Available online at http://ijim.srbiau.ac.ir/
Int. J. Industrial Mathematics (ISSN 2008-5621)
Vol. 9, No. 1, 2017 Article ID IJIM-00831, 9 pages
Research Article

Science and Research Branch (IAU)

Evaluating the Efficiency of Firms with Negative Data in Multi-Period Systems: An Application to Bank Data

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Received Date: 2015-02-21 Revised Date: 2015-12-12 Accepted Date: 2016-02-28

Abstract

Data Envelopment Analysis (DEA) is a mathematical technique to evaluate the performance of firms with multiple inputs and outputs. In conventional DEA models, the efficiency scores of Decision Making Units (DMUs) with non-negative inputs and outputs are evaluated in a special period of time. However, in the real world there are situations wherein performance of firms must be evaluated in multiple periods of time while negative data are present; for this matter the current paper proposes an approach for assessing the efficiency of multi-period systems in the presence of positive and negative measures. To illustrate, the average efficiency of firms with some negative measures are calculated in multi-period production systems. The suggested approach utilizes the Semi-Oriented Radial Measure (SORM) model (Emrouznejad et al. [4]) for incorporating some negative factors (inputs and outputs) and determining the efficiency of multi-period production systems. A real world data set related to banking sector is used to illustrate and clarify the proposed approach.

Keywords : Data Envelopment Analysis (DEA); Efficiency; Multi-period systems; Negative data.

1 Introduction

Many systems can be found in the real world whose performance must be evaluated in multiple periods of time in which negative data exist. Factors like profit, growth in the number of clients, and changes in orders can be considered as measures with positive and negative values. In the current study, the data envelopment analysis (DEA) technique is used for evaluating the performance of multi-period systems with negative data. DEA, popularized by Charnes et al. [1], is a non-parametric method for evaluating the efficiency of decision making units (DMUs) with multiple inputs and outputs. Nowadays DEA is used in many areas like banking [16], education [8], health [3], etc. In traditional DEA models, the efficiency scores of firms are usually evaluated in a specified period of time while data are deemed as non-negative inputs and outputs. Nevertheless, there are a number of cases incorporating negative values in the DEA literature.

Researchers such as Pastor [11], Lovell [9], and Seiford and Zhu [14] used data transformations in order to handle negative factors. Also, Portela et al. [13] propounded a directional distance approach, a range directional measure (RDM) model, for investigating negative measures. Then, Portela and Thanassoulis [12] developed the RDM model [13] to calculate the needed efficiency scores for the Malmquist type index and Luenberger indicator in the presence of negative data. Sharp et al. [15] suggested a

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modified slack based measure model for assessing the efficiency of DMUs in the presence of negative inputs and outputs. Afterwards, Emrouznejad et al. [4] introduced a semi-oriented radial measure (SORM) for dealing with situations in which variables can take both positive and negative numbers. Cheng et al. [2] provided a variant of radial measure in order to assess the performance of units where negative measures are present. Furthermore, there are some papers providing approaches for evaluating the efficiency of systems in multiple periods wherein input and output factors are non-negative.

Park and Park [10] indicated an aggregative efficiency of multi-period systems. Esmaeilzadeh and Hadi-Vencheh [5] provided a super-efficiency model based on the assumption of constant returns to scale (CRS) for estimating aggregative efficiency of multi-period systems. Furthermore, Kao and Liu [7] used a relational network model and calculated the overall and period efficiencies of each DMU. Jablonsky [6] modified Park and Park's model [10] and introduced approaches for determining the efficiency and ranking DMUs. Furthermore, Jablonsky [6] calculated the average efficiency in multi-period production systems.

In the current paper, an approach is proposed to estimate the average efficiency of multi-period systems where negative and positive factors present. To illustrate, Jablonsky's approach [6] which is applied for evaluating the average efficiency in multi-period systems, is extended and modified for situations that negative values exist. For negative data in two forms, all values of a variable are negative or some values are negative while others are positive, so Emrouznejad et al.'s method [4] is utilized and generalized, and then efficiency changes between two periods are measured. After this, the average efficiency of 50 branches of an Iranian bank is estimated with the use of the introduced method.

