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A Robust Scenario Based Approach in an Uncertain Condition Applied to Location-Allocation Distribution Centers Problem

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ABSTRACT: The paper discusses the location-allocation model for logistic networks and distribution centers through considering uncertain parameters. In real-world cases, demands and transshipment costs change over the period of the time. This may lead to large cost deviation in total cost. Scenario based robust optimization approaches are proposed where occurrence probability of each scenario is not known. It is supposed that in this case there would be budget constraints and also holding the products in the distribution centers until sending them to the retailers' destinations results additive cost that can be defined as inventory control cost in the model. In this paper, uncertainty is defined by different scenarios. Some robust approaches are presented that can be applied in location-allocation problem. The robust scenario based approaches like absolute robust and robust deviation are applied in location-allocation problem. Also a new robust approach is proposed that outperforms the existing classical approaches. The mean expected model has been discussed and compared to the robust proposed approaches. A numerical example illustrates the proposed model and the results have been reported. Finally the comparison of results shows the efficiency of proposed robust approach in comparison of classical approaches and also mean expected model.

Keywords: *Location-allocation (LA), Uncertain parameters, Scenario, Robust optimization, Mean expected model*

INTRODUCTION

Facility location decisions are costly and difficult to change. One of the most important issues in facility location problem and specifically supply chain management is locating and allocating the distribution centers. These strategic decisions are critical factors of whether materials will flow efficiently through the chain network. The location of distribution centers are affected by parameters such as demands and transportation costs. Owing to the fact that each parameter of this problem can change sensitively in the period of time, so decisions related to the design can be very important and effective on whole supply chain network. The best optimal location of these nodes can surely save the transportation costs. Location-allocation (LA) problem in facility location problem is to locate a set of new facilities in a fashion that the total distance from facilities to customers and consequently the total cost is minimized. LA problem has been considered for many years because of its broadly realistic application. In real cases, we should consider the uncertain parameters for LA problem as a number of factors including demands, distances even locations of customers or facilities can be affected. LA problem was studied in detail by (Gen and Cheng, 1997; Gen and Cheng, 2000). Hodey et al. (1997), presented several models discussed in LA. To solve these models, experts have proposed different algorithms such as branch-and-bound algorithms. Kuenne and Soland (1972) simulated annealing (Murray and Church, 1996), tabu search (Brimberg and Mladenovic, 1996) in addition to the location models considering the idea of inventory control (Daskin et al., 2002). Expected costs of inventory as well as costs of location and allocation have been considered in this model simultaneously.
While uncertain situations involve arbitrariness and uncertain parameters, all parameters are deterministic and known in certain situations. In some cases, probability of distribution function is known but in others no information about probabilities is known. Problems defined as a first category are identified as

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stochastic optimization problems, and in these problems, main goal is to optimize the expected value of objective functions. The problems in the second categories are considered as robust optimization problems and often consider optimizing the worst-case performance of the system. Tsiakis et al. (2001) have considered uncertainty condition in multi products supply chain system.

Manne (1961) considers stochastic problem inputs. Demands in Manne's model are probabilistically and allows backordering of unsatisfied demands. Mirchandani (1980) investigated the P-median models and un-capacitated warehouse location problem when travel characteristics and supply and demand patterns are stochastic. Louveaux (1986) introduces two-stage stochastic programs for solving simple plant location and p-median problems. Gabor et al. (2006) presented an approximation algorithm for a facility location problem with stochastic demand. They presented an expected value of a constraint that the probability that an arbitrary request lost was at most α.

The goal of both stochastic and robust optimization is to determine a solution that will perform the best under any possible realization of the uncertain parameters. Therefore, we can define random parameters either by continuous distribution functions or discrete scenarios. Uncertainty in cost parameters and demands and so on, is very common problem in most of LA problems.

As in the stochastic optimization case, uncertain parameters in robust optimization problems may be modeled as being either discrete or continuous. Discrete parameters are modeled using the scenario based approach. Scenario based planning is an approach in which uncertainty is described by determining a number of future more possible alternatives for effective parameters in the model. For a given problem under uncertainty with no probability information, the min-max cost solution is the one that minimizes the maximum cost across all scenarios. Besides, min-max regret approach is another one which minimizes the maximum deviation between optimum cost function and the objective values of each scenario. Vanston et al. (1977) discuss the use of scenario planning techniques and present a 12- step procedure for generating a set of appropriate scenario. Ghosh and McLafferty (1982) used scenario planning concept to make decisions about the location of retailers' stores in an uncertain environment.

