Change Point Estimation of a Process Variance with a Linear Trend Disturbance

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Abstract

When a change occurs in a process, one expects to receive a signal from a control chart as quickly as possible. Upon the receipt of signal from the control chart a search for identifying the source of disturbance begins. However, searching for assignable cause around the signal time, due to the fact that the disturbance may have manifested itself into the process sometimes back, may not always lead to successful identification of assignable cause(s). If process engineers could identify the change point, i.e. the time when the disturbance first manifested itself into the process, then corrective actions could be directed towards effective elimination of the source of disturbance. In this paper we develop a maximum likelihood estimator (MLE) for process change point designed to detect changes in process variance of a normal quality characteristic when the change follows a linear trend. We describe how this estimator can be used to identify the change point when a Shewhart S-control chart signals a change in the process variance. Numerical results reveal that the proposed estimator outperforms the MLE designed for step change when a linear trend disturbance is present.

Keywords: Change point estimation, Maximum Likelihood Estimator, Newton method, Shewhart S-control chart, Assignable cause, Statistical process control;

1. Introduction

Statistical process control (SPC) charts are tools that are used to monitor the state of a process by distinguishing between common causes and special causes of variability. When a control chart signals that a special cause is present, process engineers must initiate a search for identification of the special cause(s) which led to process disturbance. The search will depend on the process engineers' expertise and knowledge of their process. Due to the potential delay in generating a signal from control charts, the signal does not provide process engineers with what caused the process to change or when the process change actually occurred. Knowing the time of the change could lead to identifying the special cause more quickly, and to take the appropriate actions immediately to improve quality. Consequently, estimating the time of the process change would be useful to process engineers. Therefore, most control charting procedures and corresponding diagnostic tools are designed preliminary for step changes in the parameters of interest.

Over the last two decades, various estimators have been introduced to estimate change point of processes with ferent assumptions regarding the type of changes and distributions for quality characteristic of interest. Much of the literature on change point estimation is directed towards estimating the change point when the assumed change type is a simple step change. The change point properties of cumulative sum control chart proposed by [5] and exponentially weighted moving average control chart was investigated by Nishina [2]. References [15], [16], [13], and [14] proposed four maximum likelihood estimators for the process change points using the step change likelihood function for $\bar{X}$, S and C, np charts, respectively. Reference [3] also derived a maximum likelihood estimator for identifying step change in a geometric control chart that used to monitor high yield process. Reference [1] proposed a maximum likelihood estimator for identifying the change point when a multivariate χ^2-control chart signals a change in the process mean. References [11] and [6] compared the estimator suggested by [15] and [14] to those suggested by

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[5] and [2] following signals from CUSUM and EWMA control charts, respectively. Considering only step changes, they concluded that the estimator suggested by [15] and [14] outperforms those offered by the CUSUM and EWMA procedures across a wide range of change magnitudes. In all the above mentioned works, it is assumed that the change type is a simple step change. Although step changes are one potential change type, linear trends can also exist. For example, step changes can occur as a result of tool breakage, while linear trends can occur as a result of tool wear. References [7] and [9] proposed maximum likelihood estimators for $\bar{X}$ -chart and *C*-chart, respectively when a linear trend disturbance is present. All the reviewed models assume that change types are known and are either in a form of simple step change or changes that have linear trend. But, in practice, rarely is the type of change known a priori and, thus, any deviations in the true change type from the assumed change type is likely to affect the performance of the estimator. Moreover, a process parameter might also experience multiple step changes. For example, this type of behavior might occur as a result of one influential process input variable changing several times, or several influential process input variables changing at different times. References [8] and [10] proposed two change point estimators derived from the change likelihood function for a Poisson rate parameter and process fraction non-conforming without assuming prior knowledge of the exact change type, respectively. The only assumption was that the anticipated change type is one from a family of monotonic change types. Reference [4] proposed a maximum likelihood estimator for the change point of a normal process mean with the assumption of monotonic change types.

