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Research Article

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A New Method for Comparing Fuzzy Quantities Based on Scaler Value and Middle-Point of Fuzzy Numbers

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Abstract

The importance as well as the difficulty of the problem of ranking fuzzy numbers is pointed out. Here we consider approaches to the ranking of fuzzy numbers based upon the idea of associating with a fuzzy number a scalar value, its signal/noise ratios, where the signal and the noise are defined as the middle-point and the spread of each γ-cut of a fuzzy number, respectively. We use the value of a as the weight of the signal/noise ratio of each γ-cut of a fuzzy number to calculate the ranking index of each fuzzy number. The proposed method can rank any kinds of fuzzy numbers with different kinds of membership functions.

Keywords : Ranking; Fuzzy number; Defuzzification; Signal/noise ratios.

1 Introduction

In many applications of fuzzy set theory, particularly in decision making, we often obtain a measure of a course of action expressed as a fuzzy number, a fuzzy subset of the real line. For example the profit obtained by using the new XYZ process may be about §300,000. Essentially here we have some uncertainty as to the exact value of the profit. As noted in the literature1 this is a kind of possibilistic uncertainty. Often in these decision making environments we are faced with the problem of selecting one from among a collection of alternative actions. This selection process may then require that we rank, order, fuzzy numbers. While it is clear when considering two pure numbers which is bigger or smaller, the situation with respect to fuzzy numbers is not always obvious. It was early in the development of the fuzzy set theory that the problem of comparing fuzzy subsets of the real line was seen to be an important and difficult problem. The recent literature has also addressed this problem.4 What seems to be clear is that there exists no uniquely best method for comparing fuzzy numbers, the different methods satisfy different desirable criteria. While certain properties are necessary for any methodology that orders fuzzy numbers, user preferences account for a significant part of the performance of a preferred approach. Our focus here is to try to understand and suggest some methodologies for comparing fuzzy numbers. In this paper, we present a new approach for ranking fuzzy numbers using the γ-cut, the belief features and the signal/noise ratios of fuzzy numbers, where $\gamma \in [0, 1]$. The proposed method can overcome the drawbacks of Chen and Chen's method [3], Cheng's method [4], Murakami et al. [10], Yong and Qi's method [20] and Yager's method [19].

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2 A review of the existing methods for ranking fuzzy numbers

In this paper, we assume that the reader is familiar with basics of fuzzy set theory and fuzzy logic in the broad sense.

A fuzzy number is a convex fuzzy subset of the real line R and is completely defined by its membership function. Let A be a fuzzy number, whose membership function $f_A(x)$ can generally be defined as [1, 2, 12, 13, 14, 15, 8],

$$f_A(x) = \begin{cases} f_A^L(x) & when \quad a_1 \leq x < a_2, \\ \omega & when \quad a_2 \leq x < a_3, \\ f_A^R(x) & when \quad a_3 \leq x < a_4, \\ 0 & otherwise. \end{cases} \tag{2.1}$$

Where $0 \leq \omega \leq 1$ is a constant, $f_A^L : [a_1, a_2] \to [0, \omega]$ and $f_A^R : [a_3, a_4] \to [0, \omega]$ are two strictly monotonically and continuous mappings from R to closed interval $[0, \omega]$. When $\omega = 1$, then A is a normal fuzzy number; otherwise it is said to be a non-normal fuzzy number. If the membership function $f_A(x)$ is piecewise linear, then A is referred to as a trapezoidal fuzzy number and is usually denoted by, $A = (a_1, a_2, a_3, a_4; \omega)$. In particular, if $a_2 = a_3$, the trapezoidal fuzzy number is reduced to a triangular fuzzy number. Since $f_A^L(x)$ and $f_A^R(x)$ are both strictly monotonically and continuous functions, their inverse functions exist and should also be continuous and strictly monotonically. Let $g_A^L : [0, \omega] \to [a_1, a_2]$ and $g_A^R : [0, \omega] \to [a_3, a_4]$ be the inverse functions of f_A^L and f_A^R, respectively. Then $g_A^L(y)$ and $g_A^R(y)$ should be integrable on the closed interval $[0, \omega]$. In other words, both $\int_0^\omega g_A^L(y)dy$ and $\int_0^\omega g_A^R(y)dy$ should exist. In the case of trapezoidal fuzzy number, the inverse function $g_A^L(y)$ and $g_A^R(y)$ can be analytically expressed as:

$$g_A^L(y) = a_1 + \frac{(a_2 - a_1)y}{\omega}, \quad 0 \leq y \leq \omega, \tag{2.2}$$

$$g_A^R(y) = a_4 - \frac{(a_4 - a_3)y}{\omega}, \quad 0 \leq y \leq \omega. \tag{2.3}$$

