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Dynamic Response Analysis of Tall Buildings Under Axial Force Effects

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1- Introduction

Analysis methods of tall buildings include two general fields, approximate methods and finite elements methods. The finite element approach is based on discrete model and has to solve thousands of linear simultaneous equations to give quantitative results in detail. So, it is a powerful tool for analysis and design at the detailed and final design stage of tall buildings. It takes much more time for the modeling of a given structure using finite elements. Approximate methods are based on modelling of tall building by an equivalent replacement beam (Fig. 1).

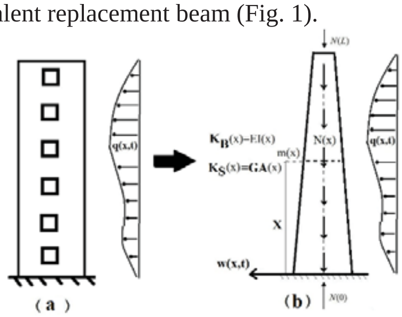

Fig. 1. Approximate analysis method of a tall building with shear-flexural stiffness under axial force, (a): Original structure, (b): Equivalent replacement beam

Approximate methods give insight into characteristics of free vibration. It is simple and accurate enough that can be routinely used for the preliminary stage of building design. In addition, to investigate the influences of some structural parameters on the static and dynamic characteristics of global structures, it would be convenient to use the analytical approximate method. In this paper, an analytical approximate approach is presented for the determination of the natural frequencies of tall structures under variable axial forces. It has been assumed that the structure has variable shear-bending stiffness and mass along the height. Through repetitive integrations, the governing partial differential equations are converted into weak form integral equations. The mode shape function is approximated by a power series. Substitution of the power series into weak form integral equations results in a system of linear algebraic equations. The natural frequencies are determined by calculation of the non-trivial solution for the resulting system of equations.

2- Conversion of the governing equation into weak form

Accounting for total potential energy of the system and applying Hamilton's principle, the governing equation of the motion for equivalent beam is given as follows:

$$\frac{d}{dx}[H^2K_S(x)\frac{d}{dx}w(x)]+\frac{d}{dx}[H^2\,N(x)\frac{d}{dx}w(x)]$$
$$-\frac{d^2}{dx^2}[K_B(x)\frac{d^2}{dx^2}w(x)]+\Omega^2H^4m(x)w(x)=0 \qquad (1)$$

In which $K_S(x), K_B(x), w(x), N(x), m(x), \Omega, H$ are: shear stiffness, bending stiffness, mode shape function, axial force function, mass per unit length, natural frequency and height of the structure, respectively. The weak form of the governing differential equation is obtained by four times repetitive integration from Eq. (1). The result is as follows:

$$\int_0^{\chi} h_1(x,s)\,w(s)ds-K_B(x)\,w(x)=\frac{C_1}{6}x^3+\frac{C_2}{2}x^2$$
$$+C_3x+C_4 \qquad (2)$$

3- Boundary conditions

$C_{i\,(i=1,2,..,4)}$ are the integration constants which can be calculated using the boundary conditions of the tall building. The boundary conditions are as follows:

$$\begin{cases} x=0 \quad w(0)=0,\ \dfrac{dw}{dx}(0)=0, \\ x=1 \quad -H^2K_S(x)\dfrac{d}{dx}w(x)-H^2N(x)\dfrac{d}{dx}w(x) \\ +\dfrac{d}{dx}\left[K_B(x)\dfrac{d^2}{dx^2}w(x)\right]=0, \\ x=1 \quad K_B(x)\dfrac{d^2}{dx^2}w(x)=0 \end{cases} \qquad (3)$$

Using the boundary conditions (3), the weak form integral equation of the governing equation is obtained as follows:

$$\int_0^{x} h_1(x,s)\,w(s)\,ds+\int_0^{1} h_2(x,s)\,w(s)\,ds-K_B(x)\,w(x)=0 \quad (4)$$

4- Calculation of the natural frequency

In the Eq. (4), the mode shape function of the vibration is approximated by the following power series:

$$w(x)=\sum_{r=0}^{p}c_r x^r \qquad (5)$$

Eq. (5) is introduced into integral equation (4) and both sides of Eq. (4) are multiplied by x^m and integrated subsequently with respect to x between 0 and 1. This results in a system of linear algebraic