The main idea in extending the robust optimization approaches has been presented by El-Ghaoui and Lebret (1997); El-Ghaoui et al. (1998); Ben-Tal and Nemirovski (1999); Ben-Tal and Nemirovski (2000). Mulvey et al. (1995), proposed the concept of robustness aiming at solving the uncertain programming.

Kouvelis and Yu (1997) discuss the use of a robustness approach to make decision in environments characterized by uncertain data. One common measure solutions in robust optimization is considering regret value, which is defined as a difference between the costs of a solution in a given scenario and the cost of the optimal solution for that scenario. Models that aim to minimize the maximum regret across all scenarios are named min-max regret models. Averbakh and Berman (1997) extended model proposed by Kouvelis and Yu (1997) and investigated the P-center problem on a network with uncertain demand values. Mausser et al. (1998) introduced general-function algorithms for min-max regret linear models considering interval-uncertain objective function coefficients, absolute regret models and also, problems modeled by relative regret approach. Mausser et al. (1999), proposed a greedy heuristic algorithm for the absolute regret problem and used some methods to avoid local optima. Velarde et al. (2004) presented notation of the robust capacitated international sourcing problem with considering a finite capacity for facilities. Assavapokee et al. (2008), presents an algorithm for solving scenario-based min-max regret and relative regret robust optimization problems for two-stage MILP formulations. A multistage stochastic programming approach is proposed by Guillen et al. (2006) for the supply chain design problem under demand uncertainty by integrating strategic and operational levels. Demand and exchange rate were the uncertain parameters which described by scenarios. Snyder and Daskin (2006) presented a novel robustness measure that combines the two objectives by minimizing the expected cost while bounding the relative regret in each scenario that named p-robust. For a comprehensive review on the facility location problems considering uncertainty refer to Snyder (2006). Recently Clibi et al. (2010), discussed Supply Chain Network (SCN) design problem under uncertainty, and presents a critical review of the optimization models proposed in the literature.

In this paper, environmental uncertainty is described by discrete scenarios where probability of occurrence in each one is not known. Therefore, robust optimization approaches are used to investigate this problem. Since most data such as demands and cost parameters are defined in discrete form in supply chain problems, scenario based robust is more proper for this type of uncertain problems. In this paper, the location-allocation problem for logistic centers is surveyed by robust scenario based approach. As it was mentioned, Daskin et al. (2002) proposed inventory-location model and suggested that in several aspects of location problems, considering inventory controlling systems as well as considering uncertainty can be an important issue can be investigated by authors.

The consideration an inventory system can be investigated as an important issue that may result more reliable and valid modeling in dealing with location-allocation logistic centers problems. In addition, a novel robustness measurer for maximum desirable regret deviation named limited min-max regret approach in which maximum value of the regret value under all scenarios are limited is another subject presented in this paper. Also, mean expected value model in which each uncertain parameter is replaced by its mean value will be proposed to illustrate effectiveness of the robust approach.

This paper is organized as follows. Sections 2 present both robust approaches and mean expected value. Proposed robust approach is defined in section 3. In section 4, the new location-allocation model considering inventory system is defined. To illustrate the proposed method, a numerical example is presented in Section 4 and the results are discussed. Finally, the last section is the conclusion of this paper.

RESEARCH METHOD

Robust Scenario Based Approaches and Mean Expected Value Model

In this section, some approaches that can be applied in location-allocation problems are defined. First, robust approaches exist in the literature are defined and then proposed as follows:

Robust Approaches

Uncertainty in the parameters of a location-allocation problem including cost parameters and demands is very common. In this paper, the uncertainty in the parameters is characterized by different scenarios such that some parameters of the location-allocation cost model are different under each scenario. Moreover, we do not know which scenario will happen in the future, in other words, there is no information about probability of occurrence of each scenario. To model robust location-allocation problem with uncertain data, we use a robust scenario based min-max (absolute robustness) optimization approach and also robust min-max regret (robust deviation) approach and also novel robust approach which is limited min-max regret approach which is presented in following section. In the first robust approach, the main goal is decreasing the worst-case scenario while robust deviation (second approach) minimizes the deviation from the optimal solutions. We suppose $\boldsymbol{x}$ *and* $\boldsymbol{u}$ as the vector of decision variables and matrix $\boldsymbol{A} = (\boldsymbol{a}_1 \dots \boldsymbol{a}_p)^T$ as different scenarios in which $\boldsymbol{a}_a^T$ s are vectors including parameters of each scenario. $\boldsymbol{x}_i$ and $\boldsymbol{u}_i$ are the vector of feasible solutions of the deterministic model and Z_a and Z_a^* are the cost and the optimum cost of a^{th} ($a = 1, \dots, p$) scenario, respectively.

