Probabilistic prediction of the next large earthquake in the Zagros Folded - Thrust Belt

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Abstract

Probabilistic methods are helpful for characterizing earthquake prediction. The seismic process can be modeled as a renewal process using a list of strong earthquakes (M $\geq$6.5) from 1900 until now which occurred in the Zagros fold -thrust belt. Two renewal models have been used. The model parameters have been specified by the method of moments and the method of maximum likelihood .We conclude that the gamma model gives the better result than the lognormal model. The probability of the occurrence of the next large earthquake during a specified interval of time can be calculated for each model.

Also by maximizing the conditional probability for each model, we estimated approximately the recurrence time of the next strong earthquake in this region. The next earthquake with M$\geq$6.5 may occur before 2012.577±5.333 (yrs) and 2012.o46±5.25 (yrs) by using the Gamma model and Lognormal Model, respectively.

Keywords: probabilistic prediction, renewal model, lognormal distribution, gamma distribution, recurrence time, zagros belt.

Introduction

Earthquake occurrence probabilities can be predicted by using different probability distributions. According to Wallace and et al. (1984), the long-term prediction predicts occurrence of an earthquake in some years or decays. A method of long–term prediction has been studied broadly in connection with earthquakes. Nishenko and Buland (1987) found that the lognormal distribution provides a better fit than Gaussian and weibull distributions and it has an appealing physical interpolation.

They found that a single worldwide value of σ=0.21 was consistent with data for the separate fault segments. Davis et al. (1989) adopted a lognormal distribution for earthquake interval times and used a locally determined rather than a generic coefficient of variation to estimate the probability of occurrence of characteristic earthquake. Mcnally and Minster (1981), have discussed that a weibull distribution is more suitable .Yilmiz and Celik (2008), attempted to find a probability distribution which is the best representation of the set of earthquake data from Turkey. They determined that the most representative probabilitymodel is the weibull distribution. The working group in California earthquake probabilities (1999) reported conditional probabilities for the San Andreas fault for the time 2000-2030.

Severalstochastic earthquake generating models have been used for seismic hazard assessment .The Poisson model assumes that the earthquake occurrences have no memory and occur independent of each other. Pasha et al. (2008) used a semi markov model in the Zagros fold thrust belt. This model assumes that the next

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earthquake occurrence is dependent both on the previous occurrence and the elapsed time interval between. Savy et al. (1980) and Grandori et al. (1984), used a renewal models which are related to a real time models, imply a time dependent accumulation of energy between major earthquakes.

RecurrenceModels of Earthquakes

As Utsu (1984) and Ferraes (2005), we have been accepted that in some seismic regions, large earthquakes occur at fairly regular intervals as graphically shown in figure1. Such a series of earthquakes is often demonstrated by a renewal process.

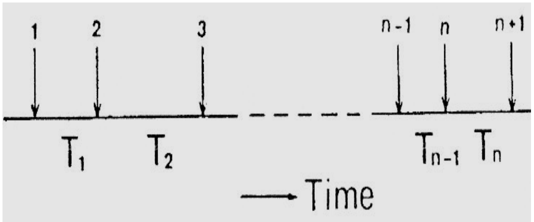


Figure 1. Occurrence times of earthquakes (arrows) and the time interval between successive earthquakes T_i ; (i=1, 2… n) Utsu T., 1984

In this model the occurrence probabilities of the next event only depends on the time since the last event, parameters of the renewal process and the time interval of the interest .Each renewal model is specified by a probability density function $\omega(t)$ for the time interval t between successive events which defines the probability of an earthquake occurrence from the time measured from the date of the last earthquake T to T+ΔT (figure 2).

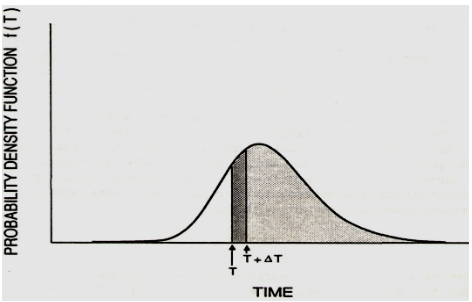


Figure 2 .The probability of an earthquake in the time interval (T,Tt+ΔT) is the shaded area.

Here we assumed the renewal model and define it by the following notations:

T: Variable representing the time interval between successive events.

T_i : Observed time interval between the ith, the (i+1) the earthquakes.

$\mu(t)dt$: Probability that the next earthquake will occur during the time interval between t and t+dt.

