

## Displacement Control Based Analytical Description of Pinching, Sliding and Degrading Hysteretic System

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### 1-Introduction

Non-linear hysteresis behavior is one of the most important inherited properties of any structural system. The shape of structural hysteretic behavior is a result of either changing material properties beyond their elastic range, or changes in structural geometry (e.g. buckling, cracks) due to subjected loads. The hysteretic response of a structure depends not only on the immediate deformation of elements, but also on the history of deformations, since this represents the energy dissipated by the structure. Increasing the displacement under strong dynamic forces, such as earthquakes, causes the hysteresis cycles to move from an elastic phase into a plastic phase. Hence, by ignoring the non-linearity in hysteresis behavior and neglecting the degradation effects, a great deal of lost energy (by dissipated energy mechanisms) is also ignored. Consequently, the designs would be uneconomical. In contrast, improper inclusion of non-linear hysteretic performance, such as neglecting sliding and strength deterioration causes non-conservative structural design.

Mostaghel [8] proposed an analytical description of the structural hysteretic model, based on a single-degree-of-freedom (SDOF) mechanical system, which consists of a combination of mass and springs. That model takes into account different structural characteristics, including pinching, stiffness degradation and load deterioration. It should be noted that Mostaghel did not address the sliding issue in his model, although it is generally considered important as it affects the rate of energy dissipation and performance of structures when seismic loads are applied. Later on, Zeynalian [13] developed the bilinear Mostaghel's model in order to consider the sliding effect. They also concluded that Mostaghel [8] equations do not satisfy some of the boundary conditions pertaining to the first cycle. Therefore, they applied some necessary corrections on the formulations proposed by Mostaghel.

It is also necessary to mention that Mostaghel[8] model is based on a force-control loading regime, which means that the response of the structural system is examined based on an applied force on the system.

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Therefore, since most experiments on real structures are performed based on a displacement-control loading regime, the model should be developed in order to be better fitted to the experimental results. Hence, in this study, the modified Mostaghel model is improved in order to generate MDOF multi-linear analytical results which can express the structural system's responses in terms of displacement-control. The cyclic loading regime which is implemented in this study is based on Method B of ASTM Standard.

### 2- Review of the Model

Mostaghel used an analytical model for formulating the hysteretic systems based on an SDOF mechanical system, which is comprised of a mass,  $m$ , two springs with stiffnesses of  $\alpha k$  and  $(1-\alpha)k$ , and deformations of  $x$  and  $u$  respectively, and a viscous damper with a damping coefficient of  $c$ . While the spring that is directly connected to the mass has the stiffness of  $\alpha k$ , the stiffness of the other spring (which is connected to the mass through a slider, with a friction coefficient of  $\mu$ ) is  $(1-\alpha)k$ . The total stiffness of the system is  $k$  and  $0 < \alpha < 1$ .  $P(t)$  is also the external applied load to the system. Pinching, such as that occurring because of strain hardening, can also be added to the system by assuming two extra springs with a stiffness of  $k_s$  and an initial gap,  $\delta_s$ , as shown in Fig.1.

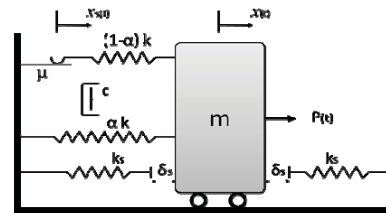


Fig. 1. Bilinear hysteretic system encountering additional stiffness

The modified non-dimensionalized dynamic equation, and the equilibrium equations for the system, which include pinching via both the inequality of strengths and stiffness hardening, can then be presented through Eqs. (1) and (2).

$$f = \alpha y + \alpha_s (|y| - \gamma_s) \text{sgn}(y) \bar{N}(|y| - \gamma_s) + (1 - \alpha)z \quad (1)$$

$$\dot{z} = \dot{y} \{ \bar{N}(\dot{y}) [ \bar{M}(z - \lambda_p) \bar{M}(y) + \bar{M}(z - 1) \bar{N}(y) ] + M(\dot{y}) [ N(z + \lambda_p) N(y) + N(z + 1) M(y) ] \} \quad (2)$$

where  $N(x)$ ,  $M(x)$ ,  $\bar{N}(x)$ , and  $\bar{M}(x)$  are derived from Signum function (Sgn) as illustrated below:

$$\text{Sgn} = \begin{cases} -1 & \text{if } x < 0 \\ 0 & \text{if } x = 0 \\ 1 & \text{if } x > 0 \end{cases} \quad (3)$$

$$N(x) = 0.5[1 + \text{Sgn}(x)] \{ 1 + 1 - [\text{Sgn}(x)] \} \quad (4)$$

$$M(x) = 0.5[1 - \text{Sgn}(x)] \{ 1 - [1 + \text{Sgn}(x)] \} \quad (5)$$

$$\bar{N}(x) = 0.5[1 + \text{Sgn}(x)] \{ 1 - [1 - \text{Sgn}(x)] \} \quad (6)$$

$$\bar{M}(x) = 0.5[1 - \text{Sgn}(x)] \{ 1 + [1 + \text{Sgn}(x)] \} \quad (7)$$

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Mostaghel subsequently expanded his model to consider stiffness degradation and load deterioration as well, employing two coefficients,  $\phi_k$  and  $\phi_l$ , which are respectively defined as:

$$\Phi_k = \frac{1}{1 + \lambda_k h(t)} \quad (8)$$

$$\Phi_l = \frac{1}{1 + \lambda_l h(t)} \quad (9)$$

where  $h(t)$  represents the total non-dimensional hysteretic energy absorbed by the friction force of the slider.