Absolute Robust (min-max)

One common objective function for absolute robust location-allocation model can be written as follows:

$$\underset{a \in \text{sets of scenarios}}{Minimize\ Z = Max} \{Z_1, \dots, Z_a, \dots, Z_p\} \tag{1}$$

Through absolute robust or min-max objective function, we want to minimize the maximum cost of all scenarios (the worst case of all scenarios) because we have no information which scenarios may happen. This criterion is suitable for the cases in which the risk is in a high level.

Robust Deviation (min-max regret)

Also another possible objective function for robust location-allocation model is robust deviation or min-max regret approach that can be written as follows:

$$\underset{a \in \text{sets of scenarios}}{Minimize\ Z = Max} \{(Z_1 - Z_1^*), \dots, (Z_a - Z_a^*), \dots, (Z_p - Z_p^*)\} \tag{2}$$

By applying this criterion, we want to select the design which has smallest deviation from the optimum solution of each scenario. Hence, the minimum cost for each scenario as a certain condition must be obtained. So this approach is applicable when we are going to find the amount of improvement in design parameters.

Mean Expected Model

In order to compare the results, we consider the mean expected value model in which each parameter is replaced by the expected value of the parameters in different scenarios. This is one of the first simple approaches to solve this type of problems in which we have some different scenarios without precise information which scenario will happen, however we show this approach is not suitable and the proposed robust approaches outperform this simple approach.

Proposed Limited Min-max Regret

Considering that in robust deviation approach, there is no limitation for the value of deviation in the cost function, modified robust model with uncertain data is proposed. The model, we present in this section is limited min-max regret approach. In this approach, the main goal is decreasing maximum value of the regret value under all scenarios while the regret values in the objective functions are limited by ω coefficient. In this model, the main robust constraints added to previous robust deviation model are $\frac{Z_a(x)-Z_a^*}{Z_a^*} \leq \omega$.

It is clear that if $\frac{Z_a(x)-Z_a^*}{Z_a^*} \leq \omega$,

we have reached the maximum allowable robustness. ω is the maximum desirable value.

So the proposed model can be defined as follows in Equation (3):

The mentioned objective function is nonlinear but it can be changed to linear form by adding suitable constraints. This objective function result is a mixed integer programming formulation.

Model Definition

In this paper, proposed location-allocation model considers budget constraints as well as limits for inventory control cost. It is supposed that the inventory system is economic order quantity (EOQ). Demand, transportation cost, inventory control parameters and budgets are uncertain and they are defined in a scenarios while the other parameters are deterministic.

Describing the Proposed Limited Min-max Robust Model

The notations used in the models are described for deterministic model as follows:

It is assumed that S denotes the set of supply nodes and Q denotes the set of possible distribution centers and R denotes the set of retailers. P denotes the type of products. A denotes the sets of scenarios. The aim of the model is sending the products type $t \in P$ from supply node $i \in S$ to distribution center $j \in Q$ and after that sending that product from distribution center node $j \in Q$ to retailer node $k \in R$.

$$\text{Minimize } Z = \text{Max } \{(Z_1 - Z_1^*), \ldots, (Z_a - Z_a^*), \ldots, (Z_p - Z_p^*)\} \qquad a \in \text{sets of scenarios}$$

$$\begin{aligned} \text{S.t} \quad & Z_1(x) \leq (1+\omega)Z_1^* \\ & \vdots \\ & Z_a(x) \leq (1+\omega)Z_a^* \\ & \vdots \\ & Z_p(x) \leq (1+\omega)Z_p^* \\ & x \in \Omega \end{aligned} \tag{3}$$