The rest of the paper is organized as follows. Section 2 reviews some concepts and formulations that are used and generalized in the current study. The introduced approach for estimating the average efficiency of multi-period systems with negative measures are provided in Section 3. An application of the introduced method in the banking sector is given in Section 4. Conclusions are presented in Section 5.

2 Preliminaries

First the SORM model, proposed by Emrouznejad et al. [4], is presented in this section. Then, Jablonsky's approach [6] to compute the average efficiency of multi-period systems is displayed.

2.1 SORM (semi-oriented radial measure) model

Consider n DMUs, $DMU_j\,(j = 1, ..., n)$, with m inputs $x_{ij}\,(i = 1, ..., m)$ and s outputs $y_{rj}(r = 1, ..., s)$. Also, assume I displays a subset of input variables in which inputs have positive values for all DMUs. L shows a subset of input variables in which inputs are positive for some DMUs and negative for others. Similarly, a subset R of output measures indicates outputs that take positive values for all DMUs while a subset K represents outputs with positive values for some DMUs and negative for others. Emrouznejad et al. [4] defined $x_{ij}, i \in L, y_{rj}, r \in K$ as follows:

$x_{ij} = x^1_{ij} - x^2_{ij},\ i \in L, \forall j$,that

$x^1_{ij} = \begin{cases} x_{ij} & if\ x_{ij} \geq 0, \\ 0 & if\ x_{ij} < 0, \end{cases},\ x^2_{ij} = \begin{cases} 0 & if\ x_{ij} \geq 0, \\ -x_{ij} & if\ x_{ij} < 0. \end{cases}$

and

$y_{rj} = y^1_{rj} - y^2_{rj},\ r \in K, \forall j$,that

$y^1_{rj} = \begin{cases} y_{rj} & if\ y_{rj} \geq 0, \\ 0 & if\ y_{rj} < 0, \end{cases},\ y^2_{rj} = \begin{cases} 0 & if\ y_{rj} \geq 0, \\ -y_{rj} & if\ y_{rj} < 0. \end{cases}$

Then they introduced the following model, variable returns to scale (VRS) SORM model in the output orientation, for calculating the efficiency of DMUs in the presence of negative data:

$$\begin{array}{ll} Max & \theta \\ s.t. & \sum_{j=1}^{n} \lambda_j x_{ij} \leq x_{io}, \forall i \in I, \\ & \sum_{j=1}^{n} \lambda_j x^1_{ij} \leq x^1_{io}, \forall i \in L, \\ & \sum_{j=1}^{n} \lambda_j x^2_{ij} \geq x^2_{io}, \forall i \in L, \\ & \sum_{j=1}^{n} \lambda_j y_{rj} \geq \theta y_{ro}, \forall r \in R, \\ & \sum_{j=1}^{n} \lambda_j y^1_{rj} \geq \theta y^1_{ro}, \forall r \in K, \\ & \sum_{j=1}^{n} \lambda_j y^2_{rj} \leq \theta y^2_{ro}, \forall r \in K, \\ & \sum_{j=1}^{n} \lambda_j = 1, \\ & \lambda_j \geq 0, \forall j. \end{array} \quad (2.1)$$

The optimal value of $\varphi^* = \frac{1}{\theta^*}$ shows the efficiency of DMU_o. Moreover, Emrouznejad et al. [4] showed another model in the input orientation. Readers can refer to Emrouznejad et al. [4] for more information in this regard.

2.2 A multi-period model

Suppose the aim is to evaluate the average efficiency of n multi-period production systems,

DMU_j $(j = 1, ..., n)$, in T periods of time $(t = 1, ..., T)$ while inputs $x_{ij}(i = 1, ..., m)$ and outputs $y_{rj}(r = 1, ..., s)$are non-negative. Jablonsky [6] suggested the following model for this purpose:

$$\begin{array}{ll} Max & \sum_{t=1}^{T} \theta_o^t/T \\ s.t. & \sum_{j=1}^{n} \lambda_j^t x_{ij}^t \leq x_{io}^t, \forall i, \forall t, \\ & \sum_{j=1}^{n} \lambda_j^t y_{rj}^t \geq \theta_o^t y_{ro}^t, \forall r, \forall t, \\ & \lambda_j^t \geq 0, \forall j, \forall t. \end{array} \tag{2.2}$$

in which x_{ij}^t indicates the ith input of jth DMU in period t and y_{rj}^t is rth output of jth DMU in period t. λ_j^t is the intensity variable. Furthermore, for ranking DMUs, $\lambda_o^t = 0$, $(t = 1, ..., T)$ was added to the aforementioned model and an additional model was also introduced for ranking weakly efficient DMUs. For more details, readers can refer to [6].

