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Research Article

Undesirable Factors and Improvement of Efficiency in Data Envelopment Analysis

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Abstract

In the present paper, determining the output levels of decision-making units (DMUs) with the preference of cone constraints, when some of the outputs are undesirable, was discussed. The output levels of a DMU are estimated when some or all of related input components are increased and the current efficiency level is improved. To estimate the output levels, the inverse data envelopment analysis (DEA) and multi objective linear programming (MOLP) models were used. The efficacy of the proposed method is indicated by using an application in bank.

Keywords : DEA; MOLP; Undesirable Output; Cone Constraints.

1 Introduction

Data envelopment analysis (DEA) is a non-parametric method for computing and assessing the relative efficiency of homogeneous decision making units ($DMUs$) with multiple inputs and outputs such as hospitals, banks, business firms, government agencies, and etc. [1, 2]. In some assessments using DEA, there may be undesirable factors among the outputs, such as: environmental assessments, modeling bank performance, combined cycle power plant performance assessment, etc. In order to improve efficiency in above mentioned cases, the good output levels and undesirable output levels should be increased and decreased respectively [3, 4, 5, 11, 14, 15]. Also, some questions may be raised concerning the assessment of $DMUs$, such as: if among a group of $DMUs$, certain inputs are increased and the efficiency level of DMU remains unchanged, how much more outputs could the unit produce? [6]. Wei et al. [6], Yan et al.,[7] and Jahanshahloo et al., [9], using inverse DEA models proposed some solutions to this question. Jahanshahloo et al, [8], proposed a multiple objective linear programming ($MOLP$) to answer the above question, when some of the inputs and outputs are undesirable. Generally, the supposed planning of organizations and companies is to improve the efficiency level so that the question is; if among a group of decision making units, certain inputs are increased, and the efficiency level of DMU is improved, how much more outputs could the unit produce? Jahanshahloo et al.[9], proposed a multi-objective programming ($MOLP$) model to address the issue. In the present paper, using

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$MOLP$ and inverse DEA models with the preference of cone constraints, this question is answered in the presence of undesirable outputs. The rest of the paper is organized as follows: Section 2 explains the output-oriented BCC model with the preference of the cone constraints; Section 3, proposes a method for determining the level of undesirable and desirable outputs of the DMU, when input components are increased, and the efficiency level of DMU is improved; Section 4, provides an application example in a bank; and the last section, elaborates on conclusion.

2 Preliminaries

Suppose that there are n $DMUs$; each one transforms m different inputs into different s outputs. The vector of input and output for DMU_j,$j = 1, 2 \ldots, n$ is denoted by (x_i, y_j). Also, suppose that K^* and $V^*, U^{g*}, \overline{U}^{b*}$ are respectively, the negative polar cons of the $V, U^g, \overline{V}^b$ and K sets where $V \subseteq R^m_+$, $U^g, \overline{V}^b \subseteq R^s_+$, and $KsubseteqR^n_+$ are the relative preference cones of the desirable and undesirable inputs and outputs of $DMUs$. We can also suppose that each $j(j = 1, 2 \ldots, n)$, $X_j \in -IntV^*$ and $Y_j \in -Int\overline{U}^{b*}$ and $Y^g_j \in -Int\overline{U}^{g*}$, respectively, denote the interior set of V^*, U^{b*} and $\overline{U}^{g*}$.

The output-oriented BCC model with the preference of the cone constraints for evaluating DMU_p is as follows [5, 6]:

$$
\begin{aligned}
\varphi^*_p = \max \varphi \\
\text{s.t.} \qquad & \qquad (2.1) \\
X\lambda - X_p \in V^* \\
-Y^g\lambda + \varphi Y^g_P \in U^{g*} \\
-\overline{Y}^b\lambda + \varphi \overline{Y}^b_P \in \overline{U}^{b*} \\
1\lambda = 1 \\
\lambda \in -K^*.
\end{aligned}
$$

Where, $\overline{Y}^b_j = Y^b_j + \eta$, $\eta - Y^b_j \in -Int\overline{U}^{b*}$ and η is selected, then for each $j = 1, 2, \ldots, n$, we have $\overline{Y}^b_j - Int\overline{U}^{b*}$. In model (2.1), $\varphi^*_p \geq 1$ and the feasible region is non-empty[4, 7, 8].

3 Proposed Method

In this section, the following question is to be answered: Suppose $\alpha_P \in R^m_+$ and the input level of the DMU_p is increased from X_P to $\alpha_P = X_P + \Delta X_P$ along the convex polar cone $-V^*$.