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equations in C_r :

$$\sum_{r=0}^{P}\left[G(m,r)+H_1(m,r)+H_2(m,r)\right]c_r=0$$

$$m=0,1,2,...,P \qquad (6)$$

In which:

$$G(m,r)=-\int_0^1 x^{r+m}K_B(x)dx\,,\ H_1(m,r)=\int_0^1\int_0^x h_1(x,s)s^r x^m dsdx$$

$$H_2(m,r)=\int_0^1\int_0^1 h_2(x,s)s^r x^m dsdx \qquad (7)$$

The system of linear algebraic equations (6) may be expressed in matrix notations as follows:

$$[A]_{(P+1)\times(P+1)}[C]_{(P+1)\times 1}=[0]_{(P+1)\times 1} \qquad (8)$$

The only unknown parameter in the coefficients matrix $[A]$ is the natural frequency of the tall building Ω. The natural frequencies are determined by calculating a non-trivial solution for the resulting system of equations. To do so, the determinant of the coefficients matrix of the system has to be vanished. The roots of the frequency equation are the natural frequencies of the tall structure.

5- Numerical examples

In the following example, a 25-story tube-in-tube structure already examined in the literature is investigated. The flexural stiffness of the outer and inner tubes are $(K_B)_o=35.2872\times10^9$ kN.m^2 and $(K_B)_i=7.5538\times10^9$ kN.m^2, respectively. The shear stiffness, mass per unit length and building height are: $K_S=3.9888\times10^7$ kN , $m=3385.728\frac{kg}{m}$ and H=75.9 m, respectively. The first two natural frequencies are calculated and compared with those in the literature. The results are presented in Table 1.

Table 1: Comparison of first two frequencies of a 25-story tube-in-tube tall building

Methods	Proposed method	Malekinejad and Rahgozar,2014
Ω_1	3.7056	3.705
Ω_2	16.1326	16.127
Methods	Wang, 1996-a	Wang, 1996-b
Ω_1	3.462	3.461
Ω_2	21.525	19.239
Methods	Youlin, 1984 (a)	Youlin, 1984 (b)
Ω_1	3.157	3.279
Ω_2	-	17.921

In order to investigate the effects of structural parameters on the natural frequencies of the structure, a basis structure with structural properties as:

$K_B=2.61\times10^{13}$ kN − m^2, $K_S=77.56\times10^8$ kN,

$m=681408$ kg.s^2 / m^2, H = 210 m,

is considered. The structural properties of the basis structure are assumed to change between 1 through 3. The effects of these changes on the natural frequencies of the basis structure are calculated as: $\%Diff=\left(\frac{\Omega-\Omega_{bs}}{\Omega_{bs}}\right)\times100$.

"bs" denotes "basis structure". The results are presented in the Figs. (2)and (3).

6-Conclusion

The application of the weak form integral equations for determining the natural frequencies of tall structures with shear-flexural deformation has been presented. The effect of structure weight on its natural frequencies has been considered. Differences between natural frequencies of proposed method and the ones obtained in the literature were in acceptable ranges. The results of numerical examples indicated that the variations of the shear stiffness is more important for the first mode, while variations of the flexural stiffness is more important for higher modes of the vibration. Also, the variations of the mass per unit length of the structure has the same effect on all three modes. The variations of the structure height has the most effect on the third mode and it has the least effect on the first mode of the vibration.

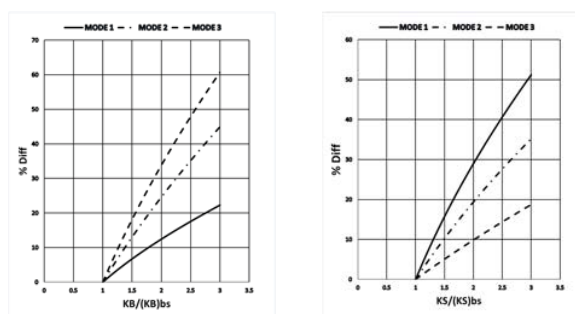

Fig. 2. The effects of the flexural stiffness and shear stiffness variations on the natural frequencies of the basis structure

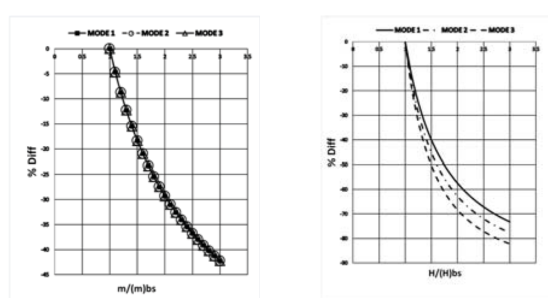

Fig. 3. The effects of mass and height variations on the natural frequencies of the basis structure