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A Comparison of the REMBRANDT System with a New Approach in AHP

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Abstract

Analytic hierarchy process (AHP) has been criticized considerably for possible rank reversal phenomenon caused by the addition or deletion of an alternative. While in many cases this is a perfectly valid phenomenon, there are also many cases where ranks should be preserved. Our findings indicate that using the geometric mean reduces the variance in ratings substantially; hence, yielding lower inconsistency in ratings. An approach is therefore proposed using the geometric mean aggregation to avoid rank reversal phenomenon. A practical example is examined using the proposed approach to demonstrate its validity and practicability in rank preservation. This paper also compares the REMBRANDT system with the proposed approach to avoid rank reversal phenomenon.

Keywords: Analytic hierarchy process (AHP); REMBRANDT system; Rank reversal; Multiple attribute decision making (MADM).

1. Introduction

Analytic hierarchy process (AHP), as a very popular multiple criteria decision making (MCDM) tool, has been considerably criticized for its possible rank reversal phenomenon, which means changes of the relative rankings of the other alternatives after an alternative is added or deleted. If the weights or the number of criteria are also changed, then rankings might be reversed. Such a phenomenon was first noticed and pointed out by Belton and Gear [3], which leads to a long-lasting debate about the validity of AHP [6,8,17,26,32,34,35,38,39], especially about the legitimacy of rank reversal [7,15,23,21,25,29].

In order to avoid the rank reversal, Belton and Gear [3] suggested normalizing the eigenvector weights of alternatives using their maximum rather than their sum, which was usually called B–G modified AHP. Saaty and Vargas [25] provided a counterexample to show that B–G modified AHP was also subject to rank reversal. Belton and Gear [4] argued that their procedure was misunderstood and insisted that their approach would not result in any rank reversal if criteria weights were changed accordingly. Schoner and Wedley [28] presented a referenced AHP to avoid rank reversal phenomenon, which requires the modification of criteria weights when an alternative is added or deleted. Schoner et al. [30] also suggested a method of normalization to the minimum and a linking pin AHP (see also [31]), in which one of the alternatives under each criterion is chosen as the link for criteria comparisons and the values in the linking cells are assigned a value of one, with proportional values in the other cells. Barzilai and Golany [1] showed that no normalization could prevent rank reversal and suggested a multiplicative aggregation rule, which replaces normalized weight vectors with weight–ratio matrices, to avoid rank reversal. Lootsma [14] and Barzilai and Lootsma [2] suggested a multiplicative AHP for rank preservation. Vargas [36] provided a practical counterexample to show the invalidity of the multiplicative AHP. Triantaphyllou [33] offered two new cases to demonstrate that the rank reversals do not occur with the multiplicative AHP, but do occur with the AHP and some of its additive variants. Leung and Cao [10] showed that Sinarchy, a particular form of analytic network process (ANP), could prevent rank reversal. As an integrative view, the AHP now supports four modes, called Absolute, Distributive, Ideal and Supermatrix

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modes, respectively, for scaling weights to rank alternatives [15,20,22,27]. In the absolute mode, alternatives are rated one at a time and there is no rank reversal when new alternatives are added or removed. The distributive mode normalizes alternative weights under each criterion so that they sum to one, which does not preserve rank. The ideal mode preserves rank by dividing the weight of each alternative only by the weight of the best alternative under each criterion. The supermatrix mode allows one to consider dependencies between different levels of a feedback network. More recently, Ramanathan [18] suggested a DEAHP, which is claimed to have no rank reversal phenomenon. But in fact, it still suffers from rank reversal.

Wang and Elhag suggested an approach in which the local priorities remained unchanged. So, the ranking among the alternatives would be preserved. We provided a practical example to show that the Wang and Elhag approach was also subject to rank reversal. Wang and Elhag maintained that in order to avoid rank reversal, original local priorities of each alternative under each criterion have to remain unchanged when an alternative is added or removed [37]. Using the arithmetic mean raise the variance in Wang and Elhag approach. The rank reversal is caused by alternation and variance of local priorities under some or all criteria before and after an alternative is added or removed.

