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Applying Stackelberg Game to Find the Best Price and Delivery Time Policies in Competition between Two Supply Chains

[1] *M. Fathian*, [*2] *M. Narenji*, [3] *E. Teymoori*, [4] *G. Jalali*

[1,2,3,4] *School of Industrial Engineering, Iran University of Science and Technology, Tehran, Iran*

ABSTRACT: In this paper, the competition between two supply chains and their elements is studied. Each chain consisted of a manufacturer and a distributor and the two chains compete in a market with single type of customer sensitive to price and delivery time. Therefore, this is a two-supply chain game and during the competition between two supply chains, elements of each supply chain (manufacturer and/or distributor) may follow either centralized or decentralized strategy; i.e. within each supply chain the elements may choose to cooperate or compete in order to achieve more profit. Combined strategies between two supply chains and their elements are of four types: i. both chains apply centralized policy; ii. The first chain chooses centralized and the second one follows decentralized policy; iii. The first chain applies decentralized and the second one chooses centralized policy; iv. Both chains follow decentralized policy. The competition between two chains was analyzed as a Stackelberg game and without loss of generality supposing that the first chain acts as leader and the second one is the follower. The profit earned by each supply chain is related to the aforementioned combination of strategies chosen by each supply chain. Finally numerical examples are provided to investigate these strategies and to determine the best combined strategy by comparing the profit of chain elements and whole profit of each chain.

Keywords: *Stackelberg game, Pricing, Delivery time, Competition, Decision making, Penalty cost*

INTRODUCTION

Two important aspects of service quality in supply chains are price and delivery time; therefore we can divide servicing strategies to two general categories according to the objective of the policy makers: moving toward an efficient supply chain or being a responsive supply chain. In efficient supply chains managers try to decrease the prices of products without serious care about delivery times, on the other hand in responsive supply chains decision makers concentrate on reducing the delivery times; hence, in responsive strategy, products are more expensive than in efficient strategy while products in responsive strategy are prepared faster than the efficient strategy. Nowadays, managers face the competition in these two aspects (price and delivery time) and should choose a hybrid strategy. Therefore, price and delivery time can be two determinant parameters in competitive environments. In some cases the market is monopolistic and competitions may be just between internal elements of a supply chain such as: a supplier, a producer, and a distributor; however, in other cases, the competition is between two supply chains.

This research studies a market which is sensitive to price and delivery time and each supply chain may choose to be either efficient or responsive. According to the market share and in turn profit earned by each supply chain, they choose their strategies (internal competition or corporation). Therefore, we have two supply

***Corresponding Author, Email:** *mnarenji@iust.ac.ir*

chains constantly compete with each other. The supply chains are consisted of two elements (a manufacturer and a distributor) that either corporate or compete, based on situation. In some previous researches, especially in (Pekgun, Griffin et al. 2008), there is just one supply chain with two competing elements (a manufacturer and a distributor) that choose Nash or Stackelberg equilibrium based on centralized or decentralized strategy. On the contrary, in the present study there are two supply chains with two elements (a manufacturer and a distributer) in each supply chain; therefore, there are four elements. On the other hand, in all previous studies there are just two same elements (supply chains, firms, producers, …) or two different elements (a supplier and a manufacturer, a marketing and a manufacturing department, …) that choose between centralized and decentralized strategies. In contrast to these studies, the present research has some distinctions (contribution) as follows:

- ✓ There is fundamental distinctionregarded as simultaneous competition between two chains and their elements. As mentioned before, we have two chains constantly compete with each other and each chain has two elements (a manufacturer and a distributor) which may have internal corporation or competition. When they have internal corporation the corresponding chain is called a centralized chain. Similarly, when they have internal competition the chain is defined as a decentralizedchain.
- ✓ The deviation cost is considered in our models. If each manufacturer produces the productions earlier than promised delivery time, it will incur stock cost and if it produces later than promised delivery time, it will pay delay penalty cost. Structure of deviation cost is non-linear

$$c^i(L^i, R_\lambda) = h\int_0^{L^i} (L^i - t)dR_{\lambda^i}(t) + b_s\int_{L^i}^{\infty} (t - L^i)d\,R_{\lambda^i}(t); \quad i\epsilon\{1,2\}$$

  and it is added to objective function of models.
- ✓ There is another cost considered in our models named "centralization cost". Centralization cost is a fixed cost and is used for sensitivity analysis of choosing the supply chain's strategies. In sensitivity analysis, we consider all scenarios of corporation and competition comparing them based on centralization cost in order to choose best strategy.
- ✓ Another contribution of our paper is decision making in three levels: strategic level (internal corporation or competition), tactical level (for production decisions), and operational level (for price and delivery time)

The exposition of the paper is as follows:

In section 2 consists of a review of the related literature. In section 3, the research method is defined in some details. Section 4 is dedicated to the problem formulation. Solution methods for all states which are extended to the case of competition between two chains are given in section 5. Numerical examples are provided in section 6 and finally the conclusion is given in section 7.

Literature Review

In supply chain's competition area of study, previous researches can be grouped into two categories. First group is related to monopolistic markets with customers who are sensitive to price and delivery time, consequently internal elements of a supply chain decide on price and delivery time to obtain more profit. The Second category is supply chains in a duopoly with sensitive customers to price and delivery time. In this kind of markets usually there are at least two supply chains trying to gain more market share.

(So and Song 1998) studied the effects of applying quoted delivery time as a decision policy in monopolistic markets. In their work, a strategy is chosen while customer demands are sensitive to both price and delivery time. Assuming some products which are different in price and delivery time is common in monopolistic markets where customers are sensitive to price and delivery time. (Palaka, Erlebacher et al. 1998) considered pricing decisions, capacity utilizing and delivery time setting in a supply chain servicing customers who are sensitive to price and delivery time. They developed a mathematical model to examine the behavior of a supply chain as an

M/M/1 queue. Their aim was to determine the best policy of price, delivery time and capacity utilization to maximize the profit of the whole supply chain. (Boyaci and Ray 2003) in their paper studied a supply chain with two substitutable products which differed only in price and delivery time (i.e. slower and faster products). The objective of supply chain was finding proper combination of price and delivery time to maximize the profit. Dedicated capacities were considered for each product. They developed an integrated model to generate some scenarios to decide about constrained capacity for none, one, or both products. (AfÃeche and Mendelson 2004) designed a model to select alternative price-service combinations for a supply chain in a monopolistic market. They considered penalty cost structure for delays relevant to type of servicing. (Katta and Sethuraman 2005) proposed a similar model to determine the price and to schedule the customer deliveries in a supply chain. They modeled the profit of each facility as an objective function to be maximized. (Boyaci and Ray 2006) designed a mechanism to formulate the role of capacity costs in finding the best policy in terms of maximizing the profit of the supply chain. This policy was shaped by some parameters such as cost, lead time, and delivery reliability.(Dobson and Stavrulaki 2007) investigated price, location, and capacity decisions on a line of time-sensitive customers. (Pekgun, Griffin et al. 2008) examined a supply chain serving to price and time sensitive customers. In their model, decisions of price and delivery time were made by marketing and production department, respectively. In this paper, there is just one supply chain with two elements (marketing department and manufacturing department) which named firms. Each firm can make decisions regarding its price and delivery time through Nash or Stackelberg equilibrium, based on centralized or decentralized strategy. Inefficiency is a parameter calculated by some researchers in order to modeling the decentralized decision making. (Pekgun, Griffin et al. 2008) also calculated the inefficiency in their paper and showed that demand is larger, delivery time is longer, price is lower, and profit is smaller in decentralized case compared with centralized decision making. (Pangburn and Stavrulaki 2008) studied capacity and price settings for dispersed, time-sensitive customer segments and (Sinha, Rangaraj et al. 2010) investigated pricing and server capacity for mean waiting time sensitive customers.