In this paper, we derive a maximum likelihood estimator to identify change point of normal process variance when a linear trend disturbance is present. The proposed estimator will be used to estimate the change point when a *S*-control chart signals a change in the process variance and its performance will be studied based on the likelihood function for the process change point. The Monte Carlo simulation is used to compare performance of our estimator and that proposed by [16] when change of normal process variance appears as a linear trend.

The paper is organized as follows. In section 2, the maximum likelihood estimator for change point of a normal process variance with linear trend is presented. Section 3 provides comparisons between the proposed estimator and the one developed for the simple step change. Our concluding remarks are presented in the final section.

2. Maximum likelihood estimator

In this section, we consider a linear trend change model for the behavior of a normal process variance. We assume that the process is initially in-control with independent observations coming from a normal distribution with a known mean μ_0 and a known variance σ_0^2. Standard deviation or *S* control chart is used to control process dispersion. However, we assume that after an unknown point in time τ (known as the process change point), the process variance changes from its in-control state of $\sigma^2=\sigma_0^2$ to an unknown out-of-control state where $\sigma^2=\sigma_i^2$ and $\sigma_i{}^2>\sigma_0^2$ for $i=\tau+1, \ldots,$. The functional form of σ_i^2 is given as follows:

$$\sigma_i^2=\sigma_0^2+\beta(i-\tau)\,, \tag{1}$$

Where, β is the magnitude or slop of the linear trend disturbance.

In the assumed *S* control chart, let S_i be standard deviation calculated for the i^{th} subgroup and X_{ij} is the j^{th} observation in subgroup *i*, $\bar{X}_i$ is the i^{th} subgroup average and *n* is the number of observations in each subgroup. We will assume that S_T is the first subgroup standard deviation to exceed a control limit and this signal is not a false alarm. Thus, $S_1,S_2,\ldots,S_\tau$ are the standard deviations from the in-control process and the process variances σ_i^2, $i=1, \ldots, \tau$ are equal to its known value σ_0^2. Whereas $S_{\tau+1},S_{\tau+2},\ldots,S_T$ are from the changed process and its variances are $\sigma_i^2=\sigma_0^2+\beta(i-\tau)$ for $i=\tau+1,\tau+2, \ldots, \mathrm{T}$.

In the assumed change model, there are two unknown parameters τ and β that represent the last subgroup taken from the in-control process and the slope parameter of the linear trend, respectively. This model can be used to derive a MLE for process change point. The estimated change point will be denoted as $\hat{\tau}$. If τ is considered as the true change point of normal process variance with linear trend, the likelihood function is given as:

$$\begin{aligned} L(\tau,\beta\mid X)=&\prod_{i=1}^{\tau}\prod_{j=1}^{n}\frac{1}{\sqrt{2\pi\sigma_0^2}}\exp\left(-\frac{\left(X_{ij}-\mu_0\right)^2}{2\sigma_0^2}\right)\\ &\times\prod_{i=\tau+1}^{T}\prod_{j=1}^{n}\frac{1}{\sqrt{2\pi\sigma_i^2}}\exp\left(-\frac{\left(X_{ij}-\mu_0\right)^2}{2\sigma_i^2}\right) \end{aligned} \tag{2}$$

The maximum likelihood estimator for τ is the value of τ that maximizes the likelihood function in (2), or equivalently, its logarithm. The natural logarithm of (2) is given below. After substituting (1) in (3) and simplification, the result will be obtained as follows.

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$$\log_e L(\tau,\beta \mid X) = -n\tau \log_e \sqrt{2\pi\sigma_0^2} - \sum_{i=1}^{\tau}\sum_{j=1}^{n}\left(\frac{(X_{ij}-\mu_0)^2}{2\sigma_0^2}\right) - n\sum_{i=\tau+1}^{T}\log_e\sqrt{2\pi\sigma_i^2} - \sum_{i=\tau+1}^{T}\sum_{j=1}^{n}\left(\frac{(X_{ij}-\mu_0)^2}{2\sigma_i^2}\right) \quad (3)$$

After substituting (1) in (3) and simplification, the result will be obtained as follows.

