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Research Article

Numerical Study of Unsteady Flow of Gas Through a Porous Medium By Means of Chebyshev Pseudo-Spectral Method

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Abstract

Unsteady flow of gas through a porous medium is a strongly nonlinear ordinary differential equation on semi-infinite interval. In this paper, we have applied the pseudo-spectral collocation method with a positive scaling factor to solve unsteady flow of gas through a porous medium. The method reduces the solution of this problem to the solution of a system of nonlinear algebraic equations. The arising system is solved by Newton's method. To confirm the accuracy and efficiency of the presented scheme, we are compared the obtained numerical results with some well-known results. Results showed a very good agreement between results of presented scheme and the numerical solutions.

Keywords : Chebyshev pseudo-spectral method; Newton iteration method; Semi-infinite interval; Chebyshev-Gauss-Lobatto points; Positive scaling factor; Chebyshev interpolation.

1 Introduction

Spectral methods are very powerful tools for obtaining the approximate solution of many problems arising in different fields of science and engineering [1]. Convenience of applying these methods and exponential convergence are two useful properties which have persuaded many authors to use them for solving many types of problems. The basic idea of spectral methods to solve functional equations is to expand the solution function as a finite series of very smooth basis functions, as given

$$y_N(x) = \sum_{n=0}^{N} y_n \phi_n(x),$$

in which, the best choice of $\phi_n(x)$, are the eigenfunctions of a singular Sturm-Liouville problem. It is well known that eigenfunctions of certain singular Sturm-Liouville problems allow the approximation of functions $C^\infty[a,b]$, where the truncation error approaches zero faster than any negative power of the number of basic functions used in the approximation, as that number (order of truncation N) tends to infinity [1]. This phenomenon is usually referred to as " spectral accuracy " [14], (for more details, refer to [1, 14, 15]). Throughout, we are using orthogonal Chebyshev polynomials of the first kind $\{T_k\}_{k=0}^{+\infty}$, which are eigenfunctions of the singular Sturm-

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Liouville problem:

$$\left(\sqrt{1-x^2}T_k'(x)\right)' + \frac{k^2}{\sqrt{1-x^2}}T_k(x) = 0.$$

Many problems in science and engineering can be modeled by a nonlinear ordinary differential equation on semi-infinite interval. Different spectral methods have been proposed for solving nonlinear problems on semi-infinite interval.

Since Laguerre polynomials are orthogonal over $[0, \infty)$ with the wight function $\exp(-x)$, Laguerre spectral method is commonly applied to solve ordinary and partial differential equations on semi-infinite interval [2, 3, 4, 5].

Another method which is named domain truncation [6], replaces semi-infinite interval with $[0, L]$ by choosing L, sufficiently large.

The third method for solving such problems is based on rational orthogonal functions. Boyd [7] defined a new spectral basis, named rational Chebyshev functions on the semi-infinite interval. Guo et al. [8] introduced a new set of rational Legendre functions which is mutually orthogonal in $L^2(0, +\infty)$. They applied a spectral scheme using the rational Legendre functions for solving the Korteweg-de Vries equation on the half line. Parand and Razzaghi [9, 10, 11] applied spectral method to solve nonlinear ordinary differential equations on semi-infinite intervals. Their approach was based on rational Tau method. They obtained the operational matrices of derivative and product of rational Chebyshev and Legendre functions and then they applied these matrices together with the Tau method to reduce the solution of these problems to the solution of a system of algebraic equations.

Guo [12] proposed a method that proceeds by mapping the original problem in semi-infinite interval to a problem in a bounded domain, and then used the appropriate Jacobi polynomials to approximate the resulting problem. Recently, Abbasbandy and Shivanian [13] used this method coupled with pseudo-spectral method and calculate multiple (dual) solutions of a model of mixed convection in a porous medium with boundary conditions on semi-infinite interval which admit dual solutions.