All the aforementioned papers in the literature studied the monopolistic conditions. However, there are some researchers who investigated competitive conditions in which two supply chains decide on the optimal policy. In such cases, managers decide on price and delivery time to achieve larger market shares. There are some ways for choosing best strategies of each chain. Sometimes both chains decide simultaneously; however, in some cases the policy making process is stepwise. In these cases, there are a leader and a follower. (Li and Lee 1994) studied Pricing and delivery-time performance in a competitive environment. (Lederer and Li 1997) considered Pricing, production, scheduling, and delivery-time competition. In a research by (So 2000), a decision model is developed to examine the effects of using quoted delivery time on competition. Like other researchers, he assumed that demands were sensitive to price and delivery time and the objective function was maximizing the operation profit. At first, he developed models for each supply chain separately and then extended the models to include two supply chains in competition mode. (Allon and Federgruen 2006) studied competition in service industries. (Pekgun, Griffin et al. 2006) analyzed two supply chains competing based on price and delivery time in a common market with common services. They examined the impact of centralization of decisions and compared some scenarios in which none, one or both of supply chains are decentralized. (Jayaswal, Jewkes et al. 2010) analyzed a supply chain serving two different types of products, differentiated in price and delivery time, in a market with two kinds of customers. The chains are assumed to be able to choose between dedicated or shared capacity in operational level and substituted products to obtain a larger share of the market. The aim was finding the best strategy for production capacity and price to maximize the whole profit of chain. (Teimoury, Modarres et al. 2011) investigated price, delivery time, and capacity decisions in an M/M/1 make-to-order/service system with segmented market. Many applications and methods for selecting best strategy of pricing and delivery time decision making are investigated in(Cachon 2003), (Wang, Jiang et al. 2004), (Liu, Parlar et al. 2007), (Xiao and Qi 2012).

In this paper, not only there are two supply chains in a duopoly market competing with each other to gain more market share, but also there is an internal competition between each supply chain's elements to achieve more profit.

RESEARCH METHOD

In this section, decision tree and four states of chains strategies is described according to their collaborations and competitions. And then, each strategy is mathematically formulated and the solution procedure is developed.

Decision Tree

To demonstrate how these two chains make decisions, decision trees are utilized. The first chain chooses its strategy of being centralized or decentralized, and then second one chooses its strategy consequently to find best policy of price and delivery time. $\mathrm{S}_\mathrm{j}^\mathrm{i}$, indicates j^{th} strategy of i^{th} chain. Therefore we will have four scenarios as figure 1.

As can be seen, we have two supply chains competing based on their individual strategies, therefore, four combined strategies are derived for the competition:

(1) $\mathrm{S}_\mathrm{c}^1 - \mathrm{S}_\mathrm{c}^2$

(2) $\mathrm{S}_\mathrm{c}^1 - \mathrm{S}_\mathrm{Dec}^2$

(3) $\mathrm{S}_\mathrm{Dec}^1 - \mathrm{S}_\mathrm{c}^2$

(4) $\mathrm{S}_\mathrm{Dec}^1 - \mathrm{S}_\mathrm{Dec}^2$

We have a Stackelberg game with two players in which the first chain is the leader and the second one is the follower.

In what follows, these combined strategies are studied and analyzed to find the best strategy.

Assumptions

Our assumptions are as follows:

- ✓ There is just one type of customers and each customer can buy from one chain.
- ✓ There is just one type of products provided by each chain for customers.
- ✓ Each chain consists of two elements: a manufacturer and a distributer.
- ✓ Customers demands are random variables which follow the exponential distribution with rate λ.
- ✓ The service facilities in centralized or decentralized strategies are modeled as queuing system ($\frac{\frac{\mathrm{M}}{\mathrm{M}}}{1}$ queue). The service time of chain i for demand is exponentially distributed with rate μ_i. Without losing generality we assume that both service rates are equal, i.e. $\mu_1 = \mu_2 = \mu$.
- ✓ Customer's demands are received by chain i according to a Poisson process with rate λ_i that depends on (1) its own price and delivery time and (2) price and delivery time of the other chain.
- ✓ In each chain customers are served based on FCFS priority discipline.
- ✓ There is penalty cost and each supply chain should pay penalty cost to customers for delay.
- ✓ Customers are sensitive to price and delivery time and they choose one chain according to these two criteria.

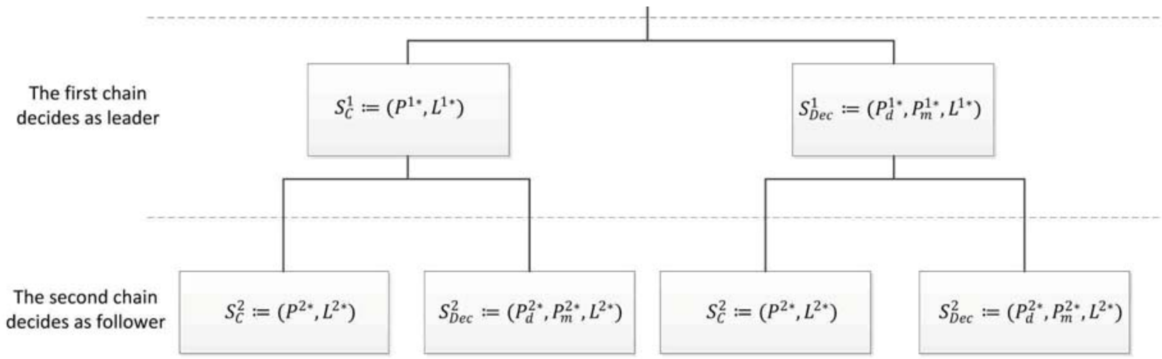

Figure 1: Decision tree for competition between two chains

Problem Formulation

Customers are sensitive to price and delivery time. Therefore, demand rate can be written as follows:

$$\lambda^i = \alpha^i - \beta_p P^i + \gamma_p (P^j - P^i) - \beta_L L^i + \gamma_L (L^j - L^i); \quad i\epsilon\{1,2\}, 3 - i \quad (1)$$

$$c^i(L^i, R_\lambda) = h \int_0^{L^i} (L^i - t) dR_{\lambda^i}(t) + b_s \int_{L^i}^{\infty} (t - L^i) dR_{\lambda^i}(t); \; i\epsilon\{1,2\} \quad (2)$$

Where λ^i and c^i are the demand rate of customers and the penalty cost of chain *i* , respectively. Other parameters and variables are as follows:

Parameters

α^i : Whole potential of the market for chain *i* ; $i\epsilon\{1,2\}$

$\beta_p >$: Sensitivity of market demand to its own price

$\beta_L >$: Sensitivity of market demand to its own guaranteed delivery time

$\gamma_p \geq$: Sensitivity of market demand to inter-chain price

$\gamma_L \geq$: Sensitivity of market demand to inter-chain delivery time

α^i :Service level of chain *i*; $i\epsilon\{1,2\}$

m_n^i : Operational cost of chain *i* that is spent by manufacturer; $i\epsilon\{1,2\}$

m_d^i : Advertising cost of chain *i* that is spent by distributor; $i\epsilon\{1,2\}$

A^i : Capacity cost of chain *i*; $i\epsilon\{1,2\}$

W^i : Whole expected time which is spent in chain *i* in steady state; $i\epsilon\{1,2\}$

$\emptyset^i$: Centralization cost for chain *i*, $i\epsilon\{1,2\}$

h : The Tardiness cost per unit per unit time

b_s : The holding cost per unit per unit time

R_{λ^i} : The distribution function of the realized delivery time for a given λ^i; $i\epsilon\{1,2\}$.

In this paper we assumed that $R_\lambda(x) = 1 - e^{-(\mu-\lambda)x}$

Variable

P^i : The price which proposed by centralized chain *i* to customers; $i\epsilon\{1,2\}$

P_d^i : The price which proposed by distributor to customers in decentralized chain *i*; $i\epsilon\{1,2\}$

P_m^i : The price which proposed by manufacturer to distributor in decentralized chain *i*; $i\epsilon\{1,2\}$

L^i : The delivery time proposed by chain *i* to customers; $i\epsilon\{1,2\}$

λ^i : Average demand rate of customers from chain *i*, $i\epsilon\{1,2\}$

μ^i : Average service rate of chain *i*, $i\epsilon\{1,2\}$

Profit Functions

$\prod_{Mnf}^{i}$:Profit of manufacturer in decentralized chain *i*; $i\epsilon\{1,2\}$

$\prod_{Dst}^{i}$: Profit of distributor in decentralized chain *i*; $i\epsilon\{1,2\}$

$\prod^{i}$: whole profit of chain *i*; $i\epsilon\{1,2\}$

Formulation

It can be seen in figure 2 that there is inter-collaboration between manufacturer (Mnf) and distributor (Dst) in the centralized supply chain. In the decentralized supply chain there is an inter-competition between manufacturer and distributor, like figure 3.Therefore, we have to solve two models as follows:

$PDTDM_c^i$ (Price and Delivery Time Decision Model with centralized elements in chain *i*).

$$\max_{P^i, L^i}$$

$$\Pi^i = \left(P^i - m_d^i - C^i(L^i, R_\lambda)\right)\lambda^i - A^i \mu^i - \emptyset^i \quad (3)$$

Subject to: $S^i(L^i) = P(W^i \leq L^i) \geq \alpha$ (4)

$P^i, L^i \geq 0$ (5)

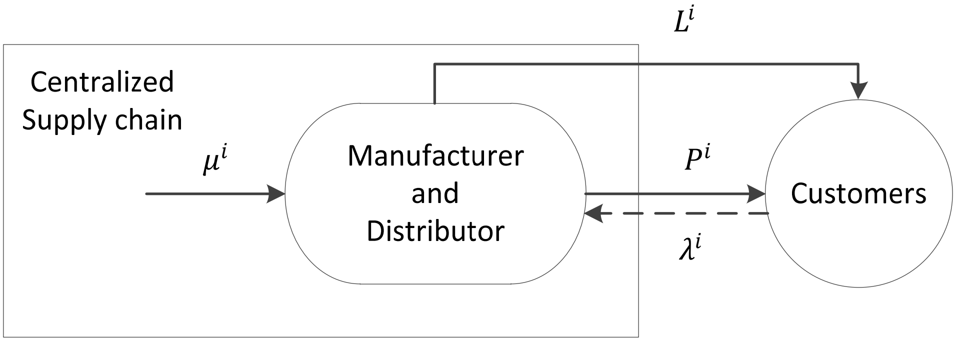

Figure 2: Centralized supply chain with collaboration between manufacturer and distributor

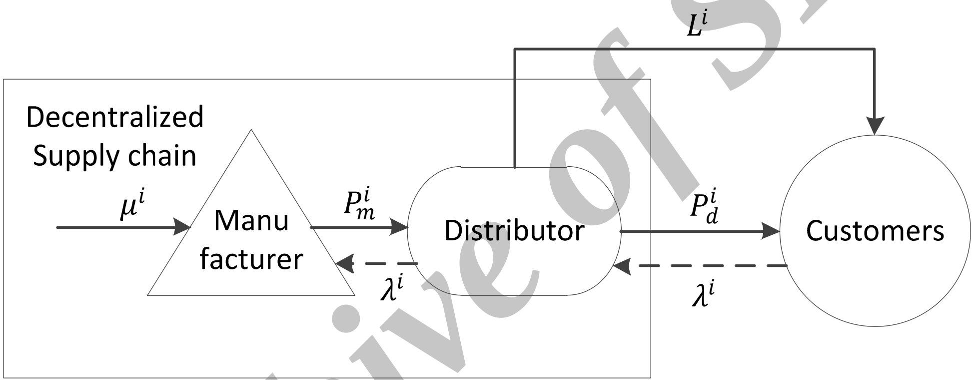

Figure 3: Decentralized supply chain with a competition between manufacturer and supplier

Where equation (3) demonstrates the objective function of the chain *i*. λ^i and C^i were given by (1) and (2), respectively. Constraint (4) shows delivery time reliability where the probability of the delivery time which is proposed by the chain being longer than the whole expected time spent in the chain is longer than the service level of the combination of manufacturer and distributor, α. Constraint (5) represents that the variables are non-negative.

$\mathrm{PDTDM_c^i}$ (Price and Delivery Time Decision Model with decentralized element sin chain *i*). (Figure 3)

$$\max_{P_m^i, L^i} \prod\nolimits_{Mnf}^{i} = \left(P_m^i - m_m^i - C^i(L^i, R_\lambda)\right)\lambda^i - A^i \mu^i - \emptyset^i \tag{6}$$

$$\text{Subject to:} \quad S^i(L^i) = P(W^i \le L^i) \ge \alpha \tag{7}$$

$$P_m^i, L^i \ge 0 \tag{8}$$

$$\max_{P_d^i} \prod\nolimits_{Dst}^{i} = \left(P_d^i - P_m^i - m_d^i\right)\lambda^i \tag{9}$$

$$\text{Subject to:} \quad P_d^i, P_m^i \ge 0 \tag{10}$$

Where equation (6) is the objective function of manufacturer which λ^i and C^i were given by equation (1) and (2), respectively. Similar to inequality (4), constraint (7) is delivery time reliability with a service level equal to α.

Constraint (8) shows non-negative variables. Equation (9) defines the objective function of distributor and constraint (10) shows non-negative variables.