3 Multi-period systems in the presence of negative data

At this moment, an approach is proposed for assessing the average efficiency of n multi-period units, DMU_j $(j = 1, ..., n)$, with m inputs x_{ij} $(i = 1, ..., m)$ and s outputs $y_{rj}(r = 1, ..., s)$ in $T(t = 1, ..., T)$ periods while inputs and outputs can take positive and negative values. Similar to section 2, a subset of input variables in which inputs have positive values for all DMUs are shown with I for each period t $(t = 1, ..., T)$. A subset of input measures with positive inputs for some DMUs and negative for others is indicated with L for each period t $(t = 1, ..., T)$. Also, a subset of output measures in which outputs take positive values for all DMUs is represented by R while a subset K of outputs contains outputs with positive values for some DMUs and negative values for others in each period $t\,(t = 1, ..., T)$. Therefore, x_{ij}^t, $i \in L$, $y_{rj}^t, r \in K$ can be defined as follows:

$$x_{ij}^t = x_{ij}^{1t} - x_{ij}^{2t},\ i \in L, \forall j, \forall t,$$

that

$$x_{ij}^{1t} = \begin{cases} x_{ij}^t & if\ x_{ij}^t \geq 0, \\ 0 & if\ x_{ij}^t < 0, \end{cases} ,\ x_{ij}^{2t} = \begin{cases} 0 & if\ x_{ij}^t \geq 0, \\ -x_{ij}^t & if\ x_{ij}^t < 0. \end{cases}$$

and

$$y_{rj}^t = y_{rj}^{1t} - y_{rj}^{2t},\ r \in K, \forall j, \forall t,$$

that

$$y_{rj}^{1t} = \begin{cases} y_{rj}^t & if\ y_{rj}^t \geq 0, \\ 0 & if\ y_{rj}^t < 0, \end{cases} ,\ y_{rj}^{2t} = \begin{cases} 0 & if\ y_{rj}^t \geq 0, \\ -y_{rj}^t & if\ y_{rj}^t < 0. \end{cases}$$

in which x_{ij}^t denotes ith input of jth DMU in period t and y_{rj}^t represents rth output of jth DMU in period t. It is clear that $x_{ij}^{1t}, x_{ij}^{2t}, y_{rj}^{1t}$,and $y_{rj}^{2t} \geq 0$. Jablonsky's approach [6] is modified for incorporating negative and positive factors (inputs and outputs). Thus, the following model is introduced for evaluating the average efficiency of multi-period systems in the presence of negative measures.

$$\begin{array}{ll} Max & e_o^M = \sum_{t=1}^{T} \theta_o^t/T \\ s.t. & \sum_{j=1}^{n} \lambda_j^t x_{ij}^t \leq x_{io}^t, \forall i \in I, \forall t, \\ & \sum_{j=1}^{n} \lambda_j^t x_{ij}^{1t} \leq x_{io}^{1t}, \forall i \in L, \forall t, \\ & \sum_{j=1}^{n} \lambda_j^t x_{ij}^{2t} \geq x_{io}^{2t}, \forall i \in L, \forall t, \\ & \sum_{j=1}^{n} \lambda_j^t y_{rj}^t \geq \theta_o^t y_{ro}^t, \forall r \in R, \forall t, \\ & \sum_{j=1}^{n} \lambda_j^t y_{rj}^{1t} \geq \theta_o^t y_{ro}^{1t}, \forall r \in K, \forall t, \\ & \sum_{j=1}^{n} \lambda_j^t y_{rj}^{2t} \leq \theta_o^t y_{ro}^{2t}, \forall r \in K, \forall t, \\ & \sum_{j=1}^{n} \lambda_j^t = 1, \forall t, \\ & \lambda_j^t \geq 0, \forall j, \forall t. \end{array} \tag{3.3}$$

