Journal of Ferdowsi Civil Engineering, Vol.28, No.2, 2017.

Improved Stability Transformation Method to Control Convergence of Structural Performance Measure Approach

B. Keshtegar

1-Introduction

Generally, there are several uncertainties in physical quantities of engineering systems. Consequently, probabilistic models should be implemented to consider these uncertainties. Design optimization methods under uncertainties have been developed to evaluate reliable performance of probabilistic constraints. Existing reliability-based design optimization (RBDO) approaches can be classified as double loop approaches (DLA), single loop approaches (SLA), and decoupled approaches. A suitable reliability method is required for evaluating the reliability level of probabilistic constraints in RBDO methods.

Commonly, the first-order reliability method (FORM) is used to estimate the reliable levels. Generally, the reliability index approach (RIA) and the performance measure approach (PMA) can be utilized for evaluating the probabilistic constraints in RBDO. The results of evaluating the performance measure illustrate that PMA provides higher efficiency and robustness in comparison with RIA. In RIA, the most probable failure point (MPFP i.e. U^*) on the limit state surface ($g(U)$) is needed to approximate the reliability index for evaluating the probabilistic constraints as follows:

$$\text{find } U^*, \quad \min \ \beta = \|U\| \qquad \text{Subjected to } \ g(U) = 0 \tag{1}$$

In PMA, the probabilistic constraint is evaluated by searching the minimum performance target point (MPTP) on the target reliability index (β_t) using the following model:

$$\text{find } U_t, \quad \min \ g(U) \qquad \text{Subjected to } \ \|U\| = \beta_t \tag{2}$$

The robustness and efficiency of iterative formula-based PMA are the important keys to implement a reliability analysis method in RBDO problems.

The advanced mean value (AMV) is commonly applied for inverse reliability analysis-based PMA due to its simplicity and efficiency. In general, the AMV scheme could converge to unstable solutions for highly nonlinear concave performance functions. The conjugate mean value (CMV), stability transformation method of chaos control (STM), modified chaos control (MCC), adaptive chaos control (ACC), self-adjusted chaos control (SACC), adjusted mean value (AMV) and conjugate gradient analysis (CGA) have been developed to enhance the robustness of MPTP search method. The CMV and AMV are adaptively combined to improve the efficiency and robustness of the FORM-based inverse reliability method in hybrid mean value (HMV). The HMV is robust for convex performance functions, but may result in unstable results for highly concave performance functions. The CC method is computationally inefficient for either convex or concave problems. The robustness and efficiency are two major challenges in reliability analysis.

In this paper, the stability transformation method-based chaos control is improved using a dynamic step size. The proposed dynamic step size is established using the performance values at the new and previous points. The sufficient descent condition is applied to adjust the proposed step size in the improved stability transformation method (ISTM). The robustness and efficiency of STM, AMV and the proposed ISTM methods are compared through several highly nonlinear mathematical and structural performance functions. The numerical study shows that the proposed ISTM algorithm is an efficient and robust FORM-based inverse reliability analysis.

2- Improve stability transformation method

In order to improve the efficiency of the STM-based CC, the iterative inverse FORM formula is given as follows:

$$U_{k+1} = U_k + \lambda_k C[f(u_k) - U_k] \tag{3}$$

where C is an $n\times n$ involutory matrix. Actually, the unit matrix is considered for C. λ_k is the dynamical step size which is suggested as follows:

$$\lambda_k = \min\{1, \left| \frac{1}{\delta\|D_k\|^2}[1 - \frac{g(d,U_{k+1})}{g(d,U_k)}] \right|\} \tag{4}$$

Where, $0<\delta<1$, $g(d,U_{k+1})$ and $g(d,U_k)$ are the performance values at the new and previous points, respectively. D_k is the search direction vector which is computed as

$$D_k = f(u_k) - U_k \tag{5}$$

In which $f(u_k)$ is a discrete nonlinear map which is given as follows:

$$f(u_k) = -\beta_t \frac{\nabla_u g(d,U_k)}{\|\nabla_u g(d,U_k)\|} \tag{6}$$

where $\nabla_u g(d,U_k)$ is the gradient vector of the performance function at point U_k. In order to control instabilities of the FORM formula-based the dynamic search direction, the sufficient descent condition i.e. $D_k < D_{k-1}$ is applied to adjust the dynamic search direction by the following relation:

$$\lambda_k = \eta\lambda_k \quad for \ \ D_k \geq D_{k-1} \tag{7}$$

[1] Assistance professor, Department of Civil Engineering, University of Zabol.
Email: Bkeshtegar@uoz.ac.ir

in which $0.5 \le \eta < 1$. The proposed step size can be adapted based on the results of the performance function at each iterations, while the step size in the STM is considered a constant smaller value at each iteration. Thus, the proposed ISTM may be converged faster than the STM for convex problems and it is more robust than the AMV for highly nonlinear problems.