We assume that each customer is served by a server in an $\frac{M}{\frac{M}{1}}$ queue system and the tail of the sojourn time distribution is known to be exponential. Therefore we can rewrite constraints (4) and (7) as follow:

$$S^i(L^i) = P(W^i \leq L^i) = 1 - e^{(\lambda^i - \mu^i)L^i} \geq \alpha$$

Solution Method and Results

Solution methods for two basic models which are introduced developed in this section:

Solution Method for $PDTDP_c^i$

The same model is studied by (Boyaci and Ray 2003) and (Jayaswal, Jewkes et al. 2010). Inspired by their solution methods , this study developed a solution which is concisely presented in the following lines.

The profit function, $\prod^i$, is decreasing function depend on service rate, μ^i. Therefore, to maximize profit, service rate should be at its minimum, i.e $\mu^i = \lambda^i - 1n\frac{1-\alpha}{L^i}$. By applying the amount of μ^i the objective function of $PDTDP_c^1$ can be written as follows:

$$\underset{P^i, L^i}{\text{Max}}$$

$$\prod{}^i = \left(P^i - m_m^i - m_d^i - A^i - C^i(L^i, R_\lambda)\right)\lambda^i + A^i \ln\frac{1-\alpha}{L^i} \tag{11}$$

$$P^i, L^i \geq 0$$

Proposition 1.The profit function in (11) is strictly convex in p^i, L^i.

Proof 1.See Appendix A.

Proposition 2.For given L^{i*}, the optimum price, P^{i*},is given by (12) as follow:

(12)

$$P^{i*} = \frac{1}{2\{\frac{\alpha^i}{\beta_p} - \frac{\beta_L}{\beta_p}L^i + \left(m_m^i - m_d^i - A^i + C^i\right)\}}$$

And the optimum guaranteed delivery time, L^{i*}, is given by unit root of:

(1)

$$\left({}^1/_2\right)[(\alpha h - (1-\alpha)b_s)\beta_P + \beta_L]\left[{}^{a^i}/_{\beta_P} - \left[{}^{\beta_L}/_{\beta_P} + h\left(1 + \frac{\alpha}{\ln(1-\alpha)}\right) - b_s\frac{(1-\alpha)}{\ln(1-\alpha)}\right]L^{i*} - \left(m_m^i + m_d^i + A^i\right)\right] + A^i \ln(1-\alpha)/{L^{i*}}^2 = 0$$

Proof 2. See appendix A

Solution Method for $PDTDP_{Dec}^i$

In this case there are two objective functions, $\prod_{Mnf}^i$ and $\prod_{Dst}^i$, which should be optimized. The profit function of manufacturer is decreasing in service rate, μ_h^i. Therefore, to maximize profit function of manufacturer, $\prod_{Mnf}^i$, service rate should be at its minimum level that guarantees the desired service level α. This implies that at optimality, the service rates are given by $\mu_h^i = \lambda_h^i - \ln\frac{1-\alpha}{L_h^i}$. Substituting μ_h^i with its obtained equivalent, the objective function of the manufacturer can rewrite as follows:

$$\underset{P_m^i, L^i}{\text{Max}}$$

$$\prod{}_{Mnf}^i = (P_m^i - m_m^i - C^i(L^i, R_\lambda) - A^i)\lambda_h^i + A^i \ln\frac{1-\alpha}{L^i} \tag{14}$$

Proposition 3.Profit functions of (6) and (9) are strictly convex in P_m^i, P_d^i, L^i.

Proof 3. See Appendix A.

Proposition 4.For given P_m^i and L^i, optimum price of distributor, P_d^{i*}, is given by (15) as follows:

(15)

$$P_d^{i*} = \frac{1}{2\{\frac{\alpha^i}{\beta_p} - \frac{\beta_L}{\beta_p}L^i + P_m^i + m_d^i\}}$$

Proof 4. See Appendix A

Proposition 5. The optimum price of manufacturer, P_m^{i*}, is given by (16) as follow:

(16)

$$P_m^{i*} = \frac{1}{2\{\frac{\alpha^i}{\beta_p} - \frac{\beta_L}{\beta_p} L^i - \left(m_d^i - m_m^i - A^i - C^i\right)\}}$$

And the optimum guaranteed delivery time, L^{i*}, is given by unit root of (17) as follow:

(17)

$$\left(\frac{1}{4}\right)\left[\left((h + b_s)R_\lambda(L^i) - b_s\right)\beta_p + \beta_L\right] \left[\frac{\alpha^i}{\beta_p} - \frac{\beta_L}{\beta_p} L^{i*} - \left(m_d^i + m_m^i + A^i + C^i\right)\right] + A^i \ln \frac{1-\alpha}{([L^{i*})]^2} = 0$$

Proof 5. See appendix A

Competition between Two Supply Chains

In the aforementioned four scenarios the first chain is the leader and the second one is the follower. In each scenario the first Chain makes its decision in an isolated environment, and then the second chain chooses its best policy according to first chain's policy. By using the concepts and propositions in previous section, the best policy for each scenario is as follows:

First Scenario: $\{\boldsymbol{S_C^1\ Vs.\ S_C^2} \coloneqq (\boldsymbol{L^{1*}, P^{1*}, L^{2*}, P^{2*}})\}$

In first scenario, both chains follow centralized strategy and their formulations are as table 1.

Best policy of two chain extracted by solution of two above models. In table 2 best policy of two chains are illustrated.

Table 1: Formulation of two chains in first scenario

Formulation of the first chain (leader)	Formulation of the second chain (follower)
$\boldsymbol{\max_{P^1,L^1} \Pi^1 = \left(P^1 - m_m^1 - m_d^1 - C^1(L^1, R_\lambda)\right)\lambda^1 - A^1\mu^1 - \phi^1}$	$\max_{P^2,L^2} \Pi^2 = (P^2 - m_m^2 - m_d^2 - C^2(L^2, R_\lambda))\lambda^2 - A^2\mu^2 - \phi^2$
$\boldsymbol{S.t{:}\quad S^1(L^1) = P(W^1 \le L^1) \ge \alpha}$	$S.t{:}\quad S^2(L^2) = P(W^2 \le L^2) \ge \alpha$
$\boldsymbol{P^1,\ L^1 \ge 0}$	$P^2,\ L^2 \ge 0$

Table 2: Solution of two chains in first scenario

Solution of the first chain (leader)	Solution of the second chain (follower)
$\boldsymbol{\left(\frac{1}{2}\right)[(\alpha h - (1-\alpha)b_s)\beta_P + \beta_L]\left[a^1/\beta_P - \left[\beta_L/\beta_P + h\left(1 + \frac{\alpha}{\ln(1-\alpha)}\right) - b_s\frac{(1-\alpha)}{\ln(1-\alpha)}\right]L^1 - \left(m_m^1 + m_d^1 + A^1\right)\right] + A^1\, ln(1-\alpha)\Big/(L^1)^2 = 0}$	$\left(\frac{1}{2}\right)[(\alpha h - (1-\alpha)b_s)(\beta_P + \gamma_P) + (\beta_L + \gamma_L)] \times \left[\frac{a^2}{(\beta_P+\gamma_P)} - \left(\frac{(\beta_L+\gamma_L)}{(\beta_P+\gamma_P)} + \left(1 + \frac{\alpha}{\ln(1-\alpha)}\right)h - (\frac{1-\alpha}{\ln(1-\alpha)})b_s\right)L^2 + \frac{(\gamma_P P^1 + \gamma_L L^1)}{(\beta_P+\gamma_P)} - (m_m^2 + m_d^2 + A^2)\right] + A^2\, ln(1-\alpha)\Big/(L^2)^2 = 0$
$\boldsymbol{P^{1*} = 1/2\,\{a^1/\beta_P - \beta_L/\beta_P\, L^1 + \left(m_m^1 + m_d^1 + A^1 + C^1\right)\}}$	$P^{2*} = 1/2\left\{\frac{a^2}{(\beta_P+\gamma_P)} - \frac{(\beta_L+\gamma_L)}{(\beta_P+\gamma_P)}L^2 + \frac{(\gamma_P P^1 + \gamma_L L^1)}{(\beta_P+\gamma_P)} + (m_m^2 + m_d^2 + A^2 + C^2)\right\}$