d_{kt}: denotes the demand that should be satisfied at each retailer nodes $k \in R$ from each types of products $t \in P$.
c_{ijt}: denotes the transmission cost of product $t \in P$ from supplier node $i \in S$ to distribution center node $j \in Q$.
c_{jkt}: denotes the transmission cost of product $t \in P$ from Distribution center node $j \in Q$ to retailer node $k \in R$.
q_{it}: denotes the supply capacity of product $t \in P$ in supply node $i \in S$.
w_j: denotes the fixed construction cost of possible distribution center nodes $j \in Q$.
B: denotes the maximum available budget.
n: denotes the necessary number of distribution centers.
h_t: denotes holding cost of product $t \in P$.
A_t: denotes ordering cost of product $t \in P$.
p_t: denotes price of product $t \in P$.
I_t :denotes inventory control budget constraint for product $t \in P$.
ω :denotes the allowable regret for all scenarios. It is assumed that the ω is identical in all scenarios.
And decision variables are as follows:
x_{ijt} : denotes the flow of product $t \in P$ from the supply node $i \in S$ to distribution center $j \in Q$.
y_{jkt} : denotes the flow of product t P from the distribution center node $j \in Q$ to retailer node $k \in R$.
u_i : denotes the location of distribution center node $j \in Q$ and is binary decision variable. If distribution center will be located at possible distribution center node $j \in Q$, u_j =1; otherwise u_j = 0.
The robust scenario based limited min-max regret model under all scenarios can be formulated as follows:
In this model:

Constraint (4) is the objective function; optimal solution of robust optimization model can be obtained considering all scenarios. Constraint (5) is the total cost under each scenario. Constraint (6) shows that the flow received to each distribution center nodes should equal to the flow sent to each retailer nodes. This constraint is flow equilibrium constrain. Constraint (7) ensures that the demand should be satisfied at each retailer node. Constraint (8) ensures that the total products that each supply node sends should not exceed their capacity. Constraint (9) is the constraint of number of distribution center nodes. Constraint (10) is inventory control cost restriction. Constraint (11) is robust constraint of solution. Constraint (12) is maximum budget available for the problem. Constraint (13) and Constraint (14) are logical constraint of the decision variables.

Objective function: $$MinZ = \underset{a \in A}{Max} \left(Z_a(x) - Z_a^* \right) \quad (4)$$

s.t:

$$Z(x)_a = \sum_{j \in Q} W_j u_j + \sum_{i \in S} \sum_{j \in Q} \sum_{t \in P} C_{ijt} x_{ijt} + \sum_{j \in Q} \sum_{k \in R} \sum_{t \in P} C_{jkt} x_{jkt} \qquad \forall a \in A \quad (5)$$

$$\sum_{i \in S} x_{ijt} = \sum_{k \in R} y_{jkt}, \qquad \forall j \in Q, \forall t \in P, \forall a \in A \quad (6)$$

$$\sum_{j \in Q} y_{jkt} \ ? \ d_{kt}, \qquad \forall k \in R, \forall t \in P, \forall a \in A \quad (7)$$

$$\sum_{i \in S} x_{ijt} \leq q_{it} \qquad \forall j \in Q, \forall t \in P, \forall a \in A \quad (8)$$

$$\sum_{j \in Q} u_j \leq n \qquad \forall a \in A \quad (9)$$

$$P_t \sum_{i \in s} \sum_{j \in Q} x_{ijt} + \frac{\sum_{k \in R} d_{kt} A_t}{\sum_{i \in s} \sum_{j \in Q} x_{ijt}} + \frac{h_t \sum_{i \in s} \sum_{j \in Q} x_{ijt}}{2} \leq I_t \qquad \forall t \in P, \ \forall a \in A \quad (10)$$

$$\frac{(Za(x) - Z^*a)}{Z^*a} \leq \omega \qquad \forall a \in A \quad (11)$$

$$\sum_{j \in Q} W_j u_j + \sum_{i \in S} \sum_{j \in Q} \sum_{t \in P} C_{ijt} x_{ijt} + \sum_{j \in Q} \sum_{k \in R} \sum_{t \in P} C_{jkt} x_{jkt} \leq B_a \qquad \forall a \in A \quad (12)$$

$$u_j \in \{0, 1\} \quad (13)$$

$$x_{ijt}, y_{jkt} \in Z^* \quad (14)$$

Other robust approaches (absolute robust and robust deviation) are the same as the above model considering the modification in the objective function and also omitting Constraint (11).