$$\begin{aligned} \dot{h} = & \Phi_l (1 - \alpha) |\dot{y}| [N(\dot{y})N(y - \gamma_p) + \bar{M}(\dot{y})M(y + \gamma_p)] \\ & + \lambda_p \bar{N}(\dot{y})M(y) + \lambda_p M(\dot{y})N(y)] \\ & \times |1 - \{\bar{N}(\dot{y})[\bar{M}(z - \lambda_p \Phi_l) \bar{M}(y) + \bar{M}(z - \Phi_l) \bar{N}(y)] + \\ & M(\dot{y})[N(z + \lambda_p \Phi_l) N(y) + N(z + \Phi_l) M(y)]\}| \end{aligned} \quad (10)$$

As mentioned earlier, Zeynalian et al. proved that Eq. (10) does not satisfy some of the system's boundary conditions. Therefore, the equation was modified. In addition, they expanded the equation in order to take into account the sliding phenomenon. The final non-dimensionalized equilibrium equation, including pinching, stiffness degradation, load deterioration, and sliding phenomena, is expressed as follows.  $\delta'_0$  is a proposed sliding function.

$$\begin{aligned} \dot{z}_i = & \dot{y} \Phi_{ki} \{N(\dot{y})[M(z_i - \lambda_{pi} \gamma_i \Phi_{li}) \bar{M}(y - \delta_0) \\ & + M(z_i - \gamma_i \Phi_{li}) \bar{N}(y - \delta_0)] \\ & + M(\dot{y})[\bar{N}(z_i + \lambda_{pi} \gamma_i \Phi_{li}) N(y - \delta_0) \\ & + \bar{N}(z_i + \gamma_i \Phi_{li}) M(y - \delta_0) \bar{M}(y + \delta_0)]\} \end{aligned} \quad (11)$$

To avoid complexity, no further detailed formulations are presented here. The next steps are to expand the system of equations for an MDOF multi-linear system, and then, to modify the equations based on a displacement-control cyclic loading regime. For this purpose, Method B of ASTM standard, which was originally developed for ISO (International Organization for Standardization) standard 16670 is applied. Therefore, instead of applying the external load function of  $P(t)$  in Eq. (1), the ASTM cycling loading regime is dictated to the system as an additional equation, as shown Fig.2.

### 3- Multi-degree-of-freedom Multi-linear Systems

Fig. 3 illustrates the schematic  $n$  degree of freedom of a multi-linear MDOF system.

For the sake of brevity, only the response of the third story from a three-story building is shown in Fig. 4 using the hysteresis loading based on Method B of ASTM E2126-07 standard. It shows the applicability of the final proposed differential equations illustrating all degrading phenomena, including stiffness-degradation, load-deterioration, pinching, stiffness-hardening, and sliding.

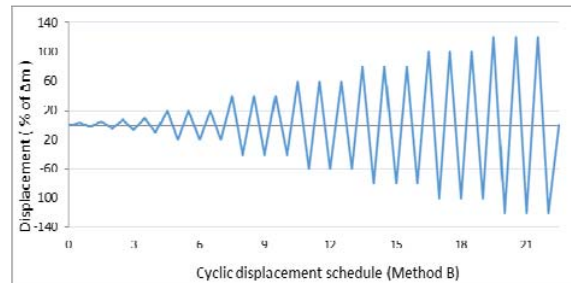


Fig. 2 Cyclic loading regime, Method B - ASTM E2126-07

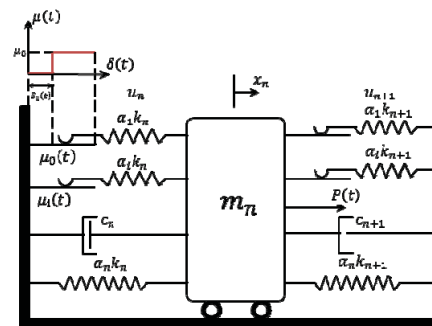


Fig. 3 Multi-linear, MDOF hysteresis mechanical system

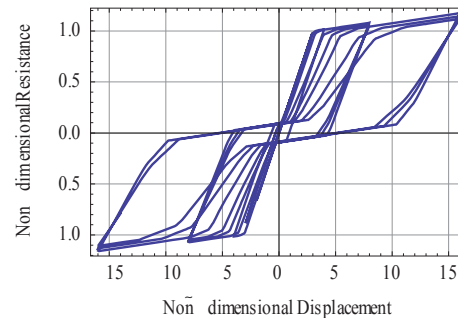


Fig. 4 Response of all degrading phenomena story 3

### 4- Conclusions

Based on the results, the following conclusions are drawn:

1. This paper presents an analytical differential model for the behavior of a general hysteretic MDOF system which includes all potential structural phenomena, such as pinching, stiffness degradation, load deterioration, and sliding.
2. The model is developed in order to take into account a displacement-control loading regime, i.e. Method B of standard ASTM E2126-07, which is usually implemented on experimental tests.
3. The results show that the proposed analytical model can provide a realistic description of the force-displacement performance of general hysteretic systems considering all degrading phenomena.