Second Scenario:

$\{\mathbf{S_C^1}\ \mathbf{Vs.}\ \mathbf{S_{Dec}^2} \coloneqq (\mathbf{L^{1*}}, \mathbf{P^{1*}}, \mathbf{L^{2*}}, \mathbf{P_m^{2*}}, \mathbf{P_d^{2*}})\}$

In second scenario, the first chain follows centralized strategy the second one follows decentralized strategy, and their formulations are as table 3.

Best policies of two chains are extracted by solution of two above models. Best policies of two chains are illustrated in table 4.

Third Scenario:

$\{\mathbf{S_{Dec}^1}\ \mathbf{Vs.}\ \mathbf{S_C^2} \coloneqq (\mathbf{L^{1*}}, \mathbf{P_m^{1*}}, \mathbf{P_d^{1*}}, \mathbf{L^{2*}}, \mathbf{P^{2*}})\}$

In third scenario, the first chain follows decentralized strategy and the second one follows centralized strategy. Formulations of two chains are as table 5.

Table 3: Formulation of two chains in second scenario

Formulation of the first chain (leader)	Formulation of the second chain (follower)
$\max_{P^1,L^1} \Pi^1 = (P^1 - m_m^1 - m_d^1 - C^1(L^1, R_\lambda))\lambda^1 - A^1\mu^1 - \phi^1$ $S.t: \quad S^1(L^1) = P(W^1 \le L^1) \ge \alpha$ $P^1,\ L^1 \ge 0$	$\max_{P_m^2,L^2} \Pi_{Mnf}^2 = (P_m^2 - m_m^2 - C^2(L^2, R_\lambda))\lambda^2 - A^2\mu^2$ $S.t: \quad S^2(L^2) = P(W^2 \le L^2) \ge \alpha$ $P_m^2,\ L^2 \ge 0$ $\max_{P_d^2} \Pi_{Dst}^2 = (P_d^2 - P_m^2 - m_d^2)\lambda^2$ $P_d^2, P_m^2 \ge 0$

Table 4: Solution of two chains in second scenario

Solution of the first chain (leader)	Solution of the second chain (follower)
$\left(\frac{1}{2}\right)[(\alpha h - (1-\alpha)b_s)\beta_P + \beta_L]\left[{a^1}/{\beta_P} - \left[{\beta_L}/{\beta_P} + h\left(1 + \frac{\alpha}{\ln(1-\alpha)}\right) - b_s\frac{(1-\alpha)}{\ln(1-\alpha)}\right]L^1 - (m_m^1 + m_d^1 + A^1)\right] + A^1\,{\ln(1-\alpha)}/{(L^1)^2} = 0$ $P^{1*} = {1}/{2}\{{a^1}/{\beta_P} - {\beta_L}/{\beta_P}L^1 + (m_m^1 + m_d^1 + A^1 + C^1)\}$	$\left(\frac{1}{4}\right)\left[(\alpha h - (1-\alpha)b_s)(\beta_p + \gamma_p) + \left(\frac{1}{2}\right)(\beta_L + \gamma_L)\right] \times \left[\frac{a^2}{(\beta_p+\gamma_p)} - \frac{(\beta_L+\gamma_L)}{(\beta_p+\gamma_p)}L^2 + \frac{(\gamma_P P^1 + \gamma_L L^1)}{(\beta_p+\gamma_p)} - (m_d^2 + m_m^2 + A^2 + C^2)\right] + A^2\frac{\ln(1-\alpha)}{(L^2)^2} = 0$ $P_m^{2*} = \frac{1}{2}\left\{\frac{a^2}{(\beta_p+\gamma_p)} - \frac{(\beta_L+\gamma_L)}{(\beta_p+\gamma_p)}L^2 + \frac{(\gamma_P P^1 + \gamma_L L^1)}{(\beta_p+\gamma_p)} - (m_d^2 - m_m^2 - A^2 - C^2)\right\}$ $P_d^{2*} = {1}/{2}\left\{\frac{a^2}{(\beta_p+\gamma_p)} - \frac{(\beta_L+\gamma_L)}{(\beta_p+\gamma_p)}L^2 + \frac{(\gamma_P P^1 + \gamma_L L^1)}{(\beta_p+\gamma_p)} + (P_m^2 + m_d^2)\right\}$

Table 5: Formulation of two chains in third scenario

Formulation of the first chain (leader)	Formulation of the second chain (follower)
$\max_{P_m^1,L^1} \Pi_{Mnf}^1 = (P_m^1 - m_m^1 - C^1(L^1, R_\lambda))\lambda^1 - A^1\mu^1$ $S.t: \quad S^1(L^1) = P(W^1 \le L^1) \ge \alpha$ $P_m^1,\ L^1 \ge 0$ $\max_{P_d^1} \Pi_{Dst}^1 = (P_d^1 - P_m^1 - m_d^1)\lambda^1$ $P_d^1,\ P_m^1 \ge 0$	$\max_{P^2,L^2} \Pi^2 = (P^2 - m_m^2 - m_d^2 - C^2(L^2, R_\lambda))\lambda^2 - A^2\mu^2 - \phi^2$ $S.t: \quad S^2(L^2) = P(W^2 \le L^2) \ge \alpha$ $P^2,\ L^2 \ge 0$

Chains best policies are illustrated by solution of two above models as table 6.

Fourth Scenario:
$\{S^1_{Dec}\ Vs.\ S^2_{Dec} \coloneqq (L^{1*}, P^{1*}_m, P^{1*}_d, L^{2*}, P^{2*}_m, P^{2*}_d)\}$

In forth scenario, both chains follow decentralized strategy and formulations of them are as table 7.