In this work, we first reformulate the unsteady flow of gas through a porous medium problem in $[0, +\infty)$ to a problem in [-1,1] by variable transformation $\mu = \frac{x-s}{x+s}$, with $s > 0$, and using spectral collocation method [15] based on Chebyshev polynomials (also called pseudo-spectral method) to approximate the resulting problem. The comparison of the results obtained by this method with results obtained by other methods shows that this method provides more accurate and numerically stable solutions.

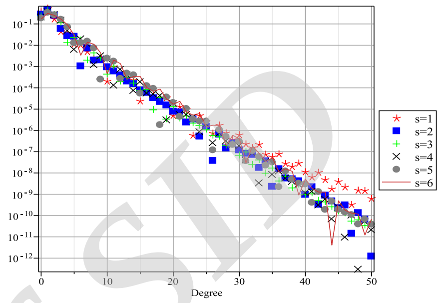

Figure 1: The coefficients $\widehat{y}_k$ versus k for six different values of s. The diagonal cross show the best choice for $N = 50$, which is $s = 4$, when $\alpha = 0.5$.

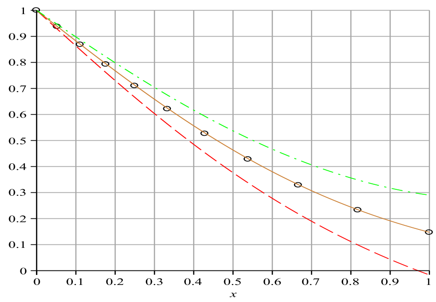

Figure 2: Dash dotted: padeè[3,3], dash line: Padeè[2,2], solid line: present method for $N = 10$, circle: numerical solutions, $\alpha = 0.5$..

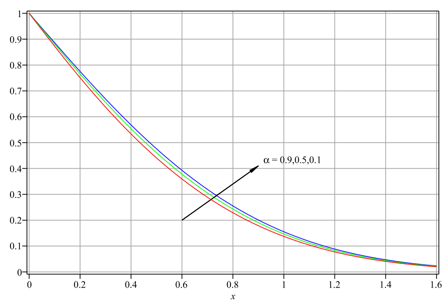

Figure 3: Approximation of y(x) for different values of α, $N = 30$ and $s = 4$.

Table 1: The maximum norm residual error of approximate solutions $y_N(x)$ for $\alpha = 0.5$

	MR errors of presented method						
N	$s = 1.00$	$s = 2.00$	$s = 3.00$	$s = 4.00$	$s = 5.00$	$s = 6.00$	CPU time
30	$2.1E-2$	$1.5E-3$	$6.0E-4$	$6.0E-4$	$8.0E-4$	$9.0E-4$	15.3s
40	$1.5E-3$	$1.4E-5$	$1.6E-5$	$1.2E-5$	$1.6E-5$	$2.5E-5$	31.5s
50	$7.0E-4$	$7.5E-7$	$7.0E-7$	$2.0E-7$	$2.9E-7$	$3.2E-7$	56.9s

Table 2: Values of $y(x)$ for $\alpha = 0.5$ and $x = 0.1$ to 1.0

		Presented method				
x	HPM[27]	MGLFM[25]	PM[15]	NS	$N = 8$	$N = 20$
0.1	0.88808651	0.90931873	0.88165883	0.88136465	0.88235049	0.88136490
0.2	0.77922351	0.81748763	0.76630768	0.76582881	0.76758847	0.76583143
0.3	0.67597925	0.71522344	0.65653800	0.65600068	0.65721502	0.65600097
0.4	0.58027292	0.60982075	0.55440240	0.55389894	0.55417725	0.55390717
0.5	0.49332936	0.51632348	0.46136503	0.46094276	0.46077216	0.46094804
0.6	0.41572078	0.41932385	0.37831093	0.37798158	0.37820424	0.37798064
0.7	0.34747118	0.40982377	0.30559765	0.30535234	0.30672155	0.30535545
0.8	0.28819396	0.31999068	0.24313255	0.24295439	0.24589590	0.24296671
0.9	0.23723457	0.20820285	0.19046237	0.19033423	0.19488803	0.19034954
1.0	0.19379708	0.21991074	0.15876898	0.14677331	0.15265189	0.14678266