Also, suppose that the current efficiency level of the DMU_p under evaluation is increased, say ρ percent of φ^*_p as such how much desirable and undesirable outputs DMU_p would produce? In order to answer to the above mentioned question, suppose that γ^g_P and γ^{-b}_P are the vectors that we expect respectively added to Y^g_P and $\overline{Y}^{-b}_P$ along the convex polar cone $-U^{g*}$ and $-\overline{U}^b$, also suppose φ^*_p increases to $(1 - \rho/100)\,\varphi^*_p$. The proposed $MOLP$ model with the preference of the cone constraints to determine the level of desired and undesired outputs is as follows:

$$
\begin{aligned}
& \max \left(\gamma^g_P, \gamma^{-b}_P\right) \\
& \text{s.t.} \qquad\qquad (3.2) \\
& X\lambda - \alpha_P \in V^* \\
& -Y^g_P\lambda + (1 - \rho/100)\,\varphi^*_p \left(Y^g_P + \gamma^g_P\right) \in U^{g*} \\
& -\overline{Y}^g_P\lambda + (1 - \rho/100)\,\varphi^*_p \left(\overline{Y}^g_P + \overline{\gamma}^g_P\right) \in \overline{U}^{g*} \\
& \gamma^g_P \in -U^{g*} \\
& \overline{\gamma}^g_P \in -\overline{U}^{g*} \\
& 1\lambda = 1 \\
& \lambda \in -K^*.
\end{aligned}
$$

φ^*_p in model (3.2), which is obtained from model (2.1), is the efficiency of DMU_p. To solve $MOLP$ model (3.2), there are different methods [10, 12, 13]. One of them is weighted sum method. In this method, for solving model (3.2) it can be considered the weight for each of the desired and undesirable outputs. Given that the model (2.1) has been used with the preference of the cone constraints, it is better to choose the weights from the cone of the relative preferences of the outputs. Suppose W^g_P and W^{-b}_P be the weighting vectors of desirable and undesirable outputs γ^g_P and γ^{-b}_P respectively. By this method, without changing the constraints, the objective function changes from $\max\left(\gamma^g_P, \gamma^{-b}_P\right)$ to single objective function $\max W^g_P\gamma^g_P + \overline{W}^b_P\overline{\gamma}^b_P$ and the optimal solution is obtained easily.

Table 1: The data of bank branches

Bank Branches	input		output		
	x_1	x_2	y_1	y_2	y_3
DMU_1	5	4	8	14	17
DMU_2	10	10	1	13	19
DMU_3	20	15	5	10	18
DMU_4	18	23	7	8	15
DMU_5	6	16	4	15	16
DMU_6	9	19	2	11	11
DMU_7	10	17	6	9	18

4 A Bank Application

This section applies our proposed method to an application in bank. Consider seven branches of a private bank in Iran, consisting two inputs and three outputs. Table 1, displays related data for this example. In the first column of Table 1, the branches of bank are named DMU_1 to DMU_7 and the definition of input and output variables for them are as follows:

$$x_1 : Personnelcostsandadministrativecosts,$$
$$x_2 : Deposit,$$
$$y_1 : Nonperformingloans(badoutput),$$
$$y_2 : Performingloans(goodoutputs),$$
$$y_3 : Profit(goodoutputs).$$

Among the outputs, the first output is undesirable and the DMU_s should decrease it as much as possible. We consider DMU_4 under evaluation. Suppose that the convert vector for the undesirable output is $\eta = 20$, and the relative preference cone of the inputs and desirable output for this DMU is $V = \{(2,2)^t v_1, v_1 \geq 0\}$ and $U^g = \{(3,1)^t u_1, u_1 \geq 0\}$ respectively. Furthermore, assume that the relative preference cone of the undesirable inputs for DMU_4 is $\overline{U}^b = R^1_+$. Therefore, the negative polar cone of the inputs and desirable outputs for DMU_4 respectively is as; $V^* = \left\{(v_1,v_2)^t : (2,2)\binom{v_1}{v_2} \leq 0\right\}$ and $U^{g*} = \left\{(u_1,u_2)^t : (3,1)\binom{u_1}{u_2} \leq 0\right\}$ where the negative polar cone of the undesirable outputs is $\overline{U}^{b*} = -R^1_+$ The Output-oriented BCC Model with the preference of the cone constraints is as follows:

$$
\begin{aligned}
&\varphi^*_D = \max \varphi \\
&\text{s.t.} \\
&\begin{pmatrix} 5 & 10 & 20 & 18 & 6 & 9 & 1 \\ 4 & 10 & 15 & 23 & 16 & 19 & 17 \end{pmatrix} \begin{pmatrix} \lambda_1 \\ \vdots \\ \lambda_7 \end{pmatrix} - \begin{pmatrix} 18 \\ 23 \end{pmatrix} \in V^* \\
&\begin{pmatrix} 12 & 19 & 15 & 13 & 16 & 18 & 14 \end{pmatrix} \begin{pmatrix} \lambda_1 \\ \vdots \\ \lambda_7 \end{pmatrix} + 13 \\
&\varphi \in U^{b*} \\
&- \begin{pmatrix} 14 & 13 & 10 & 8 & 15 & 11 & 9 \\ 17 & 19 & 18 & 15 & 16 & 11 & 18 \end{pmatrix} \begin{pmatrix} \lambda_1 \\ \vdots \\ \lambda_7 \end{pmatrix} + \begin{pmatrix} 8 \\ 15 \end{pmatrix} \\
&\varphi \in U^{g*} \\
&\lambda_1 + \lambda_2 + \cdots + \lambda_7 = 1 \\
&\lambda_1, \lambda_2, \ldots, \lambda_7 \geq 0.
\end{aligned}
\tag{4.3}
$$

The optimal solution of above model is as follows:

$$(\varphi^*_D, \lambda^*_1, \lambda^*_2, \lambda^*_3, \lambda^*_4, \lambda^*_5, \lambda^*_6, \lambda^*_7)^t$$

$$= (1.4615, , 1, 0, 0, 0, 0, 0)^t \,.$$

Suppose that the input vector of DMU_4 increased from $(18, 23)^t$ to $(20, 26)^t$ and the DMU_4 intends to improve its efficiency by $\rho = 0.2$. As such, according to model (4.3), we will have the following problem:

$$\begin{aligned}
&\max\left(\gamma_1,\gamma_2,\gamma_3\right)\\
&\text{s.t.} \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (4.4)\\
&\begin{pmatrix}5 & 10 & 20 & 18 & 6 & 9 & 1\\ 4 & 10 & 15 & 23 & 16 & 19 & 17\end{pmatrix}\begin{pmatrix}\lambda_1\\ \vdots\\ \lambda_7\end{pmatrix}\\
&-\begin{pmatrix}20\\ 26\end{pmatrix}\in V^*\\
&\begin{pmatrix}12 & 19 & 15 & 13 & 16 & 18 & 14\end{pmatrix}\begin{pmatrix}\lambda_1\\ \vdots\\ \lambda_7\end{pmatrix}\\
&+1.1692\left(13+\gamma_1\right)\in U^{b*}\\
&-\begin{pmatrix}14 & 13 & 10 & 8 & 15 & 11 & 9\\ 17 & 19 & 18 & 15 & 16 & 11 & 18\end{pmatrix}\begin{pmatrix}\lambda_1\\ \vdots\\ \lambda_7\end{pmatrix}\\
&+1.1692\left(\begin{pmatrix}8\\ 15\end{pmatrix}+\begin{pmatrix}\gamma_2\\ \gamma_3\end{pmatrix}\right)\varphi\in U^{g*}\\
&\gamma_1\in -U^{-b*}\\
&\begin{pmatrix}\gamma_2\\ \gamma_3\end{pmatrix}\in -U^{g*}\\
&\lambda_1+\lambda_2+\cdots+\lambda_7=1\\
&\lambda_1,\lambda_2,\ldots,\lambda_7\geq 0.
\end{aligned}$$

The optimal solution of above model is as follows:

$$\left(\varphi_D^*,\lambda_1^*,\lambda_2^*,\lambda_3^*,\lambda_4^*,\lambda_5^*,\lambda_6^*,\lambda_7^*,\gamma_1^*,\gamma_2^*,\gamma_3^*\right)^t$$
$$=\left(0,1,0,0,0,0,0,3.2504,3.5355,0\right)^t.$$

The optimal solution of model (4.4) reveals that: by increasing the input levels of DMU4 from $(18,23)^t$ to $(20,26)^t$ and improving its efficiency level by $\rho = 0.2$, the DMU_4 should decrease its undesirable outputs from 7 to 3.7496 and increase its undesirable outputs from $(8,15)^t$ to $(11.5355,15)^t$.

In other words, the fourth branch needs to increase its personnel and administrative costs from 18 to 20 while considering the increase of its deposit amount from 23 to 26. In this way by gaining 0.2 improvement in efficiency the level of deferred claims would decrease significantly (almost $\frac{1}{2}$ initial amount). Also, the branch can avail the bank customers with more loans in comparison to the previous times (almost 0.442 times the initial amount). Based on the method proposed in the present paper, it is suggested that the other branches of the same bank must increase their personnel and administrative, and also deposit attraction costs to function as well as the fourth branch.

5 Conclusion

The current paper aimed to address the issue of; if among a group of decision making units, certain inputs are increased and the efficiency level of DMU is improved, how much more outputs could the unit produce? by resorting to MOLP, inverse DEA and the preference of the cone constraints methods, in the presence of desirable and undesirable outputs, a solution was proposed in the same grounds. Finally, a real bank application was analyzed operationally to illustrate the applicability of the proposed method.

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