Chains best policies are extracted by solution of models. Results are shown in table 8 as follows:

Table 6: Solution of two chains in third scenario

Solution of the first chain (leader)	Solution of the second chain (follower)
$(\frac{1}{4})\left[\left((h+b_s)R_\lambda(L^1)-b_s\right)\beta_P+\beta_L\right]\left[\frac{a^1}{\beta_P}-\frac{\beta_L}{\beta_P}L^{1*}-\left(m^1_d+m^1_m+A^1+C^1\right)\right]+A^1\ln(1-\alpha)/(L^{1*})^2=0$	$\left(\frac{1}{2}\right)\left[\left((h+b_s)R_\lambda(L^2)-b_s\right)(\beta_P+\gamma_P)+(\beta_L+\gamma_L)\right]\left[\frac{a^2}{\beta_P+\gamma_P}+\frac{\gamma_P P^1_d+\gamma_L L^1}{\beta_P+\gamma_P}-\frac{\beta_L+\gamma_L}{\beta_P+\gamma_P}L^{2*}-(m^2_m+m^2_d+A^2+C^2)\right]+A^2\ln(1-\alpha)/(L^{2*})^2=0$
$P^{1*}_m=1/2\left\{\frac{a^1}{\beta_P}-\frac{\beta_L}{\beta_P}L^1-\left(m^1_d-m^1_m-A^1-C^1\right)\right\}$	$P^{2*}=1/2\{\frac{a^2}{\beta_P+\gamma_P}-\frac{\beta_L+\gamma_L}{\beta_P+\gamma_P}L^2+\frac{\gamma_P P^1_d+\gamma_L L^1}{\beta_P+\gamma_P}+(m^2_m+m^2_d+A^2+C^2)\}$
$P^{1*}_d=1/2\left\{\frac{a^1}{\beta_P}-\frac{\beta_L}{\beta_P}L^1+P^1_m+m^1_d\right\}$	

Table 7: Formulation of two chains in forth scenario

Formulation of the first chain (leader)	Formulation of the second chain (follower)
$\max_{P^1_m,L^1}\Pi^1_{Mnf}=(P^1_m-m^1_m)\lambda^1-A^1\mu^1$ $S.t$: $S^1(L^1)=P(W^1\le L^1)\ge\alpha$ $P^1_m,\ L^1\ge 0$	$\max_{P^2_m,L^2}\Pi^2_{Mnf}=(P^2_m-m^2_m)\lambda^2-A^2\mu^2$ $S.t$: $S^2(L^2)=P(W^2\le L^2)\ge\alpha$ $P^2_m,\ L^2\ge 0$
$\max_{P^1_d}\Pi^1_{Dst}=\left(P^1_d-P^1_m-m^1_d\right)\lambda^1$ $P^1_d,\ P^1_m\ge 0$	$\max_{P^2_d}\Pi^2_{Dst}=(P^2_d-P^2_m-m^2_d)\lambda^2$ $P^2_d,\ P^2_m\ge 0$

Table 8: Solution of two chains in forth scenario

Solution of the first chain (leader)	Solution of the second chain (follower)
$\left(\frac{1}{4}\right)\left[\left((h+b_s)R_\lambda(L^1)-b_s\right)\beta_P+\beta_L\right]\left[\frac{a^1}{\beta_P}-\frac{\beta_L}{\beta_P}L^{1*}-\left(m^1_d+m^1_m+A^1+C^1\right)\right]+A^1\ln(1-\alpha)/(L^{1*})^2=0$	$\left(\frac{1}{4}\right)\left[\left(\alpha h-(1-\alpha)b_s\right)\left(\beta_p+\gamma_p\right)+(\beta_L+\gamma_L)\right]\times\left[\frac{a^2}{(\beta_p+\gamma_p)}-\frac{(\beta_L+\gamma_L)}{(\beta_p+\gamma_p)}L^2+\frac{(\gamma_P P^1+\gamma_L L^1)}{(\beta_p+\gamma_p)}-(m^2_d+m^2_m+A^2+C^2)\right]+A^2\frac{\ln(1-\alpha)}{(L^2)^2}=0$
$P^{1*}_m=1/2\left\{\frac{a^1}{\beta_P}-\frac{\beta_L}{\beta_P}L^1-\left(m^1_d-m^1_m-A^1-C^1\right)\right\}$	$P^{2*}_m=\frac{1}{2}\left\{\frac{a^2}{(\beta_p+\gamma_p)}-\frac{(\beta_L+\gamma_L)}{(\beta_p+\gamma_p)}L^2+\frac{(\gamma_p P^1+\gamma_L L^1)}{(\beta_p+\gamma_p)}-(m^2_d-m^2_m-A^2-C^2)\right\}$
$P^{1*}_d=1/2\left\{\frac{a^1}{\beta_P}-\frac{\beta_L}{\beta_P}L^1+P^1_m+m^1_d\right\}$	$P^{2*}_d={}^1/_2\left\{\frac{a^2}{(\beta_p+\gamma_p)}-\frac{(\beta_L+\gamma_L)}{(\beta_p+\gamma_p)}L^2+\frac{(\gamma_P P^1+\gamma_L L^1)}{(\beta_p+\gamma_p)}+(P^2_m+m^2_d)\right\}$

Numerical Example

We consider a numerical example to compare the results of the optimal decisions for each strategy. We assume that parameters of customers are as table 9.

And the parameters of chains are as table 10.

By formulating and solving four aforementioned strategies by MATLAB 2008, variables of the first and second chain for each state extracts as table 11 and table 12 respectively:

Table 9: Parameters of customers

β_P	β_L	γ_p	γ_L	a	α	h	b_s
0.5	0.9	0.4	0.6	15	0.99	0.10	0.30

Table 10: Parameters of two chains

A^1	A^2	m_m^1	m_m^2	m_d^1	m_d^2	$\emptyset^1$	$\emptyset^2$
1.00	1.00	2.90	2.90	0.10	0.10	2.00	3.50

Table 11: Variables of the first chain

Scenarios	P^1	P_m^1	P_d^1	L^1	λ^1	μ^1	Π_{Dst}^1	Π_{Mnf}^1	Π^1
1	16.4765	-	-	0.6256	6.1987	13.5599	-	-	67.6700
2	16.4765	-	-	0.6256	6.1987	13.5599	-	-	67.6700
3	-	16.1322	22.3129	0.8924	3.0404	8.2009	18.4879	31.8152	50.3031
4	-	16.1322	22.3129	0.8924	3.0404	8.2009	18.4879	31.8152	50.3031

Table 12: Variables of the second chain

Scenarios	P^2	P_m^2	P_d^2	L^2	λ^2	μ^2	Π_{Dst}^2	Π_{Mnf}^2	Π^2
1	13.7697	-	-	0.5463	8.7538	17.1834	-	-	73.2137
2	-	13.4835	18.3444	0.7809	4.2848	10.1823	20.3991	34.9007	55.2998
3	15.1841	-	-	0.5104	10.0293	19.0522	-	-	99.2410
4	-	14.9113	20.4882	0.7280	4.9292	11.2547	26.9964	47.6673	74.6637

From the numerical results it can be concluded that generally speaking, having centralized strategy is better than following a decentralized strategy, if centralization cost be at its low level. As can be seen, in the first scenario the first chain has more profit than the second one, and it is vice versa in the second scenario. In according to Stackelberg rules, the first chain is leader, and it is better for it to choose centralized strategy, and it obtains profit as 67.6700 units. Therefore the second chain will choose centralized strategy, as follower with profit of 73.2137 units.

In third and fourth scenarios, the second chain is winner and gives more profit than the first chain. But if in any reason the first chain chooses the decentralized strategy (with 50.3031 unit profit), it is better for the second chain to choose the centralized strategy with 99.2410 unit profit.

In the table13, differences of profit sin combined strategies of two chains are presented.