$$\log_e L(\tau,\beta \mid X) = K + \sum_{i=\tau+1}^{T}\sum_{j=1}^{n}\left(\frac{(X_{ij}-\mu_0)^2}{2\sigma_0^2}\right) - \sum_{i=\tau+1}^{T}\sum_{j=1}^{n}\left(\frac{(X_{ij}-\mu_0)^2}{2(\sigma_0^2+\beta(i-\tau))}\right) - \frac{n}{2}\sum_{i=\tau+1}^{T}\log_e\left(\frac{\sigma_0^2+\beta(i-\tau)}{\sigma_0^2}\right) \quad (4)$$

Where K in (4) is a constant and it can be eliminated from the model. Since the value for β is unknown, an expression in terms of τ is required for β that maximizes (4). To do this, the partial derivative of (4) with respect to β is required which leads to (5).

As seen in (5), there is no closed form solution for β. Therefore, to provide an estimate of β for each τ without requiring an explicit closed-form expression, Newton's method is used (for more detail about Newton's method see [12]).

$$\frac{\partial \log_e L(\tau,\beta \mid X)}{\partial \beta} = \frac{1}{2}\sum_{i=\tau+1}^{T}\sum_{j=1}^{n}\left(\frac{(i-\tau)(X_{ij}-\mu_0)^2}{(\sigma_0^2+\beta(i-\tau))^2}\right) - \frac{n}{2}\sum_{i=\tau+1}^{T}\left(\frac{\sigma_0^2(i-\tau)}{\sigma_0^2+\beta(i-\tau)}\right) \quad (5)$$

Note that, if τ was known and the denominator of (6), is nonzero, Newton's method could be used to solve for β in (5). That is, $\hat{\beta}_t$ at the *(k+1)*th iteration can be written explicitly as:

$$\hat{\beta}_{t,k+1} = \hat{\beta}_{t,k} - \left\{\left[\frac{1}{2}\sum_{i=t+1}^{T}\sum_{j=1}^{n}\left(\frac{(i-t)(X_{ij}-\mu_0)^2}{(\sigma_0^2+\hat{\beta}_{t,k}(i-t))^2}\right) - \frac{n}{2}\sum_{i=t+1}^{T}\left(\frac{(i-t)}{\sigma_0^2+\hat{\beta}_{t,k}(i-t)}\right)\right] \times \left[-\sum_{i=t+1}^{T}\sum_{j=1}^{n}\left(\frac{(i-t)^2(X_{ij}-\mu_0)^2}{(\sigma_0^2+\hat{\beta}_{t,k}(i-t))^3}\right) + \frac{n}{2}\sum_{i=t+1}^{T}\left(\frac{(i-t)^2}{(\sigma_0^2+\hat{\beta}_{t,k}(i-t))^2}\right)\right]^{-1}\right\} \quad (6)$$

Where $\hat{\beta}_{t,0} = 0$. It can be proved that the denominator in (6) is always negative, which is a necessary condition for (4) to have a maximum. Even further, since the process variance is greater or equal to zero, β must be greater or equal to $-\sigma_0^2/(i-\tau)$ for any given $i = t+1,...,T$. The procedure in (6) will work well for the increasing rate case since β has no upper bound. Whereas, for decreasing rates the linear decreasing trend would eventually produce negative rates of variance which are impossible. Thus, only increasing trends are considered here.

As a result, using the procedure defined in (6), the value for β can be obtained at each potential change point value. The procedure is then repeated T times, once for each potential value of τ. Then, the estimated values of β at each potential change point value ($\hat{\beta}_\tau$) are substituted in (4) and the change point of normal process variance ($\hat{\tau}$) is obtain using (7).

$$\hat{\tau} = \arg\max_{0\le t<T}\left\{\sum_{i=\tau+1}^{T}\sum_{j=1}^{n}\left(\frac{(X_{ij}-\mu_0)^2}{2\sigma_0^2}\right) - \sum_{i=\tau+1}^{T}\sum_{j=1}^{n}\left(\frac{(X_{ij}-\mu_0)^2}{2(\sigma_0^2+\beta(i-\tau))}\right) - \frac{n}{2}\sum_{i=\tau+1}^{T}\log_e\left(\frac{\sigma_0^2+\beta(i-\tau)}{\sigma_0^2}\right)\right\} \quad (7)$$

Where $\hat{\tau}$ is the MLE for the last subgroup number obtained from the in-control process.