Table 3: The values of the initial slope $y'(0)$ for various values of α

		Presented method with	Wazwaz [23]	
α	Numerical solutions	$N = 40$ and $s = 4$	Padeè $[2, 2]$	Padeè $[3, 3]$
0.1	−1.1390072	−1.1390072	−3.5565588	−1.9572089
0.2	−1.1504755	−1.1504755	−2.4418943	−1.7864755
0.3	−1.1629415	−1.1629414	−1.9283384	−1.4782708
0.4	−1.1766157	−1.1766156	−1.6068568	−1.2318018
0.5	−1.1917906	−1.1917906	−1.3731781	−1.0255297
0.6	−1.2088942	−1.2088941	−1.1855196	−0.8400346
0.7	−1.2285985	−1.2285984	−1.0214113	−0.6612048
0.8	−1.2520838	−1.2520838	−0.8633400	−0.4776697
0.9	−1.2818813	−1.2818813	−0.6844601	−0.2772628

Table 4: The values of the initial slope $y'(0)$ for various values of α

α	MR errors of presented method $N = 40$ and $s = 4$	MR errors of perturbation method
0.1	$1.4E-5$	$1.2E-3$
0.2	$1.4E-5$	$5.5E-3$
0.3	$1.3E-5$	$1.1E-2$
0.4	$1.3E-5$	$2.2E-2$
0.5	$1.3E-5$	$3.7E-2$
0.6	$1.2E-5$	$6.0E-2$
0.7	$1.2E-5$	$9.1E-2$
0.8	$1.2E-5$	$1.4E-1$
0.9	$1.2E-5$	$2.1E-1$

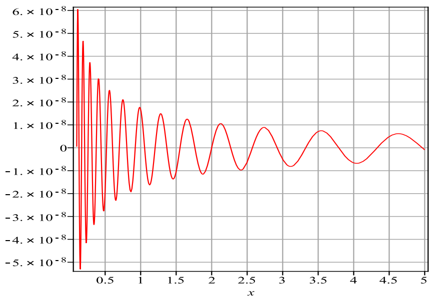

Figure 4: Residual error of (2.2) for $\alpha = 0.5$, $N = 30$ and $s = 4.$.

2 Unsteady gas equation

In this section, a brief review of unsteady flow of gas through a porous medium is presented (more details in this field can be found in [16, 17]). The description of the physical problem closely follows that of Agarwal and Regan [18]. In the study of the unsteady flow of gas through a semi-infinite porous medium initially filled with gas at a uniform pressure $\rho_0 = 0$, at time $t = 0$, the pressure at the outflow face is suddenly reduced from ρ_0 to $\rho_1 = 0$ ($\rho_1 = 0$ is the case of diffusion into a vacuum) and is, thereafter, maintained at this lower pressure. The unsteady isothermal flow of gas is described by a non-linear partial differential equation. The nonlinear partial differential equation that describes the unsteady flow of gas through a semi-infinite porous medium has been derived by Muskat in the form

$$\nabla^2 (P^2) = 2A\frac{\partial P}{\partial t} \tag{2.1}$$

where P is the pressure within porous medium and the constant A is given by the properties of the medium. In the one dimensional medium extending from $z = 0$ to $z = \infty$, Eq. (2.1) reduces to

$$\frac{\partial}{\partial z}\left(P\frac{\partial P}{\partial z}\right) = A\frac{\partial P}{\partial t},$$

with the following boundary conditions:

$$\begin{cases} P(z,0) = P_0, & 0 < z < \infty, \\ P(0,t) = P_1(< P_0), & 0 < t < \infty. \end{cases}$$

By using the following independent and dimension-free variables:

$$x = \frac{z}{\sqrt{t}}\left(\frac{A}{4P_0}\right)^{\frac{1}{2}}, \qquad y(x) = \alpha^{-1}\left(1 - \frac{P^2(z)}{P_0^2}\right),$$

introduced by Kidder [16], the problem transforms to the following nonlinear ordinary differential equation (unsteady gas equation):

$$y''(x) + \frac{2xy'(x)}{\sqrt{1-\alpha y(x)}} = 0, \quad x \geq 0, \quad 0 \leq \alpha < 1. \tag{2.2}$$