CONCLUSION

In this paper we studied the combined strategies for competition between two supply chains and their elements. The supply chains are similar, but they are different in suggested price and delivery time and both of them sell one product and compete in a common market where there is no other competitor. In our developed models, two supply chains compete under Stackelberg game. The first chain plays as leader and the second one plays as follower. To achieve more profit, the first chain makes its decision in an isolated environment and then, the other one chooses its strategy based on the first chain's strategy. We defined four scenarios and for each, we developed and solved a mathematical model. Finally a numerical example is analyzed to show that each strategy has its own effect on the chains. We found out that choosing centralized policy needs collaboration between elements of chain and it has extra costs (ϕ^i) for the whole chain.

Therefore, one parameter that directly affects on choosing better strategy is centralization cost. It means if the centralization cost of a chain be equal to zero it is better to chooses the centralized strategy. But in the real world usually there is centralization cost for collaboration of chains elements. Therefore, when the centralization cost raises according to environmental changes maybe supply chain and their elements decided to change the whole strategy of the chain. In this example it seems that the turning point for the first chain occurs in $\emptyset^1 = 19$ (figure 4).

As can be seen, without any attention to the second chain, the first chain for sustain its profit, should changes its strategy when the centralization cost exceed from 19. By considering the second chain and its strategy we can demonstrate that the first chains action is not the best decision (figure 5).

Table 13: Profits in combined strategies of two chains

		Second chain	
		Centralized Str.	**Decentralized Str.**
First chain	Centralized Str.	$(\Pi_C^{1*}, \Pi_C^{2*}) = (67.6700, 73.2137)$	$(\Pi_C^{1*}, \Pi_{Dec}^{2*}) = (67.6700, 55.2998)$
	Decentralized Str.	$(\Pi_{Dec}^{1*}, \Pi_C^{2*}) = (50.3031, 99.2410)$	$(\Pi_{Dec}^{1*}, \Pi_{Dec}^{2*}) = (50.3031, 74.6637)$

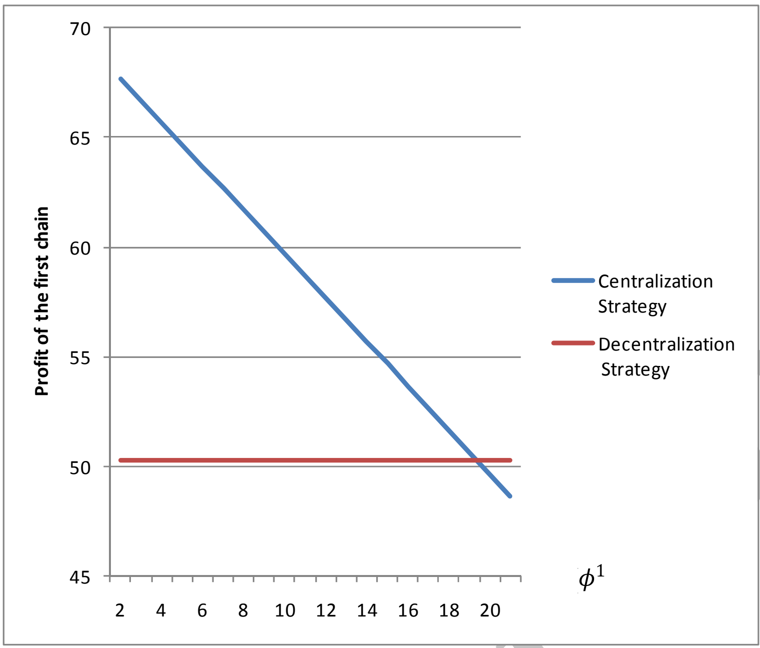

Figure 4: Turning point for the strategy of the first chain without attention to second chain

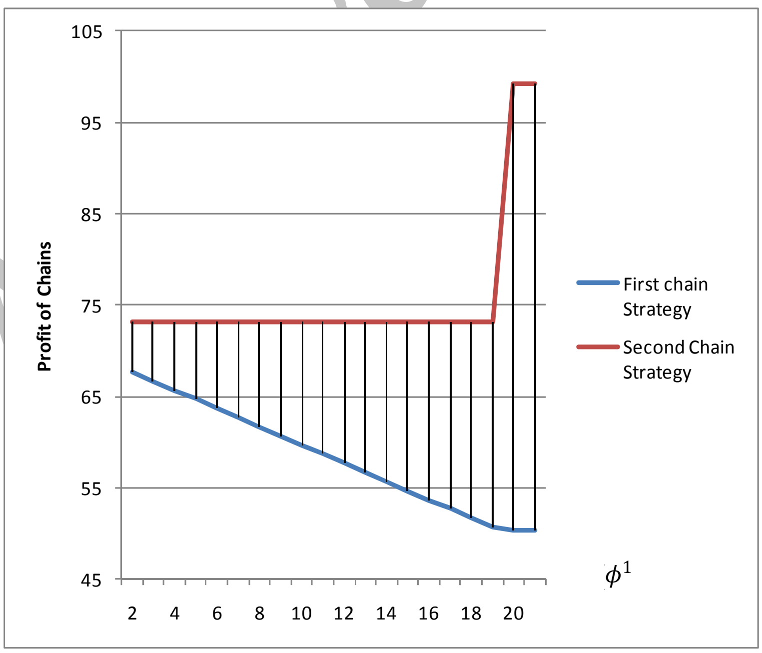

Figure 5: Turning point of the strategy of the first chain with attention to second chain

If the centralization cost of the first chains increases during the time according to environmental changes, and the first chain changes its strategy to decentralized in turning point, it can sustains just 1 unit of profit but the profit of second chain will growth significantly. In other word if the first chain continuous its centralized strategy, it loses 1 unit of profit while the profit of the second chain will be 23.54 unit more than the first one. But if the first chain changes its strategy to decentralized strategy it can hold its profit in fixed level but the profit of the second chain raises considerably and stands in 48.93 unit more than the first chain.

Appendix A.

Proof 1. Differentiating Π^i in (3) twice with respect to P^i and L^i, we have $\frac{\partial^2\Pi^i}{\partial {P^i}^2} = -2(\beta_p + \gamma_p) \leq 0$ and $\frac{\partial^2\Pi^i}{\partial {L^i}^2} = 2A^i \ln\frac{(1-\alpha)}{{L^i}^3} \leq 0$, respectively.

Proof 2. By using the calculations in proof 1 and differentiating Π^i in (11) with respect to P^i, we have $\frac{\partial \Pi^i}{\partial P^i} = a^i - 2\beta_p P^i - \beta_L L^i + \beta_p(m_m^i + m_d^i + A^i + C^i)$. By setting the right-hand side of the first order derivative to zero, we obtain

$$P^{i*} = \frac{1}{2\{\frac{a^i}{\beta_p} - \frac{\beta_L}{\beta_p}L^{i*} + (m_m^i + m_d^i + A^i + C^i)\}}.$$

Differentiating Π^i with respect to L^i, and replacing P^{i*} and C^i in the equation and setting the right hand side to zero yields $\left(\frac{1}{2}\right)[(\alpha h - (1-\alpha)b_s)\beta_P + \beta_L]\left[{a^i}/{\beta_P} - \left[{\beta_L}/{\beta_P} + h\left(1 + \frac{\alpha}{\ln(1-\alpha)}\right) - b_s\frac{(1-\alpha)}{\ln(1-\alpha)}\right]L^{i*} - (m_m^i + m_d^i + A^i)\right] + A^i \ln\frac{(1-\alpha)}{{L^{i*}}^2} = 0$, and by solving the equation we can find L^{i*}.