3- Comparative results

The robustness and efficiency of the proposed ISTM algorithm are investigated through several nonlinear performance functions. The results of ISTM with parameters of $\delta = 0.5$ and $\eta = 0.95$ are compared with PMA and STM with the parameter of $\lambda = 0.1$. For this purpose, the numbers of computations gradient vector (Iteration) and performance values are used to illustrate efficiency and robustness of the FORM-based inverse reliability methods.

Example 1: nonlinear performance function

The nonlinear performance function is considered as follows:

$$g_1 = X_1^3 + X_2^3 - 18 \tag{8}$$

where $X_1 \sim N(10,5)$, $X_2 \sim N(9.9,5)$ and $\beta_t = 3.0$ in which $N(\mu,\sigma)$ is the normal distribution with the mean of μ and standard deviation of σ. The converged results of this performance function based on the proposed ISTM are obtained after 42 iterations as $g(X_t)$= -31.066473 and X_t= [-2.89058, 2.22974].

Figure 1 illustrates the convergence histories of the performance values for different reliability methods such as AMV, STM and ISTM. It can be seen that the AMV has resulted in unstable results as chaotic solutions while the STM and ISTM are more robust than the AMV for this example. The STM has converged after 126 iterations but the proposed ISTM has converged to stable results after only 42 iterations. The results from Fig. 1 show that the proposed ISTM is slightly more efficient than the STM and the ISTM has converged about three-times faster than STM.

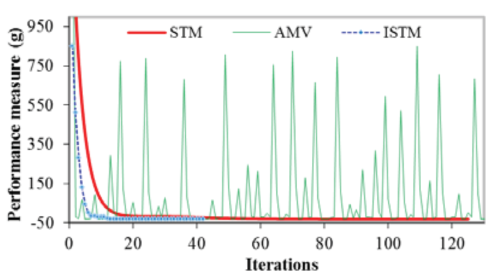

Fig. 1 Convergence histories of the performance function Example 1 for different reliability methods

Example 2: nonlinear multi-dimensional performance function

$$g_2 = -0.75 + 0.489\, X_1 X_4 + 0.843\, X_2 X_3 - 0.0432\, X_5 X_6 + 0.0556\, X_5 X_7 + 0.000786\, X_7^2 \tag{9}$$

Where $X_1 - X_4 \sim N(1, 0.05)$, $X_5 \sim N(0.3, 0.006)$, X_6, $X_7 \sim N(0,10)$ and $\beta_t = 3.0$.

The results from the CC method are extracted as performance value at MPTP of 0.07535 and MPTP of [0.97643, 0.96006, 0.96006, 0.97643, 0.301, 25.67798, -8.05467]. Based on the results of the ISTM using the dynamical step size, the performance value of Eq. (9) at MPTP and the MPTP are obtained after 12 iterations as $g(X_t)$=0.07529 and Xt= [0.97645, 0.96008, 0.96008, 0.97645, 0.3011, 25.66047, -8.14201]. As seen, the results obtained from the proposed ISTM are in close agreement with the reliability results extracted from the CC method.

The performance histories of different reliability methods for Example 2 are shown in Fig. 2. The results of Fig. 2 illustrated that the AMV method yields unstable results as periodic-2 solutions i.e. $g(X_k)$=0.5665 and $g(X_{k-1})$=1.3955. However, the STM and the proposed ISTM have more robustly converged after 86 and 12 iterations, respectively. The ISTM is as robust as the STM but it is slightly more efficient than the STM for this example. The ISTM produces stable results faster than the STM with convergence rate about seven-times larger than STM.

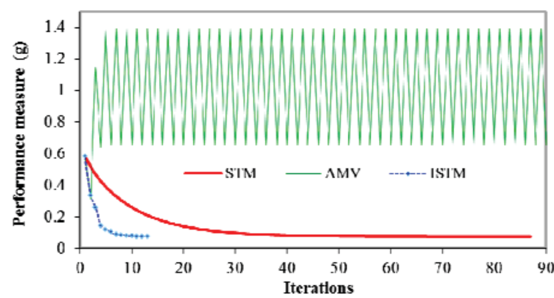

Fig.2 Convergence histories of the performance function Example 2 for different reliability methods

4- Conclusions

The stability transformation method is improved using an adaptive step size. The adaptive step size is proposed to improve the efficiency of STM using sufficient descent condition. The results indicated that the AMV provides unstable results and the STM is an inefficient method for highly nonlinear performance functions. However, the ISTM is as robust as STM but it is more efficient.

The dynamic step size can be adapted using the performance information and it is adjusted using sufficient descent condition. Therefore, this step size can be used to improve the robustness of FORM compared to AMV and can enhance the efficiency of STM for highly nonlinear problems.