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Numerical Solution of Mass Transfer Process during Drying of Apple Slices Using Pseudospectral Method

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Introduction

Convective air drying is one of the oldest and most popular drying methods. Designing and controlling the convective air drying needs the mathematical description of the moisture transfer during the drying process, known as drying kinetics. Fick's second law of diffusion can be used for modelling the moisture distribution inside the moist object during drying process.

Mathematical modeling of drying process is a very important tool, as it contributes to understand better moisture distributions inside the product which helps designing, improving and controlling drying operation in the food industry.

Implementation of the partial differential equations subject to the correspondent initial and boundary conditions is one of the main methods of mathematical modeling to describe the physical phenomena such as moisture transfer during drying. In the recent decades, considerable number of research works have been devoted to numerical solution of mass transfer phenomena during convective drying of food products by using the common numerical solution such as FDMs, FEMs and FVMs.

The spectral collocation (pseudospectral) methods is a powerful tool for the numerical solutions of smooth PDEs like mass transfer equations. Pseudospectral methods are able to achieve the high precision with using a small number of discretization points compared to FDMs and FEMs and with low computational time and computer memory.

The objective of present research is to simulate the mass transfer phenomena in one dimension during convective drying of apple slices. The validation of the presented numerical model was done by comparing experimental drying data taken from Kaya *et al.* (2007) and Zarein *et al.* (2013). For more confirming the numerical approach, a numerical example with the exact solution is provided and the related errors were evaluated.

Materials and Methods

Estimation of mass transfer coefficients

The convective mass transfer coefficient in the surface of the apple slice was obtained according to the relationship presented by Paitil (1988) and Janjai et al. (2008).

$$h_m = {D_{air}}/{\mathrm{L}}\ \left(2 + 0/522\ ({\rho_{air} u L}/{\mu_{air}})^{0.5} ({\mu_{air}}/{\rho_{air} D_{air}})^{0.33}\right) \qquad (1)$$

Estimation of effective moisture diffusivity coefficient

Fick's second law of diffusion was applied to obtain the effective moisture diffusivity coefficient of the apple slices. The analytical solution of this equation can be written as follows (Crank, 1975):

$$MR = \frac{8}{\pi^2} \sum_{n=0}^{\infty} \frac{1}{(2n+1)^2} \exp\left(-\frac{-(2n+1)^2\, \pi^2 D t}{4 l^2}\right) \qquad (2)$$

In this study, we consider the Pseudospectral methods for solving 1D mass transfer equation. In order to develop the model, the following common assumptions are considered: negligible heat changes during drying process, moisture is transferred inside the slices by diffusion, one-dimensional mass transfer in apple slices, non-shrinkage and non-deformation of the slice.

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Results and Discussion

In the field of numerical analysis, the main advantage of pseudospectral methods compared to others such as FDMs and FEMs are exponential convergence and sufficient accuracy (Sun *et al.*, 2012). The values of parameters and coefficients of mathematical model are summarized in Table 1. The comparisons between the predicted average moisture content and the experimental data are shown in Fig. 1 & 2. It can be seen, the numerical results are in good agreement with the experimental data. The values of the correlation coefficient and the root mean square error from comparison of numerical result with experimental data taken from Zarein *et al*. (2013) and Kaya *et al.* (2007) were 0.9996, 0.0729 and 0.997, 0.1561 respectively. Moreover, the running time for solving 1D mass transfer equations was about 3 seconds. This result is the evident that the presented model is successful for predicting the moisture content history during drying process.

Moreover, by using the considered numerical method the approximate solutions of defined numerical example for different discretizing points was evaluated and the associated error history are shown in Figure 3. It can be seen that the values of errors are very low and about 10^{-3} and 10^{-5}, that confirms the high accuracy, robustness and efficiency of the suggested numerical approach.

Conclusions

Spectral collocation (pseudospectral) method is presented to solve mass transfer equation in one dimensional in during convective drying process approximately. The model was validated by the reported experimental data from convective drying of apple slices. Also, a numerical example, which had an exact solution in a closed form, was provided to illustrate the high accuracy of the proposed method. The results of statistical computations (r and $RMSE$) and numerical example showed the efficiency, applicability and robustness of the presented approach.

Keywords: Apple drying, Modeling, Numerical solution, Pseudospectral method