Proof 3. Differentiating Π_{Dst}^i twice with respect to P_d^i, and differentiating Π_{Mnf}^i twice with respect to P_m^i, L^i, we can show that $\frac{\partial^2\Pi_{Dst}^i}{\partial {P_d^i}^2} = -2\beta_p \leq 0$, $\frac{\partial^2\Pi_{Mnf}^i}{\partial {P_m^i}^2} = -\beta_p \leq 0$, and $\frac{\partial^2\Pi^i}{\partial {L^i}^2} = 2A^i \ln(1-\alpha)/{L^i}^3 \leq 0$.

Proof 4. By using the calculations in proof 3 and assuming that P_m^i and L^i are known, we differentiate Π_{Dst}^i in (9) with respect to P_d^i, we have

$$\frac{\partial\Pi_{Dst}^i}{\partial P_d^i} = a^i - 2\beta_p P_d^i - \beta_L L^i + \beta_p(P_m^i + m_d^i)$$

and by setting the right-hand side to zero we extract P_d^{i*} as

$$P_d^{i*} = \frac{1}{2\{\frac{a^i}{\beta_p} - \frac{\beta_L}{\beta_p}L^i + P_m^i + m_d^i\}}$$

Proof 5. By using the results of proof 3 and differentiating Π_{Mnf}^i in (6) with respect to P_m^i and L^i, we have

$$\frac{\partial\Pi_{Mnf}^i}{\partial L^i} = a^i - \beta_p P^i - \beta_L L^i - \frac{1}{2}\beta_p(P_m^i - m_m^i - A^i - C^i)$$

and

$$\frac{\partial\Pi_{Mnf}^i}{\partial L^i} = \left(\frac{\partial P_m^i}{\partial L^i} - \frac{\partial C^i}{\partial L^i}\right)\lambda^i + (P_m^i - m_m^i - A^i - C^i) - A^i \ln\frac{(1-\alpha)}{{L^i}^2}.$$

By setting the right-hand sides to zero and replacing P_m^{i*} in the first equation and P_m^{i*}, C^i and $\frac{\partial C^i}{\partial L^i}$ in second equation and we extract P_m^{i*} and L^{i*} from equations below:

$$P_m^{i*} = \frac{1}{2\{\frac{a^i}{\beta_p} - \frac{\beta_L}{\beta_p}L^i - (m_d^i - m_m^i - A^i - C^i)\}}$$

;

$$\frac{1}{4}\left(\beta_L + \beta_p(\alpha h - (1-\alpha)b_s)\right)\left\{\frac{a^i}{\beta_p} - \frac{\beta_L}{\beta_p}L^{i*} - (m_m^i + m_d^i + A^i + C^i)\right\} + A^i \ln\frac{(1-\alpha)}{{L^{i*}}^2} = 0.$$

REFERENCES

AfÃeche, P. and Mendelson, H. (2004). Pricing and Priority Auctions in Queueing Systems with a Generalized Delay Cost Structure. *Management Science*, 50 (7), pp. 869-882.

Allon, G. and Federgruen,A. (2006). Competition in Service Industries.

Boyaci, T. and Ray, S. (2003). Product Differentiation and Capacity Cost Interaction in Time and Price Sensitive Markets. *Manufacturing and Service operations management*, 5 (1), pp. 18-36.

Boyaci, T. and Ray, S. (2006). The Impact of Capacity Costs on Product Differentiation in Delivery Time, Delivery Reliability, and price. *Production and Operations Management*, 15 (2), pp. 179-197.

Cachon, G. P. (2003). Supply Chain Coordination with Contracts. *Handbooks in Operations Research and Management Science*, 11, pp. 229-340.

Dobson, G. and Stavrulaki, E. (2007). Simultaneous Price, Location, and Capacity Decisions on a Line of Time- sensitive Customers. *Naval Research Logistics (NRL)*, 54 (1), pp. 1-10.

Jayaswal, S., Jewkes, E. and Saibal, R. (2010). Product Differentiation and Operations Strategy in a Capacitated Environment. *European Journal of Operational Research,* 210 (3), pp. 716-728.

Katta, A. K. and Sethuraman, J. (2005). Pricing Strategies and Service Differentiation in an M/M/1 Queue-A profit Maximization Perspective, Working paper 2005.

Lederer, P. J. and Li, L. (1997). Pricing, Production, Scheduling, and Delivery-time Competition. *Operations Research*, 45 (3), pp. 407-420.

Li, L. and Lee, Y. S. (1994). Pricing and Delivery-Time Performance in a Competitive Environment. *Management Science*, 40 (5), pp. 633-646.

Liu, L. and Parlar, M. (2007). Pricing and Lead Time Decisions in Decentralized Supply Chains. *Management Science*, 53 (5), pp. 713-725.

Palaka, K. O. and Erlebacher, S. (1998). Lead-time Setting, Capacity Utilization, and Pricing Decisions under Lead-time Dependent Demand. *IIE Transactions*, 30 (2), pp. 151-163.

Pangburn, M. S. and Stavrulaki, E. (2008). Capacity and Price Setting for Dispersed, Time-sensitive Customer Segments. *European Journal of Operational Research*, 184 (3), pp.1100-1121.

Pekgun, P., Griffin, P. M. and Keskinocak, P. (2006). Centralized vs. Decentralized Competition for Price and Lead-time Sensitive Demand. *Submitted to Management Science*.

Pekgun, P., Griffin, P. M. and Keskinocak, P. (2008). Coordination of Marketing and Production for Price and Leadtime Decisions. *IIE Transactions*, 40 (1), pp. 12-30.

Sinha, S. K., Rangaraj, N. and Hemachandra, N. (2010). Pricing Surplus Server Capacity for Mean Waiting Time Sensitive Customers. *European Journal of Operational Research*, 205 (1), pp. 159-171.

So, K. C. (2000). Price and Time Competition for Service Delivery. *Manufacturing and Service Operations Management*, 2 (4), pp. 392-409.

So, K. C. and Song, J. S. (1998). Price, Delivery Time Guarantees and Capacity Selection. *European Journal of Operational Research*, 111 (1),pp. 28-49.

Teimoury, E., Modarres, M., Kazeruni Monfared, A. and Fathi, M. (2011). Price, Delivery Time, and Capacity Decisions in an M/M/1 Make-to-order/Service System with Segmented Market. *The International Journal of Advanced Manufacturing Technology*, 57 (1-4), pp. 235-244.

Wang, Y., Jiang, L. and Jun-Shen, Z. (2004). Channel Performance under Consignment Contract with Revenue Sharing. *Management Science*, 50 (1), pp. 34-47.

Xiao, T. and Qi, X. (2012). A Two-stage Supply Chain with Demand Sensitive to Price, Delivery Time, and Reliability of Delivery. *Annals of Operations Research*, pp. 1-22.