Our literature review shows that the rank reversal phenomenon has not been perfectly resolved and there still exist debates about the ways of avoiding rank reversals. So, this paper offers an approach to avoid rank reversal.

A group in the Netherlands, led by F.A. Lootsma, has developed a system which uses Ratio Estimation in Magnitudes or deci-Bells to Rate Alternatives which are Non-DominaTed [12,13]. This system is intended to adjust for three contended flaws in AHP. First, direct rating is on a logarithmic scale [12], which replaces the fundamental 1-9 scale presented by Saaty [11]. Second, the Perron-Frobenius eigenvector method (EM) of calculating weights is replaced by geometric mean, which avoids potential rank reversal [1]. Third, aggregation of scores by arithmetic mean is replaced by the product of alternative relative scores weighted by the power of weights obtained from analysis of hierarchical elements above the alternatives.

Then, this paper compares the REMBRANDT system with the proposed approach. A practical example is examined using the REMBRANDT system and the proposed approach to verify their validity and practicability in rank preservation.

2. The REMBRANDT System

The REMBRANDT system has been designed to address the three criticized features of AHP. The first issue addressed by Lootsma is the numerical scale for verbal comparative judgment. Saaty presented a verbal scale for the ratio of relative value between two objects where 1 represents roughly equal value, 3 represents the base objects as being moderately more important than the other objects, 5 reflects essential advantage, 7 very strong relative advantage, and 9 the ultimate overwhelming relative advantage. Lootsma perceives that relative advantage is more naturally concave, and presents a number of cases where a more nearly logarithmic scale would be appropriate, such as planning horizons, loudness of sounds, and brightness of light. Therefore, Lootsma presents a geometric scale where the gradations of decision maker judgment are reflected by the scale as follows:

1/16: strict preference for object 2 over base object.
1/4: weak preference for object 2 over the base object.
1: indifference.
4: weak preference for the base object over object 2.
16: strict preference for the base object over object 2.

The ratio of value r_{jk} on the geometric scale is expressed as an exponential function of the difference between the echelons of value on the geometric scale δ_{jk}, as well as a scale parameter y. Lootsma considers two alternative scales y to express preferences. For calculating the weight of criteria, $y = \ln\sqrt{2} \approx 0.347$ is used. For calculating the weight of alternatives on each criterion, $y = \ln 2 \approx 0.693$ is used. The difference in echelons of value δ_{jk} is graded as in Table 1.

Table 1
AHP scale and corresponding REMBRANDT scale

Verbal description	Saaty ratio w_j / w_k	REMBRANDT δ_{jk}
Very strong preference for object k	1/9	-8
Strong preference for object k	1/7	-6
Definite preference for object k	1/5	-4
Weak preference for object k	1/3	-2
Indifference	1	0
Weak preference for object j	3	2
Definite preference for object j	5	4
Strong preference for object j	7	6
Very strong preference for object j	9	8

The second suggested improvement is the calculation of impact scores. The arithmetic mean is subject to rank reversal of alternatives. The geometric mean is not subject to rank reversal, nor is logarithmic regression. Note that Saaty [19] argues that rank reversal when new reference points are introduced is a positive feature. Barzilai, Cook and Golany [1], taking an opposing view, argued that the geometric mean was more appropriate for calculation of relative value (through weights) than the arithmetic mean used by Saaty.

Lootsma proposes logarithmic regression, minimizing $\sum_{j<k}(\ln r_{jk} - \ln v_j + \ln v_k)^2$ where r_{jk} are the ratio

comparisons made by the decision maker for base object j and compared object k, and the weight for j (w_j) is represented by $\ln v_j$. The analysis is to calculate these weights. Since $r_{jk} = \frac{w_j}{w_k}$, error is represented minimizing the squared error yields the set of weights w_i which best fit the decision maker expressed preferences. Solving this is complicated by the fact that the resulting data set is singular. However, a series of normal equations can be solved to yield the desired weights.