Model (3.3) is an output-oriented model with the assumption of VRS. The proposed model in the input orientation is given as follows:

$$\begin{array}{ll} Min & e_o^m = \sum_{t=1}^{T} \theta_o^t/T \\ s.t. & \sum_{j=1}^{n} \lambda_j^t x_{ij}^t \leq \theta_o^t x_{io}^t, \forall i \in I, \forall t, \\ & \sum_{j=1}^{n} \lambda_j^t x_{ij}^{1t} \leq \theta_o^t x_{io}^{1t}, \forall i \in L, \forall t, \\ & \sum_{j=1}^{n} \lambda_j^t x_{ij}^{2t} \geq \theta_o^t x_{io}^{2t}, \forall i \in L, \forall t, \\ & \sum_{j=1}^{n} \lambda_j^t y_{rj}^t \geq y_{ro}^t, \forall r \in R, \forall t, \\ & \sum_{j=1}^{n} \lambda_j^t y_{rj}^{1t} \geq y_{ro}^{1t}, \forall r \in K, \forall t, \\ & \sum_{j=1}^{n} \lambda_j^t y_{rj}^{2t} \leq y_{ro}^{2t}, \forall r \in K, \forall t, \\ & \sum_{j=1}^{n} \lambda_j^t = 1, \forall t, \\ & \lambda_j^t \geq 0, \forall j, \forall t. \end{array} \tag{3.4}$$

$e_o^{M*}$and e_o^{m*} indicate the efficiency scores of DMU_o(i.e. the unit under evaluation) in models (3.3) and (3.4), respectively. The optimal value of model (3.3) is not less than one, that is $e_o^{M*} \geq 1$ and DMU_o is efficient in all periods if and only if $e_o^{M*} = 1$. If $e_o^{M*} > 1$, then DMU_o is inefficient at least in one period. Furthermore, the efficiency score of model (3.4) is not greater than one, that is $e_o^{m*} \leq 1$. Provided that $e_o^{m*} < 1$, DMU_o is the inefficient unit at least in one period, and it is efficient in all periods if and only if $e_o^{m*} = 1$. We determine the average efficiency of multi-period systems in the presence of negative data because we believe it is more logical and rational in comparison with assessing the efficiency from optimistic and pessimistic points of view. Also, for calculating the efficiency changes of a DMU between two periods, the Malmquist

Productivity Index (MPI) can be used. The following formulas are presented for estimating the efficiency changes of a DMU between periods t and $t+k$:

$$MPI_o^{M(t,t+k)} = \frac{e_o^{M(t+k)}}{e_o^{M(t)}}, \quad (3.5)$$

$$MPI_o^{m(t,t+k)} = \frac{e_o^{m(t+k)}}{e_o^{m(t)}}, \quad (3.6)$$

$e_o^{M(t+k)}$and $e_o^{M(t)}$(i.e. $\theta_o^{t+k}$and θ_o^t, in model (3.3)) are the efficiency scores of DMU_o in period $t+k$ and period t that are obtained from model (3.3). In formula (3.5) if $MPI_o^{M(t,t+k)} > 1$, the performance of DMU_o has been deteriorated. If $MPI_o^{M(t,t+k)} < 1$, the performance of DMU_o has been improved and the efficiency is without change if $MPI_o^{M(t,t+k)} = 1$. In formula (3.6), $e_o^{m(t+k)}$and $e_o^{m(t)}$(i.e. $\theta_o^{t+k}$and θ_o^t, in model (3.4)) show the efficiency scores of DMU_o in periods $t+k$ and t that are calculated by model (3.4). In this case, changes in efficiency are interpreted as follows:

If $MPI_o^{m(t,t+k)} > 1$ then the efficiency of DMU_o has been improved,

If $MPI_o^{m(t,t+k)} < 1$, the efficiency of DMU_o has been deteriorated,

If $MPI_o^{m(t,t+k)} = 1$, the efficiency is without change.