$\phi(t)$: Probability that the next earthquake will occur at a time later than t.

$p(\tau|t)$: Conditional probability that the next earthquake will occur during the time interval between t and $t+\tau$.

$\omega(T)$: Density for the distribution of T.

R_t : Time of the last rupture of the fault or fault segment.

τ : Next expected prediction recurrence time.

These relations have been used in seismology (Utsu T., 1984)

$$\omega(T) = \mu(t)\,\phi(t) \quad (1)$$

$$\phi(t) = \exp\left\{-\int_0^t \mu(t)d_t\right\} = \int_t^\infty \omega(T)d_t \quad (2)$$

$$p(\tau|t) = 1\text{-}\exp\left\{-\int_t^{t+\tau} \mu(t)d_t\right\} = 1\text{-}\frac{\phi(t+\tau)}{\phi(t)} \quad (3)$$

Utsu (1984) examined the recurrence of large earthquakes in Japan by using four distributions and concluded that all four models seem to be acceptable. Here we compare the Lognormal and the Gamma models; the corresponding functions $\omega(T), \phi(t), \mu(t)$ and $p(\tau|t)$ are expressed by the following equations:

(1) Gamma model:

$$\omega(T) = \frac{c}{\Gamma(t)}\,(CT)^{r-1}\,e^{-cT} \quad c>0,\ r>0 \quad (4)$$

$$\phi(t) = \Gamma(r,cT)\big/\Gamma(r) \quad (5)$$

$$\mu(t) = c^r\,t^{r-1}\,e^{-ct} \big/ \Gamma(r,ct) \quad (6)$$

$$p(\tau|t) = 1\text{-}\frac{\Gamma(r,c(t+\tau))}{\Gamma(r,ct)} \quad (7)$$

Where $\Gamma(k,x)$ represents the incomplete gamma function of the second kind.

(2)Lognormal model:

$$\omega(T)=\frac{1}{\sqrt{2\pi}\sigma T}\exp\left\{-\frac{(\ln T-m)^2}{2\sigma^2}\right\} \quad m>0, \sigma>0 \tag{8}$$

$$\phi(t)=1-\Phi\left(\frac{\ln t-m}{\sigma}\right) \tag{9}$$

$$\mu(t)=\frac{1}{\sqrt{2\pi}\sigma t}\exp\left\{-\frac{(\ln t-m)^2}{2\sigma^2}\right\}\Big/\left\{1-\Phi\left(\frac{\ln t-m}{\sigma}\right)\right\} \tag{10}$$

$$p(\tau|t)=1-\left\{1-\Phi\left(\frac{\ln(t+\tau)-m}{\sigma}\right)\right\}\Big/\left\{1-\Phi\left(\frac{\ln t-m}{\sigma}\right)\right\} \tag{11}$$

Where $\Phi(x)$ represents the error integral (Utsu, 1984). Sornette and Knopoff (1997),demonstrated that For the lognormal model the longer it has been since the last earthquake , the shorter the time until the next event; but for large elapsed times since the last earthquake, the longer the time until the next one (figure 3).

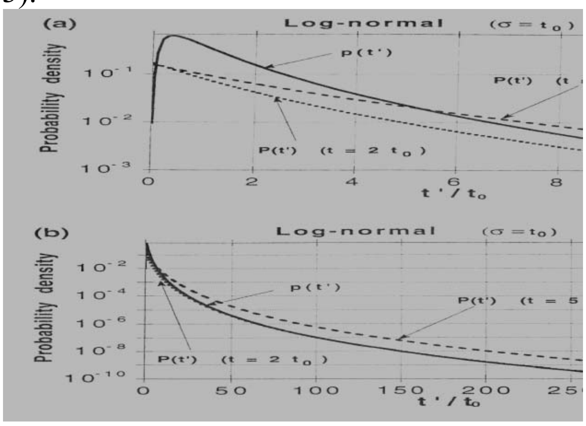


Figure 3. Lognormal distribution with $\sigma = t_0$: $t=2t_0$ and $t=5t_0$. (a) Short times and (b) long times [9].