In order to determine the centroid point $(\overline{x}_0, \overline{y}_0)$ of a fuzzy number A, Wang et al. [18] provided the following centroid formulae:

$$\overline{x}_0(A) = \frac{\int_{a_1}^{a_2} x f_A^L(x)dx + \int_{a_2}^{a_3} (x\omega)dx + \int_{a_3}^{a_4} x f_A^R(x)dx}{\int_{a_1}^{a_2} f_A^L(x)dx + \int_{a_2}^{a_3} (\omega)dx + \int_{a_3}^{a_4} f_A^R(x)dx}, \tag{2.4}$$

$$\overline{y}_0(A) = \frac{\int_0^\omega y(g_A^R(y) - g_A^L(y))dy}{\int_0^\omega (g_A^R(y) - g_A^L(y))dy}. \tag{2.5}$$

The ranking value $R(A)$ of the fuzzy number A is defined as follows [4]:

$$R(A) = \sqrt{\overline{x}_0^2(A) + \overline{y}_0^2(A)}. \tag{2.6}$$

The larger the value of $R(A)$, the better the ranking of A.

In [7], the authors presented a centroid-index ranking method for ordering fuzzy numbers. The centroid point of fuzzy number A, is $(\overline{x}_A, \overline{y}_A)$ where $\overline{x}_A$ and $\overline{y}_A$ are the same as formula 2 and 3 in [7]. The ranking value $S(A)$ of the fuzzy number A is defined as follows:

$$S(A) = \overline{x}_A \times \overline{y}_A. \tag{2.7}$$

The larger the value $S(A)$, the better the ranking of A. In [3], Chen et al. proposed a simple method to obtain COG point of fuzzy numbers. If A is a generalized fuzzy number, where $A = (a_1, a_2, a_3, a_4; \omega)$, then the COG point (x_A^*, y_A^*) of A is as follows:

$$x_A^* = \frac{y_A^*(a_2 + a_3) + (a_1 + a_4)(1 - y_A^*)}{2}, \tag{2.8}$$

$$y_A^* = \begin{cases} \frac{\omega(\frac{a_3 - a_2}{a_4 - a_1} + 2)}{6}, & a_1 \neq a_4, \\ \frac{1}{2}, & a_1 = a_4. \end{cases} \tag{2.9}$$

After obtaining the COG point of fuzzy number A where $A = (a_1, a_2, a_3, a_4; w_A)$, the ranking value *Rank(A)* can be calculated as

$$Rank(A) = x_A^* + (w_A - y_A^*)^{\hat{s}_A} \times (y_A^* + 0.5)^{1 - w_A}, \tag{2.10}$$

where,

$$\hat{s}_A = \sqrt{\frac{\sum_{i=1}^4 (a_i - \overline{a})^2}{3}}, \tag{2.11}$$

and,

$$\overline{a} = \frac{a_1 + a_2 + a_3 + a_4}{4}. \tag{2.12}$$

The larger the value *Rank(A)*, the better the ranking A. However, this method has a drawback in that it cannot correctly rank generalized fuzzy numbers in some situations. The example is used to show the drawback Chen's method.

Example 2.1 *Two generalized fuzzy number A and B are shown as follows (Fig. 1):*

$$A = (-0.01, -0.01, -0.01, -0.01; 1),$$

$$B = (0.01, 0.01, 0.01, 0.01; 0.8).$$

It can be easily to obtain the COG points of fuzzy numbers A and B respectively, as follows, $(x^*_A, y^*_A) = (-0.01, 0.5)$ *and* $(x^*_B, y^*_B) = (0.01, 0.4)$. *By applying Chen method, we have* $R(A) = 0.99$ *and* $R(B) = 0.989$. *The ranking result shows that ranking order is* $A \succ B$. *However, it can be easily seen that the correct order is* $A \prec B$.