4 Efficiency measurement of Iranian bank branches

The banking sector is one of the most significant sectors in countries. Banks play the important role in financial systems and economic development. Therefore, efficiency estimation of banks as notable financial institutes is essential for economic progress. Furthermore, the performance changes and comparing the efficiency of a bank between two periods are important issues for management and future decisions.

For these reasons, 50 branches of an Iranian bank are evaluated in two years, 2014 and 2015 in the current section. Input and output data for the years 2014 and 2015 are shown in Tables 1 and 2, respectively. Input variables chosen for the analysis are:

- The number of employees,
- Expenses and
- Costs.

Outputs are

- Loans,
- Profits,
- Deposits and
- The number of clients.

The profit factor is considered as a measure that can take positive and negative values. Indeed, with regard to the profit as an output factor, the loss is deemed as a negative value. In this empirical application we have used the approach suggested in the output orientation because our purpose is to maximize the output factors.

At first, model (3.3) is calculated for estimating the average efficiency of branches in two years. Results can be found in Table 3. Column 2 of Table 3 shows the efficiency of branches in 2014 while the efficiency scores of branches in 2015 year are presented in column 3 of Table 3. Furthermore, the average efficiency of the two periods is presented in column 4. As can be seen, 32 branches are efficient in 2014 while this number decreases to 19 in 2015. Nevertheless, 15 branches are efficient averagely. Actually, they are efficient in both years, 2014 and 2015. Also, branch 36 is the most inefficient DMU with a score of 1.4643. Moreover, formula (3.5) is utilized for obtaining changes of efficiency between the two years. The results can be seen in columns 5 and 6 of Table 3. 15 branches have the fixed performance between the two years while 7 branches have improved their performance. Nonetheless, the performance has been worsened in 28 branches. It seems the majority of branches should change their schemes for improving the efficiency.

GAMS (General Algebraic Modeling System) software on an Intel (R) Core 2, 3 GB RAM, 2.20 GHz PC has been applied in this study in order to run the proposed model for the data set of bank branches.

5 Conclusions

In the real world there are situations that the efficiency of units with some negative data must

Table 1: Input and output data for 2014.

