان عملکرد الگوریتم ها با هم مقایسه خواهند شد.

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