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Variable Damping in Geometric Nonlinear Plate Bending Analysis Using Dynamic Relaxation

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1- Introduction

Bending plate structures are important in mechanical and civil engineering. Therefore, a great deal of research has been done to describe their behavior. Moreover, closed-form and numerical solutions are proposed for these structures.

The analysis of elastic plates with large deformations is very complicated and there are a few approaches for finding an exact solution. Numerical and approximate solution procedures have been developed for large displacements with the increasing processing power of modern computers. One of these tactics is called dynamic relaxation (DR). This method employes the Second-Order Richardson approach. This scheme was developed by Frankel. The static equilibrium equations are converted to a fictitious dynamic system in this way.

In this study, geometrically nonlinear bending analysis of plates is performed using DR and finite difference methods. At the first stage, the damping factor is obtained so that the convergence is achieved. It should be added that in the common DR formulation, the damping is constant in the DR iterations. Here, the variable damping is used. In this kind of damping, the constant mass and the variable mass are applied. The constant mass does not change during analysis, while the variable mass is updated at each iteration. In many problems, researchers applied the velocity criterion for the convergence of the plates' analysis by utilizing the DR and finite difference. Based on the velocity criterion, the structure will reach a static equilibrium if the velocity is less than the allowable error in the DR iterations. In this paper, three criteria are used to stop the analysis; the velocity, the kinetic energy and the displacement ratio of the two successive iterative steps. Afterwards, a comparison study is performed between them.

2- The dynamic relaxation method

Dynamic relaxation is one of the explicit methods to solve a system of simultaneous equations. In this process, a fictitious mass and a fictitious damping are added to static structural equations to obtain the following fictitious dynamic system:

$$[M]^n\left\{\ddot{X}\right\}^n + [C]^n\left\{\dot{X}\right\}^n + [S]^n\{X\}^n = \{f\}^n = \{P\} \qquad (1)$$

The terms M and C are the fictitious mass matrix and the damping matrix, respectively. Both are diagonal. The displacement, velocity and acceleration are demonstrated by X, $\dot{X}$, and $\ddot{X}$, correspondingly. Moreover, the number of iterations is denoted by n. The stiffness matrix; external internal load vectors are shown by S, P and f, respectively. To solve Eq. (1), the inertia and damping forces should become zero.

In the DR approach, the velocity variations are assumed to be linear and the acceleration is supposed to be constant for each time step t. Thus, the following equalities can be obtained for the iterative relations of this tactic by using central finite differences:

$$\dot{X}_i^{n+\frac{1}{2}} = \frac{2m_{ii}^n - C_{ii}^n t^n}{2m_{ii}^n + C_{ii}^n t^n}\dot{X}_i^{n-\frac{1}{2}} + \frac{2t^n}{2m_{ii}^n + C_{ii}^n t^n}(p_i - f_i^n) \qquad (2)$$

$$X_i^{n+1} = X_i^n + t^{n+1}\dot{X}_i^{n+\frac{1}{2}}, \quad i = 1, 2, ..., ndof \qquad (3)$$

3- DR and bending plates

In this section, the DR finite difference formulas are obtained for geometric nonlinear bending plates. For this purpose, equilibrium differential equations, the strain-displacement relation, the curvature-displacement relation and internal forces' formula are applied. It should be emphasized that the middle surface has a strain in the large-deflection theory in contrast to the small-deflection theory. Eq. (4) shows the velocity for each degree of freedom.

$$\left(\frac{\partial\alpha}{\partial t}\right)^a = \frac{1}{1+C_\alpha^*}\left[\left(1-C_\alpha^*\right)\left(\frac{\partial\alpha}{\partial t}\right)^b + \frac{t^a}{m_\alpha}\left(EQ_i\right)\right] \qquad (4)$$

Here, the equilibrium differential equation at α direction is shown by EQ_i. The symbol α can be u, v and w, which are displacements in X, Y and Z directions, respectively. The in-plane displacements are denoted by u and v. Moreover, the lateral displacement (deflection) is w. Furthermore, $C_\alpha^* = \frac{c_\alpha t^a}{2m_\alpha}$, in which the term c_α is damping factor.

4- The DR Stability

Stability of dynamic relaxation iterations depends on the estimation of appropriate mass and damping matrices. In this research, the absolute values of the stiffness matrix entries in a row are calculated and summed to find the nodal mass in each iteration. This summation is established by two parts. One of them is constant during the analysis, whereas the other one should be calculated in each iteration. The in-plane forces' sign plays an important role in the numerical stability of sum of entries in the out-of-plane equilibrium differential equation. In other words, the stiffness is decreased if the in-plane force's sign is minus.