#Branch	Inputs			Outputs			
	Employees	Expenses	Costs	Loans	Profits	Deposits	Clients
1	23	341553.6	27475.90446	15759.58	8462.3113	599370.1	46048
2	19	266656.6	22767.88216	28998.29	20219.545	369099.2	49969
3	10	124764.1	11471.75027	11448.98	2603.7317	175013.1	31932
4	16	263170.1	20075.31399	34958.05	-2756.598	455357.5	32814
5	12	141968.6	12922.78883	7160.744	-4238.179	274747.2	36807
6	13	159763.9	14254.2599	12169.06	13512.079	214638.3	29385
7	14	219514.8	15985.86957	7161.087	7072.5923	484201.7	26042
8	7	111588.9	8899.005053	21406.83	4911.4066	116912	23492
9	18	293543.2	19729.09533	9203.314	11933.063	440286.6	27606
10	13	87890.53	14518.40462	9002.474	-13100.41	344004.9	30199
11	15	213154	16696.48565	11212.32	10761.876	288747.9	40547
12	11	128932.6	12608.80289	6718.22	-8898.567	312628.1	32853
13	9	99391.69	10877.17805	15447.08	6637.8251	182111.1	23072
14	6	36780.9	6250.53117	957.3781	-8485.414	161352.6	15265
15	13	102348.4	15441.72033	25669.19	4458.6534	176351.1	8594
16	8	160316.3	9338.397872	15936.14	17603.501	122899.3	27296
17	8	101995	9594.827157	11784.65	4641.2722	171497.7	22740
18	8	92269.86	8568.514489	11041.62	13123.504	115930.2	19444
19	9	145643.7	9317.355439	3065.821	11519.893	163285.1	24521
20	9	144294.8	10765.07639	16879.29	10106.332	208565.7	19863
21	14	155055.2	17620.38934	29818.95	10134.409	392619.7	15998
22	5	50530.51	5342.754074	1929.848	1996.3737	90864.58	11383
23	11	111124.2	11932.90858	7465.225	4524.7582	186078.6	20191
24	13	101283.2	14623.20417	11884.04	-1967.574	193930.1	23552
25	13	106377.2	14859.47837	8233.518	-9070.798	346692.1	21666
26	37	1547020	63598.44384	310911.3	111200.9	1474744	66206
27	5	44285.72	5270.868106	2761.976	-1168.767	117439.3	15436
28	5	65808.05	5196.055186	892.1892	1962.5556	123157.7	12797
29	7	80196.59	8654.531133	16582.12	6622.5184	74643.7	15560
30	5	41888.71	5262.01995	664.5616	-7698.613	131533.3	9864
31	6	61101.9	6945.290131	9320.273	846.05276	110750.5	8886
32	6	67351.62	6612.901467	4281.546	1983.8216	129783.9	17426
33	5	36316.26	5482.210432	3506.984	619.56076	80610.9	12836
34	6	56910.11	6440.679649	3917.869	-2613.403	122478.8	19959
35	6	74653.93	6827.922441	9093.735	3325.1662	119424.8	18104
36	7	107827.9	7525.574056	483.2824	13833.684	119063.3	12126
37	7	76469.83	7937.535498	9096.752	-1527.559	147274.9	30961
38	10	87981.71	11373.03691	13602.42	-5760.168	229123.3	25296
39	6	69897.14	7163.657875	7550.696	-1680.533	142416.7	14395
40	7	32692.83	7470.866604	2330.389	867.3898	83382.88	14263
41	4	66270.46	4893.031295	1551.293	3818.4618	97054.66	16739
42	5	44771.54	5009.610757	1413.792	2879.8245	82666.38	9244
43	6	70298.75	6353.1055	6630.648	2987.5124	131630.5	12682
44	4	87894.9	4909.28451	2326.202	7915.0523	145392.6	20928
45	6	59241.94	6411.074654	4583.839	4872.7078	102044	18813
46	10	199919.6	16036.00784	61465.92	28209.542	212863.6	22836
47	5	33578.11	5471.760532	3222.479	-3909.97	103821	11115
48	8	79230.6	8433.796763	2969.224	239.50353	161719.1	23960
49	9	97576.99	10050.60266	6376.547	2439.0705	202022.8	20298
50	5	45761.6	5323.4467	1617.114	-732.1983	112166.3	17470

be evaluated in multiple periods of time. For instance, a factor like profit can take negative val-

Table 2: Input and output data for 2015.