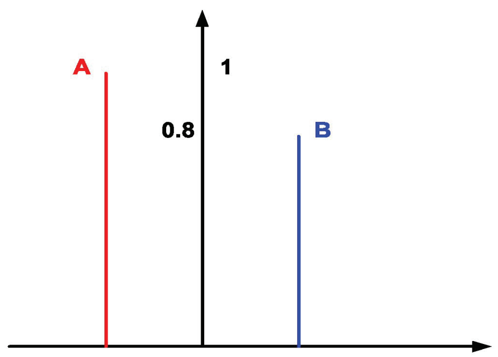

Figure 1

3 A novel method for ranking fuzzy numbers

In this section, we present a new method for ranking fuzzy numbers. The proposed method integrates many concepts, such as the approximate area measure [5], the belief feature [6] and the signal/noise ratio [9]. Assume that a decision maker wants to determine the ranking order of m fuzzy numbers $A_1, A_2, \cdots$, and A_m. The kth γ-cut $A_i^{\gamma_k}$ of fuzzy number A_i is defined as follows:

$$A_i^{\gamma_k} = \{x | f_{A_i}(x) \geq \gamma_k, x \in X\}, \; \gamma_k = \frac{k}{n}, \; k \in \{0, 1, \cdots, n\}, \quad n \in N \tag{3.13}$$

where n denotes the number of γ-cuts.
The minimal value $l_{i,k}$ and the maximal value $r_{i,k}$ of the kth γ-cut of the fuzzy number A_i are defined as follows:

$$l_{i,k} = \inf_{x \in X} \{x | f_{A_i}(x) \geq \gamma_k\}. \tag{3.14}$$

$$r_{i,k} = \sup_{x \in X} \{x | f_{A_i}(x) \geq \gamma_k\}. \tag{3.15}$$

respectively. The maximal barrier U and the minimal barrier L of the m fuzzy numbers $A_1, A_2, \cdots$, and A_m are defined as follows:

$$U = \max_{\forall i} \{x | x \in A_i^{\gamma}, \; 0 \leq \gamma \leq h_{A_i}, \; 1 = 1, 2, \cdots, m\}, \tag{3.16}$$

$$L = \min_{\forall i} \{x | x \in A_i^{\gamma}, \; 0 \leq \gamma \leq h_{A_i}, \; 1 = 1, 2, \cdots, m\}. \tag{3.17}$$

where A_i^{γ} denotes the γ-cut of the fuzzy number A_i and h_{A_i} denotes the height of A_i defined as follows:

$$h_{A_i} = \sup_{x \in X} f_{A_i}(x). \tag{3.18}$$

The signal/noise ratio $\eta_{i,k}$ of the kth γ-cut of the fuzzy number A_i used in the proposed method is defined as follows:

$$\eta_{i,k} = \frac{m_{i,k} - L}{\delta_{i,k} + c}, \tag{3.19}$$

where $m_{i,k}$ and $d_{i,k}$ denote the middle-point and the spread of $A_i^{\gamma_k}$, respectively, defined as follows:

$$m_{i,k} = \frac{r_{i,k} + l_{i,k}}{2}, \tag{3.20}$$

$$\delta_{i,k} = r_{i,k} - l_{i,k}. \tag{3.21}$$

L denotes the minimal barrier of the m fuzzy numbers $A_1, A_2, \cdots, A_m$ defined by Eq. (3.17), c is a parameter, and $c > 0$. The parameter $c > 0$ is used to avoid the case that if the fuzzy number A_i is the crisp value "0", the signal/noise ratio will be indeterminate. From Eq. (3.19), we can find that the larger the value of c, the smaller the influence of $\delta_{i,k}$ on the signal/noise ratio $\eta_{i,k}$. Therefore, we think that the influence of $\delta_{i,k}$ on $\eta_{i,k}$ should be smaller than the influence of $m_{i,k}$ on $\eta_{i,k}$. The value of c should be greater than the value of $R - L$ in order to avoid the special case that if we want to obtain the ranking order of two equal crisp values A_1 and A_2, the values of $R-L$ and $\delta_{i,k}$ of the kth γ-cut of the fuzzy number A_1 and A_2 will be all zero and the signal/noise ratio will be indeterminate or undefined, where $\gamma_k \in [0, 1]$. In the following, we present a new approach for comparing fuzzy numbers based on the