The typical boundary conditions imposed by the physical properties are

$$y(0) = 1, \qquad y(\infty) = 0. \tag{2.3}$$

Several methods have been applied for the analytical and numerical solution of this problem. Kidder [16], used perturbation technique to solve problem (2.2). He assumed that

$$y(x) = y_0 + \alpha y_1 + \alpha^2 y_2 + \cdots, \tag{2.4}$$

and set

$$(1-\alpha y)^{-\frac{1}{2}} = 1 + \frac{1}{2}\alpha y + \frac{3}{8}\alpha^2 y^2 + \cdots. \tag{2.5}$$

Substituting (2.4) and (2.5) into (2.2) and organizing it based on coefficients of $1, \alpha, \alpha^2$ gives:

$$\begin{aligned}
1 \quad &: \quad y_0'' + 2xy_0 = 0, \ \ y_0(0) = 1, y_0(\infty) = 0 \\
&\longrightarrow \ \ y_0 = 1 - erf(x), \\
\alpha \quad &: \quad y_1'' + xy_0y_0' + 2xy_1 = 0, y_1(0) = y_1(\infty) = 0 \\
&\longrightarrow y_1 = -\frac{1}{2\pi}\left\{y_0\left[1 + \sqrt{\pi}xe^{-x^2}\right] - e^{-2x^2}\right\}, \\
\alpha^2 \quad &: \quad y_2'' + xy_0'\left\{y_1 + \frac{6}{8}y_0^2\right\} + xy_1'y_0 + 2xy_2 = 0, \\
&y_2(0) = y_2(\infty) = 0 \longrightarrow \ \ y_2 = -\frac{1}{\pi}y_1 - \frac{1}{2\pi}y_0 \\
+ \quad &\frac{1}{8\pi^{3/2}}xe^{-3x^2} - \frac{1}{16\sqrt{\pi}}x(5-2x^2)e^{-x^2}\,(y_0)^2 \\
&+\frac{1}{4\pi}(2-x^2)e^{-2x^2}y_0 + \\
&\frac{3^{3/2}}{16\pi}\left[erf(\sqrt{3}x) - erf(x)\right].
\end{aligned}$$

The convergence of the expansion (2.4) for $0 < \alpha < 1$ is guaranteed [16] through physical properties of y. It is easily seen that the complexity of the calculations increases rapidly with increasing the number of terms [16] and also as perturbation quantity α and variable x increases from 0 to 1, the exact solution decreases (see numerical results in section 4).

Agarwal and Regan [18], presented the various existence results for Eq. (2.2). They show that the boundary-value problem (2.2) has a solution $y \in C^2[0, +\infty)$ with $0 < y(x) \leq 1$ for $x \in [0, +\infty)$. So far, several analytical and numerical methods have been developed for solving problem in Eq. (2.2). Wazwaz [23], solved this problem by modifying the decomposition method and Padé approximation. Noor and Mohyud-Din [24], the variational iteration method using He's polynomials and Padé approximation for solving this problem. Parand et al. [25, 26, 27] applied the Lagrangian method, generalized Laguerre polynomials, rational Chebyshev collocation method and homotopy perturbation method for solving nonlinear problem (2.2). This equation has been recently solved by Abbasbandy [28] with two different numerical approaches based on finite-difference scheme known as the Keller-box method and the shooting method.