The proposed MLE framework can be applied when any process dispersion control chart gives an out-of-control signal, including the CUSUM, EWMA and Shewhart *S* chart. In the following section we use Monte Carlo simulation to compare the performance of the proposed estimator with a step change estimator proposed by [16].

3. Comparison of the change point estimators

In this section, Monte Carlo simulation is conducted to make a comparison between the proposed estimator with the one suggested by [16] when linear trend disturbance is present. In the following comparisons, τ and $\hat{\tau}_{SC}$ are used to show the results of the derived estimator for linear trend and the simple step change estimator, respectively. In the simulation study, *S*-chart is used to monitor process dispersion.

3.1. False alarm

The simulation modeling of false alarms needs to be carefully addressed. So, in this subsection the handling of false alarms in the simulation model is discussed. Following the path in the literature of change points, when a control chart signals an out-of-control condition at subgroup, then the signal is considered as a false alarm because no change has actually taken place in the process. When a false alarm is encountered in a simulation run, it is treated in the same way that a false alarm would be treated on an actual process. Namely, if one determines that a

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signal is indeed a false alarm, then one is affirming that the process is currently in-control and could restart their monitoring of the process. Thus, when a false alarm is encountered at subgroup, the control chart is restarted at subgroup while not altering the scheduled change point. This is the same approach considered by researchers including [9].

3.2. Comparisons of accuracy performances and Precision performances of the estimators

In this subsection, to make comparisons between the two mentioned estimators, the simulation study is organized as follows; for subgroup $1, 2, ..., \tau = 50$, observations are generated randomly from a normal distribution with known mean $\mu_0 = 100$ and variance $\sigma_0^2 = 25$. Then, starting with subgroup 51, observations are generated randomly from a normal distribution with mean $\mu_0 = 100$ and changed variance σ_i^2; until the S-chart issued a signal at T that is not a false alarm. Recall that after subgroup τ, the variance changes with linear trend based on (1). For given value of T, the change point estimators, $\hat{\tau}$ and the one developed for step change, i.e. $\hat{\tau}_{SC}$, are computed and recorded. This procedure is repeated a total of 10,000 times for different values of slope parameter β and size of subgroup n (here only the results of estimators for subgroups with size $n = 5$ and 15 are presented). Afterwards, the average of $\hat{\tau}$ and $\hat{\tau}_{SC}$ (denoted as $\bar{\hat{\tau}}$ and $\bar{\hat{\tau}}_{SC}$) over all simulations are determined along with the mean square errors (denoted as $MSE(\bar{\hat{\tau}})$ and $MSE(\bar{\hat{\tau}}_{SC})$). The results of simulation for accuracy performance study with $n = 5$ are shown in Table 1.As seen in Table 1, the results show that when size of subgroup is $n = 5$, the derived estimator for linear trend disturbance provides much better accuracy performance over all ranges of β values in comparison with the simple step change estimator suggested by Samuel, Pignatiello, and Calvin [16].

Table 1
Accuracy performances for two different MLEs of the change point following a genuine signal from a *S*-control chart. $n = 5$, $\tau = 50$ and $N = 10,000$ independent runs

β	$E(T)$	$\bar{\hat{\tau}}$	$\bar{\hat{\tau}}_{SC}$	$MSE(\bar{\hat{\tau}})$	$MSE(\bar{\hat{\tau}}_{SC})$
0.05	151.8	103.79	113.38	2893.60	4017.60
0.20	102.3	69.51	77.59	380.54	761.00
0.35	88.6	62.83	69.17	164.60	367.57
0.65	77.4	57.89	62.42	62.23	154.32
1.00	71.2	55.87	58.92	34.47	79.52
1.50	66.7	53.91	56.63	15.27	43.92
2.00	64.0	53.13	55.36	9.78	28.69
3.00	61.0	52.20	53.89	4.82	15.10

Afterwards similar Monte Carlo simulations are conducted to estimate precision performances for simple step change and the proposed estimators. In the precision performance study, the probability of correct estimation of true change point will be estimated for situations that estimated change points are within specific observations of the true change point. The results of precision performance study of two mentioned estimators for subgroups with size *n*=5 are presented in Table 2.