The first stage decision variable is the location of distribution center nodes and the second stage decision variables the products flow through each possible rout between nodes.

Describing the Mean Expected Model

We consider the mean expected value model in which each parameter is replaced by the expected value of the parameters in different scenarios to compare the results. The parameters involve in the model can be defined as follows:

$\overline{d}_{kt}$: denotes the average demand that should be satisfied at each retailer nodes $k \in R$ from each types of product $t \in P$.

$\overline{c}_{ijt}$: denotes the average transmission cost of product $t \in P$ from supplier node $i \in S$ to Distribution center node $j \in Q$.

$\overline{c}_{jkt}$: denotes the average transmission cost of product $t \in P$ from Distribution center node $j \in Q$ to retailer node $k \in R$.

$\overline{q_{it}}$: denotes the average supply capacity of product $t \in P$ in supply node $i \in S$.

$\overline{w_j}$: denotes the average fixed construction cost of possible distribution center nodes $j \in Q$.

$\overline{B}$: denotes the average maximum available budget.

n : denotes the necessary number of distribution centers.

$\overline{h_t}$: denotes average holding cost of product $t \in P$.

$\overline{A_t}$: denotes average ordering cost of product $t \in P$.

$\overline{p_t}$: denotes average price of product $t \in P$.

$\overline{I_t}$: denotes average inventory control budget constraint for product $t \in P$.

x_{ijt} : denotes the flow of product $t \in P$ from the supply node $i \in S$ to distribution center $j \in Q$.

y_{jkt} : denotes the flow of product $t \in P$ from the distribution center node $j \in Q$ to retailer node $k \in R$.

u_j : denotes the location of distribution center node $j \in Q$ and is binary decision variable. If distribution center will be located at possible distribution center node $j \in Q$, u_j =1; otherwise u_j =0.

The model can be presented as follows:

The objective function (15) minimizes total expected cost. Constraints (16)-(23) have same definitions such constraints (6)-(14) with eliminating scenarios concept, respectively.

RESULTS AND DISCUSSION

Numerical Example and Results

For better illustration of proposed approach a hypothetical example is described.

Suppose that there are five supply nodes, four possible distribution nodes, six retailer nodes. It is assumed that three types of product are going to be transferred between nodes. Figure 1 illustrates the case better.

$$\textit{Objective function}_{mean}: \text{Min } Z(x) = \sum_{j\in Q}\overline{W_j}u_j + \sum_{i\in S}\sum_{j\in Q}\sum_{t\in P}\overline{C_{ijt}}x_{ijt} + \sum_{j\in Q}\sum_{k\in R}\sum_{t\in P}\overline{C_{jkt}}x_{jkt} \quad (15)$$

s.t:

$$\sum_{i\in S}x_{ijt} = \sum_{k\in R}y_{jkt} \qquad \forall j \in Q, \forall t \in P \quad (16)$$

$$\sum_{j\in Q}y_{jkt}\ ?\ \overline{d_{kt}} \qquad \forall k \in R, \forall t \in P \quad (17)$$

$$\sum_{i\in S}x_{ijt} \le \overline{q_{it}} \qquad \forall j \in Q, \forall t \in P \quad (18)$$

$$\sum_{j\in Q}u_j \le n \quad (19)$$

$$\overline{p_t}\sum_{i\in S}\sum_{j\in Q}x_{ijt} + \frac{\sum_{k\in R}\overline{d_{kt}}\overline{A_t}}{\sum_{i\in S}\sum_{j\in Q}x_{ijt}} + \frac{\overline{h_t}\sum_{i\in S}\sum_{j\in Q}x_{ijt}}{2} \le \overline{I_t} \qquad \forall t \in P \quad (20)$$

$$\sum_{j\in Q}\overline{W_j}u_j + \sum_{i\in S}\sum_{j\in Q}\sum_{t\in P}\overline{C_{ijt}}x_{ijt} + \sum_{j\in Q}\sum_{k\in R}\sum_{t\in P}\overline{C_{jkt}}x_{jkt} \le \overline{B} \quad (21)$$