3 Pseudo-spectral method

Considering differential equation (2.2) with boundary conditions (2.3), by the change of variable

$$y(x) = Y(\mu), \;\; with \;\; \mu = \frac{x - s}{x + s}, \qquad s > 0, \tag{3.6}$$

where the parameter s is a scaling factor which can be used to tune the spacing of collocation points.
We have

$$\begin{aligned} x &= \frac{s(1+\mu)}{1-\mu}, \\ \frac{dy}{dx} &= \frac{1}{2s}(1-\mu)^2\frac{dY}{d\mu}, \\ \frac{d^2y}{dx^2} &= \frac{1}{4s^2}(1-\mu)^4\frac{d^2Y}{d\mu^2} - \frac{1}{2s^2}(1-\mu)^3\frac{dY}{d\mu}. \end{aligned}$$

Hence, Eqs. (2.2) and (2.3) are converted to the differential equation with boundary conditions on interval $[-1, 1]$, i.e.

$$\frac{1}{4s^2}(1-\mu)^3\frac{d^2}{d\mu^2}Y(\mu) - \frac{1}{2s^2}(1-\mu)^2\frac{d}{d\mu}Y(\mu) + \tag{3.7}$$

$$\frac{(\mu+1)\frac{d}{d\mu}Y(\mu)}{\sqrt{1-\alpha Y(\mu)}} = 0,$$

$$Y(-1) = 1, \qquad Y(1) = 0.$$

Now, we apply the pseudo-spectral method for solving the boundary value problem (3.7). This method involves using the Chebyshev-Gauss-Lobatto points

$$\mu_j = \cos\left(\frac{\pi j}{N}\right), \qquad j = 0, 1, \cdots, N.$$

The unknown function $y(\mu)$ in problem (3.7) for any $s > 0$, can be approximated by a truncated series of Chebyshev polynomials,

$$Y_N(\mu) = \sum_{k=0}^{N} \widehat{Y}_k T_k(\mu), \tag{3.8}$$

where $T_k(\mu)$ are orthogonal Chebyshev polynomials of the first kind,

$$T_0(\mu) = 1, \qquad T_1(\mu) = \mu,$$

and in general,

$$T_{k+1}(\mu) = 2\mu T_k(\mu) - T_{k-1}(\mu), \qquad k \geq 1,$$

and $\widehat{Y}_k$ are the Chebyshev coefficients which are determined by

$$\widehat{Y}_k = \frac{2}{N\widehat{c}_k}\sum_{i=0}^{N}\frac{1}{\widehat{c}_i}Y(\mu_i)\cos\left(\frac{\pi i k}{N}\right), \tag{3.9}$$

where $\widehat{c}_0 = \widehat{c}_N = 2$ and $\widehat{c}_i = 1$ for $1 \leq i \leq N-1$. As it is well-known in Chebyshev pseudo-spectral method, derivatives formulae of the function $y(\mu)$ at the collocation points are presented as

$$\frac{dY}{d\mu}(\mu_i) = \sum_{j=0}^{N} D_{ij}Y(\mu_j), \tag{3.10}$$

$$\frac{d^2Y}{d\mu^2}(\mu_i) = \sum_{j=0}^{N} D_{ij}^2 Y(\mu_j), \tag{3.11}$$

where D is the Chebyshev differentiation matrix and $N+1$ is the number of collocation nodes. The entries of the differentiation matrix D are [15]

$$\begin{aligned} D_{00} &= -D_{NN} = \frac{2N^2+1}{6}, \\ D_{ii} &= -\frac{1}{2}\frac{\mu_i}{1-\mu_i^2}, \qquad i \neq 0, N, \\ D_{ij} &= -\frac{1}{2}\frac{\widehat{c}_i}{\widehat{c}_j}\frac{(-1)^{i+j}}{\mu_i - \mu_j}. \end{aligned} \tag{3.12}$$

Using [20], when the number of nodes increases, accordingly the condition number of operational matrix of derivative grows. But, the governing differential equation in 3.7 is defined on the interval $[-1,1]$ and therefore, we do not need to large numbers of nodal points (Based on spectral accuracy property) for obtaining an appropriate approximate solution. By employing derivatives formulations (3.10) and (3.11), Eq. (3.7) is transformed to the following expressions

$$\frac{1}{4s^2} \quad (1-\mu_i)^3 \left(\sum_{j=0}^{N} D_{ij}^2 Y(\mu_j)\right) - \frac{1}{2s^2}(1-\mu_i)^2 \left(\sum_{j=0}^{N} D_{ij} Y(\mu_j)\right) + \frac{(\mu_i+1)\left(\sum_{j=0}^{N} D_{ij} Y(\mu_j)\right)}{\sqrt{1-\alpha\, Y(\mu_i)}} = 0, \quad (3.13)$$