The ratio matrix in REMBRANDT for criteria is transformed through the operator $e^{0.347r(jk)}$ to generate the set of values transformed to the logarithmic scale. Krovac [9] notes that the geometric mean of row elements of such a matrix yields the solution minimizing the sum of squared errors of the form $\sum_{i=1}^{n}\sum_{i=1}^{n}(\ln r_{jk} - w_j + w_k)^2$.

This solution is normalized by product. It is a simple matter to normalize by sum, simply dividing each element by the total.

The third improvement proposed by Lootsma is aggregation of scores. This lowest level is normalized multiplicatively, so that the product of components equals 1 for each of the k factors over which the alternatives are compared. Therefore, each alternative has an estimated relative performance w_k for each of the k factors. The components of the hierarchical level immediately superior to this lowest level are normalized additively, so that they add to 1, yielding weights *O(i)*. The aggregation rule for each alternative j is

$$w_j = \prod_{i=1}^{k} w_i^{o(i)} \qquad (1)$$

Where *i* is the number of criteria.

3. An Approach for Rank Preservation

Sometimes, it may be argued that rank reversal is a normal phenomenon in some situations where implying that it does not make sense to avoid it.

Harker and vargas [8], Saaty [21], and Saaty and Takizawa [24] argued that an exact replica or a copy of an alternative should not be added to the choice set because it adds nothing to the choice set. If an alternative is added as a new one, which is not an exact replica or a copy of an alternative and does add new information to choice set, then the original ranking must be ignored [21]. Saaty and Takizawa [24] also argued that if an apple and an orange were being compared and one adds another apple to the set, a new criterion such as "the number of elements of a criterion type (number of apples and number of oranges)" should be added to the hierarchy to preserve one's expectations, and thus one should alter the criteria set and the priorities assigned to them.

As is known, the weights of criteria are usually assumed to be independent of the number of alternatives in most of the real world MCDM problems and MCDM approaches. Although this assumption is also under debate in the AHP [5,28], it is not easy to accept the assumption that the weights or the number of criteria should vary with the number of alternatives. As a matter of fact, if the weights or the number of criteria are changed, then there will be no need to preserve rank. The rank reversal should be acceptable in this situation. So, if there is an approach that can preserve rank without the need of changing the weights or the number of criteria when an alternative is added or removed, it will be much easier to be accepted.

Based on our previous study, we might come to the conclusion that the rank reversal is caused by alternation and variance of local priorities under some or all criteria before and after an alternative is added or removed. Therefore, in order to avoid rank reversal, the original local priorities of each alternative under every criterion have to remain unchanged when an alternative is added or removed. In what follows, we discussed how to keep the original priorities unchanged when an alternative is added.

Let $A=(a_{ij})_{n\times n}$ be a comparison matrix with respect to some criterion and $B=(b_{ij})_{(n+1)\times(n+1)}$ be the augmented comparison matrix with the same criterion after the $(n+1)^{th}$ alternative is added. Their geometric mean weights are denoted by $W_A=(w_{1A}, \ldots, w_{nA})^T$ and $W_B=(w_1, \ldots, w_{n+1})^T$, respectively. Since W_B is the normalized geometric mean vector of the comparison matrix B, namely, $BW_B=\lambda W_B$, it follows that $B(kW_B)=\lambda_{max}(kW_B)$ for any $k \succ 0$, which means kW_B is also a geometric mean vector of B. The only difference between W_B and $\hat{W}_B=kW_B$ is that $\sum_{i=1}^{n+1} w_{iB} = 1$ while $\sum_{i=1}^{n+1} \hat{w}_{iB} = k \neq 1$. In order to keep the original priorities of the first n alternatives unchanged, the following condition has to be met:

$$\sum_{i=1}^{n} w_{iA} = \sum_{i=1}^{n} kw_i \qquad (2)$$

Since $\sum_{i=1}^{n} w_{iA} = 1$, we get from equation (2)

$$k = \frac{1}{\sum_{i=1}^{n} w_i} \qquad (3)$$

Accordingly,

$$\widehat{W}_B = kW_B = \left(\frac{w_1}{\sum_{i=1}^{n} w_i}, \frac{w_2}{\sum_{i=1}^{n} w_i}, \ldots, \frac{w_{n+1}}{\sum_{i=1}^{n} w_i}\right)^T \qquad (4)$$

Where $\hat{W}_B$ can be interpreted as the normalization with respect to the original n alternatives. Therefore, as long as we use the rescaled geometric mean vector $\hat{W}_B$ instead of the geometric mean W_B, the original priorities of the first n alternatives under each criterion will be kept unchanged. Accordingly, the ranking among them will be able to be preserved. If an alternative is going to be removed, then the remaining alternatives should keep unchanged their original local priorities with respect to each criterion. Accordingly, their composite weights will not change and there will be no rank reversal to happen in this situation.

4. A Practical Example

This example demonstrates the rank reversal phenomenon in Wang and Elhag approach, which involves three comparison matrices over nine alternatives with respect to three criteria a, b and c, respectively. Then alternative D is added. We also have a comparison matrix over criteria with respect to the objective. This example has been examined using the REMBRANDT system and the proposed approach. The aim is to use geometric mean reduce the variance in ratings substantially, thereby yielding lower inconsistency ratings [16]. This study shows that the scale used to have less impact than the aggregation rule about rank reversal phenomenon.

4.1. AHP Calculations – Wang and Elhag approach

Table 2 shows the local and composite weights for the alternatives before and after the addition of D. As can been seen from Table2, the ranking between A,...,J is $B \succ E \succ H \succ G \succ F \succ C \succ A \succ I \succ J$ before D is introduced, but becomes $B \succ E \succ D \succ G \succ H \succ F \succ C \succ A \succ I \succ J$ after D is added. The ranking is reversed. Such a phenomenon is referred to as rank reversal, which many occur not only when an alternative is added, but also when an alternative is removed. Wang and Elhag thought the reason for rank reversal to happen any changes in local priorities. It is observed from Table 2 that the Wang and Elhag approach fails to keep unchanged the priorities. The cause of the rank reversal is the variance in ratings.
Table 3 shows comparison matrix of criterion relative to the objective. Inconsistency rate is equally to 0.01.

4.2. The proposed approach calculations

It can be observed from Table 4 that geometric mean weights do preserve the rank for this example. In multiple criteria decision analysis (MCDA), these priority values are seen as utilities. There is no wonder that any changes in utilities may result in the changes of final ranking. The reason for the proposed approach to preserve the ranking is because geometric mean has very low inconsistency index, indicating high consistency. Obtained weights of criteria using this approach are equal to AHP.

Table 2
Final weights before and after the addition of D

Alternatives	Eigenvector weights relative to <u>*a*</u>	Eigenvector weights relative to <u>*b*</u>	Eigenvector weights relative to <u>*c*</u>	Composite weights	Rescaled composite weights	Priority
A	0.059	0.047	0.087	0.064	----	7
B	0.227	0.047	0.281	0.216	----	1
C	0.051	0.320	0.036	0.083	----	6
E	0.227	0.139	0.040	0.170	----	2
F	0.059	0.170	0.117	0.088	----	5
G	0.059	0.047	0.324	0.121	----	4
H	0.187	0.139	0.040	0.145	----	3
I	0.072	0.047	0.036	0.060	----	8
J	0.059	0.047	0.038	0.052	----	9
A	0.060	0.046	0.086	----	0.065	8
B	0.227	0.046	0.282	----	0.214	1
C	0.049	0.321	0.037	----	0.09	7
D	0.072	0.169	0.282	----	0.150	3
E	0.227	0.141	0.041	----	0.158	2
F	0.060	0.169	0.115	----	0.094	6
G	0.060	0.046	0.322	----	0.136	4
H	0.184	0.141	0.041	----	0.135	5
I	0.072	0.046	0.037	----	0.057	9
J	0.060	0.046	0.038	----	0.051	10