$$u_j \in \{0,1\} \quad (22)$$

$$x_{ijt}, y_{jkt} \in Z^* \quad (23)$$

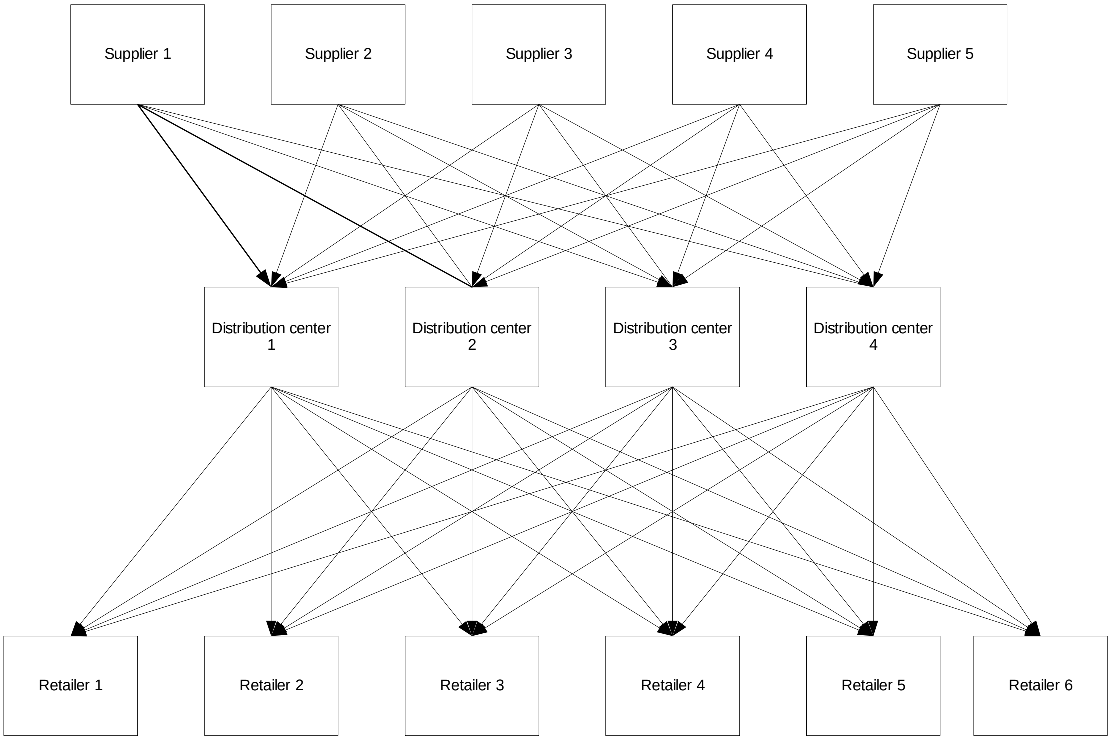

Figure 1: Location-allocation distribution centers problem

Tow robust approaches proposed in previous section, proposed limited min-max regret approach and mean expected model are applied in this section. The results are given and the comparison between the proposed robust approaches is done.

According to different situation we have three scenarios in which the demand of each retailer nodes, transportation cost, maximum budget and also inventory parameters differ. The data are given in table 1, 2.

The acceptable regret for all scenarios is 0.3. Actually the necessary number of distribution center nodes is 2. The deterministic model, robust optimization models and also mean expected value model solved by lingo 8.0.

To solve the robust models based on the robust min-max regret or robust deviation approach and proposed limited min-max regret, optimum solution for each scenario should be computed. Hence, we optimized each scenario separately and obtained the optimum decision variables which are the amount of products flow through specified routs and defining which distributor nodes are selected. The locating-allocating costs are given in table 3. The optimum objective value of deterministic model is shown by Z_a^*

The results of locating-allocating costs for tne proposed robust approach and the mean expected one as well as two other robust approaches are given in table 4.

The results show that, after obtaining the solution of the model and calculating objective function of each scenario, the results are worse than optimum cost of each scenario when they computed separately, and it is rational because we have uncertainty in parameters and amount of loss in objective owing to lack of information about probability of happening each scenario. But the point is the solutions obtained by solving the model should result in minimum loss in the cost and because of this reason robust approaches are considered. It is clear that robust approach outperform mean expected value approach. Besides, the proposed limited min-max regret approach performs better than other robust approaches, because of considering specific regret deviation. This value should be defined considering that the solution region will be feasible.