$$i = 1, 2, \cdots, N-1,$$

where

$$Y(\mu_0) = 0, \qquad Y(\mu_N) = 1.$$

Eqs. (3.13) are actually a system of nonlinear equations. The system of nonlinear algebraic equations, has motivated many theoretical developments including the fact that solution formulas do not in general exist and hence, it must be solved numerically by iteration methods [21, 22]. As is well-known, a disadvantage of such methods is that the initial approximation must be chosen sufficiently close to the exact solution in order to guarantee their convergence. After solving system (3.13), we have $Y(\mu_k)$ yielding the Chebyshev coefficients $\widehat{Y}_k$ by (3.9). Now we denote the approximate solution of Eq. (2.2) by

$$y_N(x) = Y_N(\mu) = \sum_{k=0}^{N} \widehat{Y}_k T_{k,s}(x), \qquad x \in [0,+\infty),$$

where $T_{k,s}(x) = T_k\left(\frac{x-s}{x+s}\right)$.

4 Comparison study

The system (3.13), can be solved by using Newton's method [21] with initial value $Y^{(0)} = (0,0,\cdots,0)^T$ and 40 iterations. All the computations associated with the method have been performed by a personal computer having the Intel Pentium 4, 2.2 GHz processor and using Maple 13 with 30 digits precision.

We denote the maximum norm of residual error of approximate solutions $y_N(x)$ for Eq. (2.2), in the following form:

$$MR = \max_{x\in[0,+\infty)} |A(y_N(x))|,$$

where $A(y) = y'' + 2xy'(1-\alpha y)^{-\frac{1}{2}}$.

In, Table 1, some values of N and s, the CPU time and the MR errors are listed. Fig. 1 shows the absolute values of Chebyshev coefficients as computed for several s. Fig. 1 and Table 1 show that, $s = 4$ can be a proper value for scaling parameter s when $30 \le N \le 50$ but is smaller for smaller N: note that the curves for small s are well below those of the diagonal cross for $s = 4$. Also, Fig. 1 indicates that the decay rate of the absolute values of Chebyshev coefficients is geometric. Fig. 2 shows comparison of unsteady gas equation graph obtained by present method when $N = 10$ and $s = 2$, with perturbation method [16], Adomian decomposition method [23] and numerical solutions [28]. Fig. 3 shows the values of $y(x)$ obtained by 10th-order approximation of presented method, for different values of α. See Fig. 4 for residual error of Eq. (2.5) obtained by presented method. Table 2 shows the approximations of $y(x)$ for unsteady gas equation with $\alpha = 0.5$ obtained by the proposed method for $N = 8, 20$ when $s = 2$, the perturbation method(PM) [16], the Lagrangian interpolation of modified generalized Laguerre functions (MGLF) [25], homotopy perturbation method (HPM) [27] and numerical solutions (NS). For comparison purposes, Table 3 shows the values of the initial slope $y'(0)$ by using the padé $[2,2]$ and padé $[3,3]$ by Wazwaz [23], by numerical solutions and by presented method for some values of α. Table 4 shows the values of the MR errors by presented method when $N = 40$ and $s = 4$, 2-term of perturbation method [16] for some values of α. It indicates that as perturbation quantity α increases from 0 to 1, the rate of convergence decreases.

5 Conclusion

One knows that calculation of the solution of the unsteady gas through a porous medium is difficult even by numerical methods. On the other hand, it is quite valuable in order to its importance in investigating gas-solid processes. So, finding accurate solution of this equation has become an important task in physics and engineering. In this paper we constructed an approximation to the solution of nonlinear unsteady gas equation by a Chebyshev pseudo-spectral. With this method we can approximate the unbounded differential equations like unsteady gas, Blasius and Falkner-Skan. The numerical results show the reliability and efficiency of the presented method compared with some well-known methods which have applied.

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