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5- Boundary conditions

To satisfy equilibrium equations in each grid node, boundary conditions should be considered. In other words, compatibility relations should be satisfied in addition to the equilibrium equations for structural analysis. Compatibility relations are obtained from boundary conditions. These equations can be written according to displacements or forces. In general, compatibility relations consist of two parts. One of them is displacement's boundary conditions, while the other one is the force's compatibility relations.

If displacements of nodes are specified, their equilibrium equations are ignored in the system. In other words, DR relations are not written in these nodes. For example, equilibrium equations are not written in simple and clamp supports. The support displacements are considered by this method.

If load's value is specified in the boundaries, their effect should be considered in the equilibrium equations. These loads are called forces compatibility. In the finite difference procedure, the solution process is performed in two steps, which are the overall and exact approach to satisfy boundary conditions. At the first stage, problem is solved without force compatibility conditions. Force compatibility conditions are applied at the second step.

There are two methods for establishing force compatibility conditions. At the first technique, the specified boundary forces are directly applied to equilibrium equations of adjacent nodes. Another method is alteration force conditions with equivalent displacement relations.

6- The F factor

In this section, various bending plates are analyzed. The F factor is changed from 0.1 to 3. The variable damping is used for all samples. Note that the constant damping does not cause any change in the results. The convergence criterion is the velocity error, and the DR iterations are stopped when the velocity is less than 10^{-5} in *X*, *Y* and *Z* directions.

Numerical samples show that the following value can be used to estimate the F factor based on the plate deflection in the large-displacement theory. If the ratio of deflection to the thickness is less than 1.5, F=2-3. For the ratio between 1.5 to 2, F=1.5-2 and for the ratio greater than 2, F=0.5-1.

7- The constant and variable damping

Here, the constant and variable damping are compared. Previous samples are used. The convergence criteria utilized in this study are the velocity error, kinetic energy error and the displacement ratio of the two successive iterative steps. For the velocity error, DR iterations are stopped when the velocity is less than 10^{-5} in *X*, *Y* and *Z* directions. Based on the previous section, the F factor is set 0.75. In this paper, the numbers of total iterations and total analysis duration are compared.

Numerical results show that the variable damping decreases the number of iterations and analysis time based on the velocity convergence criterion. The samples utilized in this study were ordinary plates in which the analysis duration is reduced 10% by using both the variable damping and the velocity criterion. If the plate's geometry is complex, this 10% reduction is very beneficial and significant.

8- Conclusion

The finite difference approach doesn't require the employment of stiffness matrix, element force vectors as well as assembling process. These are the advantages of finite difference procedure to other methods. Therefore, the number of iterations and the duration of analysis decrease. Moreover, computers with low processing power can be used. The finite difference scheme with the DR process is used to analyze geometric nonlinear bending plates. The fictitious mass is calculated by Cassell and Hobbs's method. Both constant and variable damping is assumed in this study. The damping is estimated by $c_{\alpha_{i,i}} = m_{\alpha_{i,i}}(F)^{-1}$. A value was determined for F so that the convergence criteria is satisfied. It is worth emphasizing that the convergence rate is dependent on the load value and mesh, in addition to F. Results show that the convergence is satisfied for any load with fine mesh and F=0.75. In small-displacement theory, F factor can be chosen very greater than 0.75 so that the convergence rate increases.

Furthermore, samples were analyzed with the constant and variable damping by F=0.75. Three criteria for convergence of the variable damping method were utilized by the authors. These are the velocity error, kinetic energy error and the displacement ratio of the two successive iterative steps. The velocity criterion was applied for the constant damping. The loads were applied into forty increments. The previous increment results were used to increase the convergence rate. The allowable error was obtained by a trial-and-error process for the kinetic energy criterion and the displacement ratio of two successive iterative steps so that the deflection at the maximum load equals to the deflection based on the velocity criterion. Numerical results show that the proposed algorithm reduced the number of iterations and the analysis duration. The minimum decrease was 10%. Hence, it is recommended that both the variable damping and the velocity criterion be applied in the DR procedure with the finite difference scheme. Moreover, it was concluded that the displacement criterion had the minimal effect, and the answers were the worst ones.