Table 1: Demands of the retailer nodes, cost of transmission between nodes and maximum budget under each scenario

		Demands of the retailer nodes						Cost of transmission between supply nodes and distribution centers	Cost of transmission between distribution centers nodes and Retailers	Maximum budget	Fixed cost for allocating the distributors nodes			
		1	2	3	4	5	6				1	2	3	4
Scenario1	product1	250	200	550	650	100	250	20	20	300000	20000	30000	35000	25000
	product2	150	100	50	100	200	50	25	25					
	product2	600	400	700	500	400	300	30	30					
Scenario2	product1	500	400	1100	1300	200	500	50	40	600000	20000	30000	35000	25000
	product2	300	200	100	200	400	100	75	25					
	product3	1200	800	1400	1000	800	600	90	60					
Scenario3	product1	125	100	225	325	50	125	70	40	500000	20000	30000	35000	25000
	product2	75	50	25	50	100	25	95	25					
	product3	300	200	350	250	200	150	50	60					

Table 2: Inventory control parameters under each scenario

	Type of product	Price cost	Holding cost	Ordering cost	Inventory budget
Scenario 1	product1	10	2	50	75000
	product2	15	1	40	65000
	Product3	14	3	30	70000
Scenario 2	product1	12	2	40	95000
	product2	16	1	35	95000
	product3	9	3	24	90000
Scenario 3	product1	9	2	45	65000
	product2	16	1	18	55000
	product3	20	3	21	60000

Table 3: Results of optimum cost of location-allocation problem for three scenarios separately

	Optimum cost of each scenario (Z_a^*)
Scenario 1	201640
Scenario 2	568000
Scenario 3	275150

Table 4: Comparison between the results of absolute robust, robust deviation, limited min-max regret approaches and mean expected value in location-allocation distributor problem

	Cost of scenario1	Cost of scenario2	Cost of scenario3
Absolute robust approach	259390	638630	436650
Robust deviation approach	250390	658000	460650
Limited min-max regret	247300	609000	426640
Mean expected approach	378910	837560	763760

Table 5 also shows comparison between the solution obtained from robust approaches and mean expected approach. As we can see, robust solutions are better than mean expected value solution. The percentage of gap between these two solutions is computed with the ratio:

(Mean expected value objective – Robust objective value) / Robust objective value×100. The computed values are given in table 5. These comparisons show efficiency of robust modeling particularly proposed limited min-max regret rather than mean expected value modeling.

The improvements obtained by proposed robust approach is greater than other robust approaches, considering all of the robust approaches outperform mean expected value model.

The sensitivity analysis based on changing the allowable regret value is given in table 6 as follows in the next page.

As it is clear from the results, if the regret coefficient increase the objective value of each scenario is increasing and this procedure continues up to the cost value of each scenario obtained by robust regret approach. Figure 2 shows the behavior of the regret coefficient better.

Table 5: Improvement in cost of each scenario by applying robust approaches

	Improvement in cost of scenario1	Improvement in cost of scenario 2	Improvement in cost of scenario 3	Average improvement
Absolute robust approach Mean expected approach	46%	31.2%	74.5%	50.56%
Robust deviation approach Mean expected approach	51.3%	27.2%	65.8%	48.1%
Limited min-max regret Mean expected approach	53.2%	37.5%	79%	56.6%

Table 6: Sensitivity analysis

	Regret coefficient	Cost of scenario1	Cost of scenario2	Cost of scenario3
1	0.3	247300	609000	426640
2	0.4	247390	609180	426750
3	0.5	247410	609300	426820
4	0.7	247450	609370	426910
5	1	247550	609420	426980