From the occurrence times of n+1 earthquakes in the past, we obtain the time interval length T_i (i=1, 2… n), their mean and the variance. The methods of moments and maximum likelihood are used to estimate the values for parameters of these models. The formulas for these methods and $L=\prod_{i=1}^{n}\omega(T_i)$ are as follows:

(1) Gamma model:

Moments:

$$E[T_i] = r/c \quad , \quad V[T_i] = r/c^2 \tag{12}$$

Maximum Likelihood:

$$\text{Ln } L = n\{r \ln c - \ln \Gamma(r) + (r-1) E[\ln T_i] - cE[T_i]\} \tag{13}$$

$$E[T_i] = r/c \tag{14}$$

$$E[\ln T_i] = \frac{\Gamma'(r)}{\Gamma(r)} - \ln c \tag{15}$$

(2) Lognormal model:

Moments:

$$E[T_i] = e^{m+\sigma^2/2},\ V[T_i] = e^{2m+\sigma^2}(e^{\sigma^2}-1) \tag{16}$$

Maximum likelihood:

$$\text{Ln } L = -n/2\ \{\ln 2\pi\sigma^2 + 2\ E[\ln T_i] - \frac{V[\ln T_i]}{\sigma^2}\} \tag{17}$$

$$E[\ln T_i] = m \tag{18}$$

$$V[\ln T_i] = \sigma^2 \tag{19}$$

Conditional probabilities for recurrence times of large earthquakes are a practical and powerful form for estimating the probability of future large earthquakes. If we define F (t) =1-$\phi(t)$ as Ferraes (2005), we obtain:

$$F(\tau|T \ge t) = \frac{F(\tau)-F(t)}{1-F(t)} \quad , \quad \tau > t \tag{20}$$

By differentiating with respect to τ :

$$f(\tau|T \ge t) = \frac{f(\tau)}{1-F(\tau)} \quad , \quad \tau > t \tag{21}$$

A reasonable prediction criterion for the occurrence interval τ between the last and the next earthquake is the one which maximizes the conditional probability density $f(\tau|T \ge t)$ Ferraes (2005)

$$\frac{\partial}{\partial T} f(\tau|T \ge t) = 0 \tag{22}$$

Therefore estimator $\hat{T}$ is the solution of equation (22).

If we consider the MME parameters for our models and find the solution of equation (22), then we can conclude that:

(1) Gamma model:

$$\hat{T}_g = \frac{r-1}{c} \qquad r \geq 1 \tag{23}$$

(2) Lognormal model:

$$\hat{T}_L = e^{m-\sigma^2} \tag{24}$$

Any predictor $\hat{T}$ has a square error [2]:

$$\varepsilon^2 = E[(T-\hat{T})^2] = Var[T] + (E[T_i] - \hat{T})^2 \tag{25}$$

Example from Earthquake Data in Zagros Fold-Thrust Belt

The Zagros fold-thrust belt with NW-SE trend is the most seismic zone in Iran (figure 4).It extends for about 1500 kilometers from southeastern Turkey through northern Syria and southern Iran.

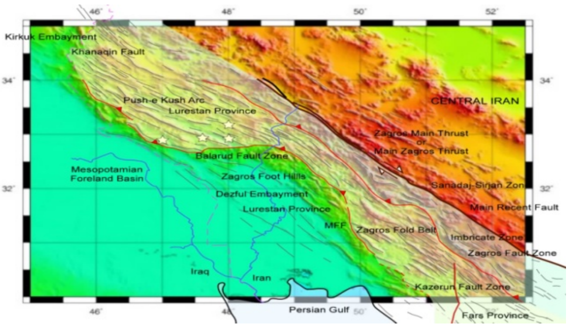


Figure 4 .The Zagros fold-thrust belt.

Talebian and Jackson (2004), proposed a segmentation of the Zagros fold-thrust belt to develop specific responses to plate convergence (figure 5).

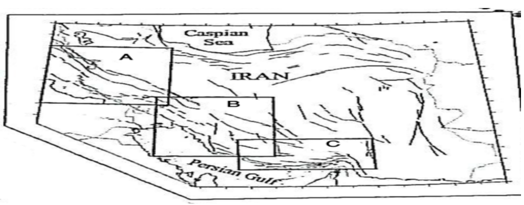


Figure 5 .The area of investigation [10].

The area under investigation experienced many great earthquakes. We consider all earthquakes with ($M \geq 6.5$) Richter from 1900 that occurred in this region(www.iiee.ac.ir:/) (Table 1). Following Olsson (1982), Utsu (1984) and Ferraes (2005), in the case where more than one large earthquake has taken place within a short time interval, we may consider all but the largest as aftershocks and eliminate them from table. Therefore three earthquakes in 1957, 1958, and 1961 are regarded as one event and the two earthquakes in 1999 and 2002 are counted as one event.