distance method. The method not only considers the signal/noise ratio of a fuzzy number, but also considers the minimum crisp value of fuzzy numbers. The proposed method for ranking fuzzy numbers $A_1, A_2, \cdots, A_m$ is now presented as follows:
Use the point $(RI(A_j), 0)$ to calculate the ranking value $sn/r(A_j) = D(RI(A_j), x_{min})$ of the fuzzy numbers A_j, where A_j , where $1 \leq j \leq m$, as follows:

$$D(RI(A_j), x_{min}) = \|RI(A_j) - x_{min}\| \quad (3.22)$$

From formula (3.22), we can see that $sn/r(A_j) = D(RI(A_j), x_{min})$ can be considered as the Euclidean distance between the point $(RI(A_j), 0)$ and the point $(x_{min}, 0)$. We can see that the larger the value of $sn/r(A_j)$, the better the ranking of A_j, where $1 \leq j \leq m$. When ranking n fuzzy numbers $A_1, A_2, \cdots, A_m$, the minimum crisp value $x_{\min}$ is defined as:

$$x_{\min} = min\{x | x \in Domain(A_1, A_2, \cdots, A_m)\}. \quad (3.23)$$

The index $RI(A_j)$ of fuzzy numbers A_i is calculated as $RI(A_j) = \frac{h_{A_i} \sum_{k=1}^{n} \gamma_k \times \eta_{i,k}}{\sum_{k=1}^{n} \gamma_k}$, where $\gamma = h_{A_i} \times \frac{k}{n}$, $k \in \{1, 2, \cdots, n\}$, $n \in N$, and n denotes the number of γ-cuts.

3.1 An Application

Chen and Chen [3] proposed a method to handle fuzzy multi-criteria decision making problems based on fuzzy number induced ordered weighted averaging (FN-IOWA) operator and applied the algorithm to a human selection problem. In this section, we use the same example illustrated in Chen and Chen to show the efficiency of the proposed ranking method. For more detailed information about the FN-IOWA operator, (see [11, 16, 17, 19]). Here we just pay attention to the fuzzy ranking step in the final decision making process.
A new manager will be recruited among three candidates, X, Y and Z. The final scores, which can be obtained by an FN-IOWA operator, are fuzzy numbers and are listed as follows:

$$S_X = (0.2501, 0.7727, 2.2501; 1),$$
$$S_Y = (0.0667, 0.5000, 1.8750; 1),$$
$$S_Z = (0.1667, 0.6592, 2.2500; 1).$$

By applying the proposed ranking method, the index radius of gyration of each alternative can be obtained as follows:

$$sn/r(X) = 1.10,$$
$$sn/r(Y) = 0.09,$$
$$sn/r(Z) = 1.01.$$

We can see that their ranking order $X > Z > Y$. Therefore, Candidate X is more suitable than Candidate Z, and Candidate Z is more suitable than Candidate Y. The result are the same as the one presented in Chen and Chen.

4 Conclusion

In this paper, we have presented a new approach for ranking of fuzzy numbers. First, we present a new method for ranking fuzzy numbers based on the γ-cuts, the belief features and the signal/noise ratios of fuzzy numbers. The proposed method calculates the signal/noise ratio of each γ-cut of a fuzzy number to evaluate the quantity and the quality of a fuzzy number, where the signal and the noise are defined as the middle-point and the spread of each γ-cut of a fuzzy number, respectively. We use the value of a as the weight of the signal/noise ratio of each γ-cut of a fuzzy number to calculate the ranking index of each fuzzy number. The proposed fuzzy ranking method can rank any kinds of fuzzy numbers with different kinds of membership functions.

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A New Method for Comparing Fuzzy Quantities Based on Scaler Value and Middle-Point of Fuzzy Numbers

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یک روش جدید برای مقایسه اعداد فازی بر اساس مقدار اسکالر و نقطه میانی از اعداد فازی

چکیده:

هدف این مقاله مطالعه ی توزیع نسبت آماره های ترتیبی تعمیم یافته برای توزیع پارتو می باشد. برآورد پارامترهای توزیع پارتو برپایه ی آماره های ترتیبی تعمیم یافته و نسبت هایی از آنها بدست آمده اند. بعنوان برآورد نقطه ای از روش گشتاوری و برآورد نااریب استفاده شده است. سازگاری برآوردگر نااریب تشریح شده است. برای مقایسه ی عملکرد برآوردگرهای بکارگرفته شده ، نتایج عددی محاسبه شده اند. مثالی نیز به کمک داده های واقعی آورده شده است.