#Branch	Inputs			Outputs			
	Employees	Expenses	Costs	Loans	Profits	Deposits	Clients
1	21	348141.48	20088.802	24186.748	17003.115	678464.6	46964
2	17	256772.6	15824.631	36775.618	-6355.392	390342.6	51047
3	12	217626.58	11179.451	12455.401	-6506.136	245562.5	32769
4	15	267778.62	14093.802	33385.022	1803.1985	515440.4	33917
5	12	153063.69	11193.739	9390.3137	9272.7597	343989.1	37730
6	14	185056.27	13111.831	18365.788	-254.9367	254456.5	30141
7	12	228904.29	11224.741	8114.2416	-2005.332	411479.4	26497
8	7	117698.4	6532.332	14316.368	-4604.91	141451.2	24050
9	15	314039.67	14020.841	25399.623	-2231.14	506956.2	29132
10	12	86136.126	11182.012	11647.497	20777.03	324342	30915
11	14	289895.45	13084.286	12014.048	-7155.507	347699.8	41523
12	13	134586.13	12166.12	8663.9261	16477.51	383487.2	33671
13	9	107759.8	8397.8681	13569.103	1370.5174	255353.2	23806
14	6	38556.556	5630.8613	2204.8069	15688.123	186299.8	15628
15	10	97821.211	9374.0074	24863.022	1901.074	192741	9194
16	8	164193.21	7485.6404	16933.107	-10753.39	141052.8	28113
17	8	98915.387	7460.0489	10669.764	2395.0058	201135.9	23624
18	7	103752.04	6554.2749	11284.416	-3226.879	143490.2	20225
19	9	151090.33	8405.3637	5989.6116	-11990.24	205073.7	25405
20	8	147549.84	7471.8181	7513.2192	-6285.209	235494.9	20550
21	8	163025.07	7475.0539	46962.009	12160.743	237015.5	16340
22	5	52756.682	4693.9952	4363.4922	-563.2497	104635.1	11639
23	10	127094.98	9394.4598	10614.788	586.88746	218924.6	20978
24	11	105006.47	10335.376	18707.249	2788.3464	209725.5	24246
25	13	98961.203	12147.959	8477.1308	16668.957	385418.6	22308
26	36	1666329.3	34437.236	179323.07	-139656.1	1923135	67699
27	5	53317.986	4661.9942	1909.2215	3623.1971	143609.6	15882
28	5	79935.602	4665.5352	1309.4256	2298.7278	161601.3	13111
29	7	82430.765	6527.023	17109.286	-7069.052	89291.98	16036
30	5	41984.158	4692.164	1566.5893	9922.4888	167436.4	10207
31	6	59268.11	5608.3144	10678.969	620.22839	122325.7	9203
32	7	90333.142	6556.0096	6115.7884	4900.2383	163833.7	18174
33	5	50485.613	4665.4142	6306.434	1701.8963	104589.8	13328
34	6	56098.196	5594.5964	3806.8001	4040.1075	144299.6	20559
35	6	70842.171	5602.4035	11821.422	293.65229	124291.2	18543
36	8	108156.11	7498.5619	2735.5313	-6782.928	138239.6	12541
37	7	82561.821	6509.3823	12616.65	3406.1063	183837.5	31555
38	10	94016.22	9327.1783	24089.005	9989.2619	287008	26094
39	6	91565.474	5616.1304	5982.938	1193.479	172611.7	14954
40	7	36059.454	6551.2161	3416.5737	1664.9014	101290.1	14719
41	6	67289.407	5618.7319	2898.0483	459.55294	131617.4	17289
42	4	39317.811	3762.8899	4826.6473	1052.6853	105153.1	9505
43	5	77671.608	4662.9742	7700.2647	3421.9334	166928.2	13242
44	6	105531.49	5603.0014	3423.2815	-1029.178	181130.3	21410
45	6	65564.8	5598.1084	7225.4043	1146.8331	143728	19350
46	10	196129.53	9412.4933	43300.632	-24475.74	240365.7	23816
47	5	38359.386	4676.7752	3878.4632	6197.1179	126795.4	11542
48	8	96638.591	7480.4909	4809.0912	2603.8739	207639.3	24837
49	7	88133.115	6556.9786	9495.0527	7153.7979	239989.6	20927
50	6	48733.019	5614.7654	2257.6831	3291.4909	127860.5	18016

ues in evaluating the efficiency of banks in several periods. Nevertheless, traditional DEA mod-

Table 3: Results.