Occurrence Date (yrs.)	Magnitude
1909.15	7.4
1929.62	6.5
1949.40	6.5
1958.04	6.5
1977.30	6.9
1990.47	7.7
1999.26	6.7

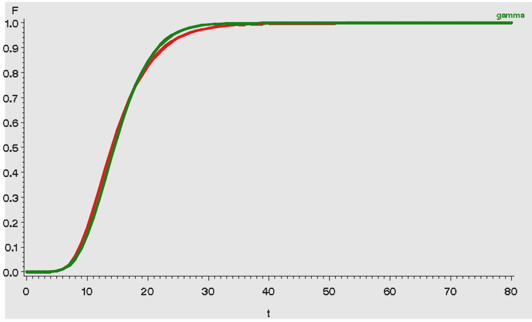
Table 1. Strong earthquakes in Zagros fold-thrust belt ($M \geq 6.5$).

In table 2, the estimated parameter values and the corresponding Ln L for the two models were listed.

Model	MOM	MLE
Gamma	c=0.588 r= 8.829 Ln L=-18.002	c=0.611 r=9.182 Ln L =-17.893
Lognormal	m=2.656 σ =0.328 Ln L=-18.502	m=2.645 σ =0.369 Ln L=-18.408
Mean interval **S. D.**	15.018 5.054	

Table 2 .Model parameters for data in table 1.

There are no considerable differences between the values estimated by the two methods. The cumulative distribution of the time intervals for our data is shown in figure 6 .Curves of F(t) for the two models which are computed on the basis of the parameters determined by the MLE are drawn in the same figure .It is difficult to decide which model fits best .The value of Ln L is an indication of how well the model fits . The Gamma model gives the largest (best) Ln L for our data. The conditional probabilities $p(\tau|t)$ are calculated for various combinations of t and τ. Table 3 and Table 4 lists parts of the results for our data based on the Gamma and the lognormal models, respectively.



- Lognormal
- Gamma

Figure 6. Cumulative distribution of T_i and the curves of 1-$\phi(t)$ for great earthquakes in Zagros belt by two models.

t / τ	0	5	10	15	20
2	0.00000	0.02302	0.16976	0.32000	0.41717
4	0.00076	0.09106	0.36590	0.56784	0.67653
6	0.01084	0.21377	0.55260	0.74123	0.82804
8	0.05345	0.37402	0.70620	0.85295	0.91198
10	0.14904	0.54085	0.81893	0.92018	0.95644
12	0.29351	0.68778	0.89447	0.95839	0.97908
14	0.46041	0.80158	0.94143	0.97907	0.99022
16	0.61929	0.88119	0.96886	0.98980	0.99554
18	0.74998	0.93248	0.98406	0.99517	0.99801
20	0.84592	0.96335	0.99211	0.99777	0.99913
22	0.91020	0.98089	0.99621	0.99810	0.99962
24	0.95016	0.99039	0.99822	0.99955	0.99984
26	0.97350	0.99532	0.99919	0.99981	0.99993
28	0.98644	0.99778	0.99964	0.99992	0.99997
30	0.99329	0.99897	0.99984	0.99996	1.00000

Table 3. Probabilities $p(\tau|t)$ based on Gamma model.

t / τ	0	5	10	15	20
2	0.00000	0.02675	0.18884	0.29394	0.33633
4	0.00033	0.11049	0.38474	0.51701	0.56486
6	0.01045	0.24991	0.55670	0.67681	0.71684
8	0.06289	0.41279	0.69234	0.78693	0.81652
10	0.17701	0.56673	0.79206	0.86088	0.88132
12	0.33242	0.69410	0.86199	0.90971	0.92322
14	0.49365	0.79074	0.90951	0.94155	0.95027
16	0.63517	0.85997	0.94112	0.96221	0.96769
18	0.74680	0.90769	0.96185	0.97554	0.97896
20	0.82887	0.93973	0.97532	0.98413	0.98621
22	0.88642	0.96088	0.98403	0.98968	0.99094
24	0.92553	0.97468	0.98966	0.99327	0.99404
26	0.95154	0.98363	0.99328	0.99558	0.99603
28	0.96860	0.98940	0.99563	0.99708	0.99737
30	0.97969	0.99312	0.99713	0.99808	0.99825

Table 4. Probabilities $p(\tau|t)$ based on lognormal model.

Ten years have passed since the last destructive earthquake in the Zagros fold-thrust belt. The column of t=10 years points out the present situation.

By maximizing the conditional probability method, we can estimate or predict the recurrence time $\hat{T}_g$ and $\hat{T}_L$ as follow:

$\hat{T}_g$ =13.317 (yrs.)