In the next simulation, the subgroups are generated with size $n = 15$. The results of this simulation are shown in Table3.

Table 2
Estimated precision performances over a range of β values for $\hat{\tau}$ and $\hat{\tau}_{SC}$ (shown in parenthesis) following a genuine signal from a *S*-control chart. $n = 5$, $\tau = 50$ and $N = 10,000$ independent runs.

β	**0.05**	**0.20**	**0.35**	**0. 65**	**1**	**1.5**	**2**	**3**
$\hat{P}(\lvert\hat{\tau}-\tau\rvert = 0)$	0.01 (0.01)	0.01 (0.01)	0.02 (0.01)	0.02 (0.02)	0.04 (0.02)	0.4 (0.03)	0.05 (0.03)	0.07 (0.05)
$\hat{P}(\lvert\hat{\tau}-\tau\rvert \le 1)$	0.02 (0.02)	0.04 (0.02)	0.05 (0.03)	0.07 (0.05)	0.10 (0.06)	0.13 (0.08)	0.16 (0.10)	0.20 (0.14)
$\hat{P}(\lvert\hat{\tau}-\tau\rvert \le 2)$	0.03 (0.02)	0.07 (0.04)	0.08 (0.05)	0.12 (0.08)	0.17 (0.11)	0.21 (0.14)	0.26 (0.17)	0.33 (0.25)
$\hat{P}(\lvert\hat{\tau}-\tau\rvert \le 3)$	0.04 (0.03)	0.09 (0.05)	0.11 (0.07)	0.17 (0.11)	0.23 (0.15)	0.29 (0.20)	0.36 (0.26)	0.45 (0.36)
$\hat{P}(\lvert\hat{\tau}-\tau\rvert \le 4)$	0.05 (0.04)	0.11 (0.07)	0.14 (0.09)	0.21 (0.14)	0.29 (0.19)	0.36 (0.27)	0.44 (0.34)	0.55 (0.47)
$\hat{P}(\lvert\hat{\tau}-\tau\rvert \le 5)$	0.06 (0.04)	0.13 (0.08)	0.18 (0.11)	0.26 (0.17)	0.34 (0.25)	0.44 (0.34)	0.52 (0.43)	0.64 (0.57)
$\hat{P}(\lvert\hat{\tau}-\tau\rvert \le 10)$	0.10 (0.08)	0.23 (0.16)	0.32 (0.23)	0.47 (0.37)	0.60 (0.53)	0.73 (0.68)	0.82 (0.80)	0.91 (0.91)

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Similar to Table 1, the results in Table 3 indicated that, when the size of the subgroups is $n = 15$, the derived estimator for the linear trend provides much better accuracy performance over all ranges of β values in comparison with the simple step change estimator suggested by Samuel et al [16].

Table 3
Accuracy performances for two different MLEs of the change point following a genuine signal from a *S*-control chart. $n = 15$, $\tau = 50$ and $N = 10,000$ independent runs.