#Branch	$\theta^{1*}(e^{M1})$	$\theta^{2*}(e^{M2})$	Average efficiency(e^{M*})	$MPI^{M(1,2)}$	Changes of efficiency
1	1	1	1	1	Fixed
2	1	1	1	1	Fixed
3	1	1.2494	1.1247	1.2494	Worsened
4	1.0021	1	1.0011	0.9979	Improved
5	1	1	1	1	Fixed
6	1.0677	1.1864	1.1271	1.1112	Worsened
7	1	1.0215	1.0107	1.0215	Worsened
8	1	1.184	1.092	1.1840	Worsened
9	1.1745	1.0331	1.1038	0.8796	Improved
10	1	1	1	1	Fixed
11	1	1.063	1.0315	1.0630	Worsened
12	1	1	1	1	Fixed
13	1	1.0969	1.0484	1.0969	Worsened
14	1	1	1	1	Fixed
15	1	1.0196	1.0098	1.0196	Worsened
16	1	1.1456	1.0728	1.1456	Worsened
17	1.0344	1.19	1.1122	1.1504	Worsened
18	1	1.3853	1.1927	1.3853	Worsened
19	1.1263	1.3409	1.2336	1.1905	Worsened
20	1.1487	1.2098	1.1793	1.0532	Worsened
21	1	1	1	1	Fixed
22	1.0946	1.3368	1.2157	1.2213	Worsened
23	1.1891	1.4061	1.2976	1.1825	Worsened
24	1.1906	1.166	1.1783	0.9793	Improved
25	1.0127	1	1.0064	0.9875	Improved
26	1	1	1	1	Fixed
27	1	1	1	1	Fixed
28	1	1.0234	1.0117	1.0234	Worsened
29	1.0769	1.1241	1.1005	1.0438	Worsened
30	1	1	1	1	Fixed
31	1.0323	1.1071	1.0697	1.0725	Worsened
32	1.0305	1.3375	1.184	1.2979	Worsened
33	1	1.1313	1.0657	1.1313	Worsened
34	1.1065	1.0051	1.0558	0.9084	Improved
35	1.0117	1.0874	1.0496	1.0748	Worsened
36	1	1.9287	1.4643	1.9287	Worsened
37	1	1	1	1	Fixed
38	1.0115	1	1.0058	0.9886	Improved
39	1.1021	1.1673	1.1347	1.0592	Worsened
40	1	1	1	1	Fixed
41	1	1.2937	1.1469	1.2937	Worsened
42	1	1	1	1	Fixed
43	1	1	1	1	Fixed
44	1	1.0022	1.0011	1.0022	Worsened
45	1	1.1635	1.0817	1.1635	Worsened
46	1	1.0304	1.0152	1.0304	Worsened
47	1	1	1	1	Fixed
48	1	1.1443	1.0722	1.1443	Worsened
49	1.059	1	1.0295	0.9443	Improved
50	1	1.0305	1.0153	1.0305	Worsened

els usually evaluate the efficiency of DMUs in a specific period of time while measures are non-negative. Therefore, the current paper has been suggested an approach for determining the per-

formance of DMUs with negative data in multi-period systems. Actually, the average efficiency of multi-period production systems has been assessed while some negative factors (inputs and/or output) exist. Also, the changes of efficiencies between the two periods have been estimated via the presented formula. Because of the major role of banks in financial systems and countries, data set of the branches of an Iranian bank has been used to demonstrate and clarify the approach.
Furthermore, ranking efficient DMUs is significant for many systems. Thus, ranking and distinguishing DMUs in multi-period systems in the presence of negative and positive data seems to be an interesting subject for future research. Further research should be conducted to find the average efficiency scores in multi-period two-stage production systems when some negative factors are present.

Acknowledgement

Financial support by Lahijan Branch, Islamic Azad University Grant No. 1235, 17-20-5/3507 is gratefully acknowledged.

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Evaluating the Efficiency of Firms with Negative Data in Multi-Period Systems: An Application to Bank Data

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ارزیابی کارایی واحدها با حضور داده‌های منفی در سیستم‌های چند دوره‌ای: کاربردی برای داده‌های بانک

چکیده:

تحلیل پوششی داده‌ها یک روش ریاضی به منظور ارزیابی کارایی واحدها با چندین ورودی و خروجی است. در مدل‌های متداول تحلیل پوششی داده‌ها، عملکرد واحدهای تصمیم‌گیرنده با ورودی و خروجی‌های نامنفی در یک دوره خاص ارزیابی می‌شود. اگرچه در واقعیت موقعیت‌هایی وجود دارند که عملکرد واحدها باید در چندین دوره از زمان تعیین گردند در حالی که داده‌های منفی حضور دارند. بنابراین در این مقاله روشی به منظور تعیین کارایی سیستم‌های چند دوره‌ای در حضور عامل‌های مثبت و منفی ارایه می‌شود. به عبارت دیگر کارایی متوسط واحدها با تعدادی اندازه منفی در سیستم‌های چند دوره‌ای محاسبه می‌شود. روش پیشنهادی مدل اندازه شعاعی نیمه ماهیت (SORM) را به منظور جادادن عامل‌های منفی (ورودی و خروجی) و تعیین کارایی سیستم‌های تولید چند دوره‌ای به کار می‌برد. یک مجموعه داده واقعی از بخش بانکداری برای توضیح روش پیشنهادی استفاده می‌شود