$\hat{T}_L$ = 12.786 (yrs.)

We add the predicted recurrence times $\hat{T}_g$ and $\hat{T}_L$ to the occurrence time of the last observed earthquake R_t =1999.26 to estimate the occurrence time of the next expected strong earthquake. Therefore, we conclude that the next earthquake with ($M \geq 6.5$) in the Zagros belt may occur approximately before the year 2012.577 by the Gamma model and before the year 2012.046 by the Lognormal model. Using equation (25), we estimate the error for the predicted Gamma and Lognormal recurrence time as follow:

$\varepsilon_g = \pm 5.333$ (yrs.)

$\varepsilon_L = \pm 5.525$ (yrs.)

Using this value error, the Gamma and Lognormal occurrence time of the expected large earthquake ($M \geq 6.5$) in the Zagros fold-thrust belt can be written as follow, respectively:

t = 2012.577 ± 5.333 (yrs.)

t = 2012.046 ± 5.525 (yrs.)

Conclusion

This paper describes the estimation of parameters for recurrence model of earthquakes on large earthquakes in Zagros fold-thrust belt. We have compared two renewal models and found that the Gamma model is better than Lognormal model for our data. However, the difference between the two models is small and it is hard to say which the best model is.

We predicted the occurrence time of the next strong earthquake ($M \geq 6.5$) in this zone by using the criterion that the conditional probability density of earthquake is a maximum when the event occur. We concluded that for the Gamma model, a damaging earthquake ($M \geq 6.5$) may occur approximately before

2012.577 ± 5.333 (yrs.).For the Lognormal model, a strong earthquake ($M \geq 6.5$) may occur nearly before 2012.046 ± 5.525 (yrs.).

References

– Davis, P. M., Jackson D. and Kagan Y. Y., 1989, The longer it has been since the past earthquake, the longer the expected time till the next?, Bull. Seism. Soc. Am., Vol. 79, No. 5, pp 1439-1456.

– Ferraes S. G., 2005, A probabilistic prediction of the next strong earthquake in the Acapulco-SanMarcos segment, Mexico. Geofisca Internacional, Vol. 44, Num. 4, pp 347-353.

– Grander G., e. Guargenti and V. petrimi, 1984, on the use of renewal processes in the seismic hazard analysis. In: proceeding of the eighth world conference on earthquakes engineering (prentice – Hall .Englewood cliffs, N. J., pp 287-291.

– Mcnally K. B. and J. B. Minster, 1981, Non uniform seismic slip rates along the Middle American Trench. J. Geophs. Res., 86, pp 4949-4959.

– Nishenko S. P. and R. Buland, 1987, A generic recurrence interval distribution for earthquake for casting, Bull. Seismol. Soc. Am., 77, pp 1382-1399.

– Olsson, R., 1982, the randomness of major global earthquake.Tectonophysics, 82, T11-T15.

– Pasha E., Mostafaei, H., Kordnourie, Sh., 2008, Prediction of earthquake occurrences in the most seismic zone in the middle east by a semi markov model, Journal of the earth, Vol. 3, No. 2, summer, pp 1-10.

– Savy, J. B., Shah H. C. and Beore D., 1980, Nonstationary risk model with Geophysical input, J. struct. Div. Asee, 106, pp 145-164.

– Sornette, D. and Knopoff, L., 1997, Bulletin of the Seismological Society of America, Vol. 87, No. 4, August, pp 789-798.

– Talebian M. and Jackson J., 2004, A reappraisal of earthquake focal mechanisms and active shortening in the Zagros mountains of Iran, Geophysics J. int., 156, pp 506-526.

– Utsu, T., 1984, Estimation of parameters for recurrence models of earthquakes, Bulletin of the earthquake research institute University of Tokyo, Vol. 59, pp 53-66.

– -Wallace, R. E., Davis J. F., Mcnally K., 1984, Terms for expressing earthquake potential, prediction and probability, Bulletin of the seismological society of American, Vol. 74, No. 5, pp 1819-1825.

– Working group on California earthquake probabilities, 1999, Earthquake probabilities in the San Francisco Bay region: 2000 to 2030-Asummary of findings. Open- file report 99-517.

– Yilmaz V., Celik H. E., 2008, A statistical approach for estimating the probability of occurrence of earthquake on the Northern Anatolian fault zone, International journal of natural and engineering sciences 2(2), pp 81-86.

– www.iiee.ac.ir:/Earthquake Catalog search.