β	$E(T)$	$\bar{\hat{\tau}}$	$\bar{\hat{\tau}}_{SC}$	$MSE(\bar{\hat{\tau}})$	$MSE(\bar{\hat{\tau}}_{SC})$
0.05	147.3	81.64	99.12	1001.40	2412.80
0.20	93.7	60.90	69.87	118.81	394.90
0.35	81.3	56.75	63.33	45.56	177.58
0.65	71.1	54.07	58.34	16.59	69.63
1.00	66.0	52.76	55.87	7.60	34.41
1.50	62.2	51.88	54.34	3.53	18.81
2.00	60.1	51.41	53.41	1.98	11.61
3.00	57.7	50.99	52.26	0.99	5.10

In the next simulation we choose subgroups with size *n=15* and compute the precision performances of the two mentioned estimators. These results are shown in Table 4. As similar to Table 2, the results of Table 4 indicate that the derived estimator for the linear trend estimates the true change point of process with a higher precision, specifically when the slope parameter is small. As seen in Table 4, when the slope parameter increases, the probability of correct estimation using step change estimator begins to become better.

In this paper, the results for a two values of sample size, i.e. *n*=5 and 15 are reported. Although not shown here, simulation studies show that the same relative performances are obtained regardless of the value of τ, σ_0^2 and β. Therefore, in general, based on these results, we conclude that proposed estimator ($\hat{\tau}$) outperforms the simple step change estimator ($\hat{\tau}_{SC}$) suggested by Samuel, Pignatiello, and Calvin [16] with regards to accuracy and precision performances when a linear trend disturbance is present.

4. Conclusion

In this paper, a maximum likelihood estimator for identifying the time of linear trend change in a normal process variance is proposed. We used the Newton's method to estimate the slope parameter β at each potential change point value. Afterwards, the Monte Carlo simulation was applied to compare the performance of the proposed estimator to the one developed by Samuel, Pignatiello, and Calvin [16] for a simple step change in terms of accuracy and precision when a linear trend disturbance is present. The results of simulation showed that the proposed estimator outperforms the MLE designed for simple step changes especially in the cases of small slope parameters.

Table 4
Estimated precision performances over a range of β values for $\hat{\tau}$ and $\hat{\tau}_{SC}$ (shown in parenthesis) following a genuine signal from a *S*-control chart. $n = 15$, $\tau = 50$ and $N = 10,000$ independent runs.

β	**0.05**	**0.20**	**0.35**	**0.65**	**1**	**1.5**	**2**	**3**
$\hat{P}(\lvert\hat{\tau}-\tau\rvert = 0)$	0.01 (0.01)	0.02 (0.01)	0.03 (0.02)	0.05 (0.02)	0.06 (0.03)	0.08 (0.05)	0.09 (0.06)	0.12 (0.08)
$\hat{P}(\lvert\hat{\tau}-\tau\rvert \le 1)$	0.03 (0.01)	0.06 (0.03)	0.09 (0.04)	0.13 (0.06)	0.17 (0.09)	0.21 (0.13)	0.25 (0.16)	0.34 (0.24)
$\hat{P}(\lvert\hat{\tau}-\tau\rvert \le 2)$	0.04 (0.02)	0.09 (0.05)	0.14 (0.07)	0.21 (0.11)	0.27 (0.16)	0.34 (0.22)	0.40 (0.28)	0.51 (0.41)
$\hat{P}(\lvert\hat{\tau}-\tau\rvert \le 3)$	0.06 (0.03)	0.13 (0.06)	0.19 (0.10)	0.28 (0.15)	0.36 (0.23)	0.45 (0.32)	0.53 (0.41)	0.66 (0.59)
$\hat{P}(\lvert\hat{\tau}-\tau\rvert \le 4)$	0.07 (0.04)	0.16 (0.08)	0.24 (0.13)	0.35 (0.21)	0.44 (0.31)	0.55 (0.43)	0.65 (0.54)	0.77 (0.73)
$\hat{P}(\lvert\hat{\tau}-\tau\rvert \le 5)$	0.09 (0.04)	0.20 (0.10)	0.28 (0.16)	0.42 (0.26)	0.52 (0.39)	0.64 (0.54)	0.74 (0.66)	0.86 (0.83)
$\hat{P}(\lvert\hat{\tau}-\tau\rvert \le 10)$	0.15 (0.09)	0.36 (0.21)	0.49 (0.34)	0.68 (0.57)	0.82 (0.77)	0.92 (0.91)	0.96 (0.96)	0.98 (0.98)

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