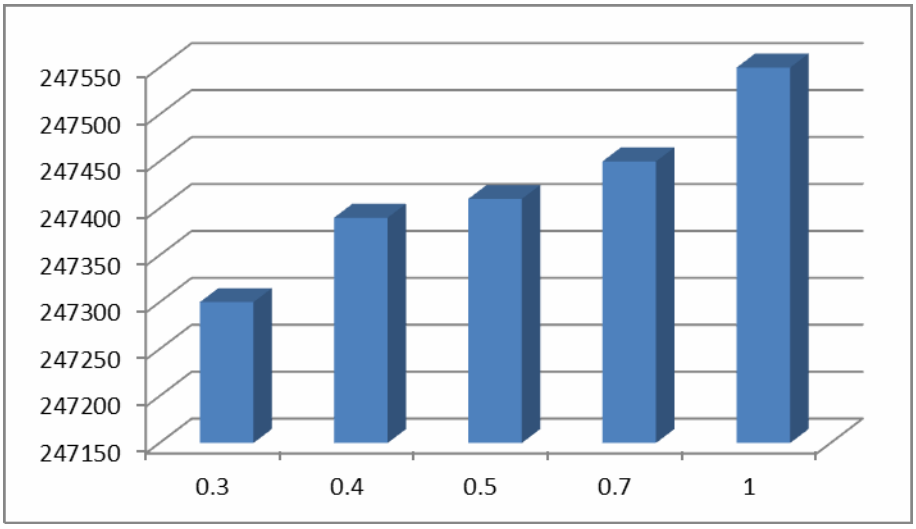

Figure 2: Sensitivity analysis of regret coefficient - Scenario 1

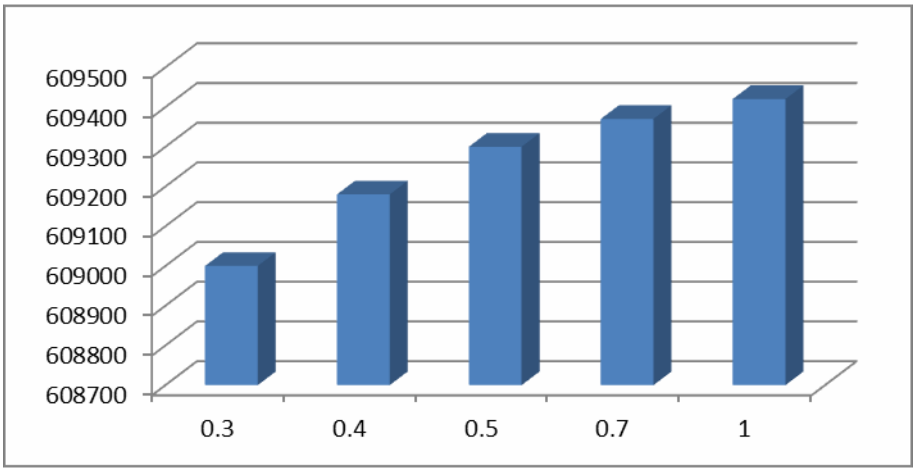

Figure 2: Sensitivity analysis of regret coefficient - Scenario 2

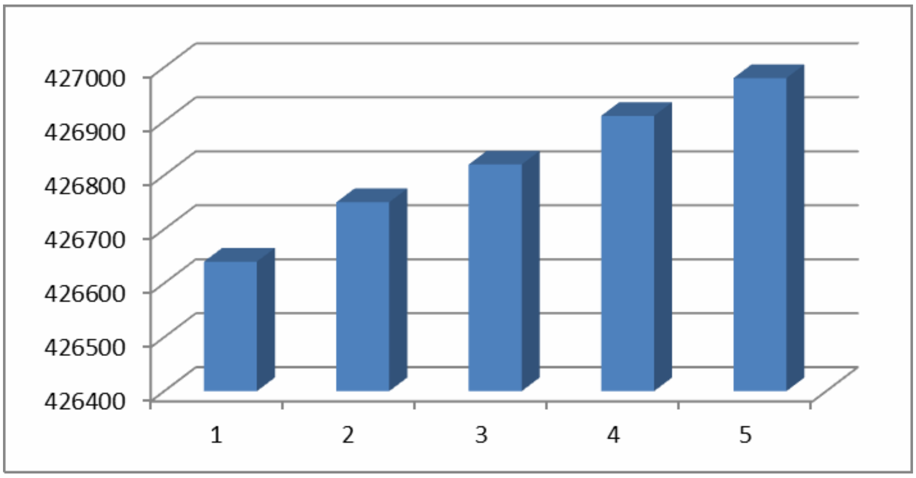

Figure 2: Sensitivity analysis of regret coefficient – Scenario 3

CONCLUSION

In this paper, we proposed a novel robust model for location-allocation problem. In the mentioned model, uncertainty of the parameters was described by discrete scenarios which the probability of each scenario occurrence was not known. For this reason, we considered limited min-max regret methodology to analyze the computational results and compared it with mean expected value model. Computational results showed effectiveness of the proposed robust modeling in comparison with other robust approaches and specially mean expected value model.

As a future research, we suggest considering more constraints that yields to more flexibility for the model and results. Also, applying this contribution in supply chain design and considering risk analysis can be an interesting area to develop the proposed model.

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