Table 3
Pairwise comparison matrix of criterion relative to the objective

Criteria	a	b	c	Priority
a	*	3	2	0.54
b	1/3	*	1/2	0.163
c	1/2	2	*	0.297
Inconsistency rate = 0.01				

Table 4
Final weights before and after the addition of D

Alternatives	Geometric mean weight relative to ***a***	Geometric mean weight relative to ***b***	Geometric mean weight relative to ***c***	Composite weights	Rescaled composite weights	Priority
A	0.059	0.047	0.087	0.0653	----	7
B	0.228	0.047	0.282	0.2143	----	1
C	0.050	0.318	0.036	0.0899	----	6
E	0.228	0.139	0.041	0.1576	----	2
F	0.059	0.170	0.116	0.0940	----	5
G	0.059	0.047	0.323	0.1356	----	3
H	0.187	0.139	0.041	0.1355	----	4
I	0.071	0.047	0.036	0.0569	----	8
J	0.059	0.047	0.038	0.0509	----	9
A	0.061	0.046	0.085	----	0.066	8
B	0.227	0.046	0.284	----	0.215	1
C	0.049	0.318	0.037	----	0.089	7
D	0.072	0.169	0.284	----	0.151	3
E	0.227	0.141	0.042	----	0.158	2
F	0.061	0.169	0.114	----	0.094	6
G	0.061	0.046	0.321	----	0.136	4
H	0.183	0.141	0.042	----	0.134	5
I	0.072	0.046	0.037	----	0.057	9
J	0.061	0.046	0.039	----	0.052	10

4.3. REMBRANDT Calculations

The impact and final scores are shown in Table 5, from which it can be seen very clearly that the REMBRANDT system preserves the ranking between G and H in this example when D is added. Comparative results shown in Table 5 indicate that results obtained by REMBRANDT were very closer to those obtained using the proposed Approach before the alternative D was added. Both were quite different from those yielded by conventional AHP.
The pairwise comparisons of criteria relative to the objective use an exponential multiplier of ln√2. This is yielded:
These are then aggregated to obtain weighted scores for each of the alternatives. For example:
A: $1^{0316} * 0.540^{0.423} * 0.397^{0.261} = 0.606$

Table 5
Final weights before and after the addition of D

Alternatives	Impact score relative to ***a***	Impact score relative to ***b***	Impact score relative to ***c***	Final scores	Final scores	Priority
A	0.54	0.397	1	0.606	----	7
B	3.999	0.397	12.692	3.154	----	1
C	0.397	10.074	0.25	0.798	----	6
E	0.54	0.397	0.34	1.5	----	3
F	3.999	1.852	1.852	1.241	----	4
G	0.54	2.939	21.758	1.605	----	2
H	0.54	0.397	0.34	1.234	----	5
I	2.519	1.852	0.25	0.445	----	8
J	0.735	0.397	0.27	0.4	----	9
A	0.574	0.354	0.812	----	0.565	8
B	3.999	0.354	9.844	----	2.824	1
C	0.406	8.57	0.19	----	0.707	7
D	0.758	2.638	9.844	----	2.363	2
E	3.999	1.741	0.268	----	1.368	4
F	0.574	2.638	1.516	----	1.162	5
G	0.574	0.354	15.991	----	1.45	3
H	2.462	1.741	0.268	----	1.115	6
I	0.758	0.354	0.19	----	0.401	9
J	0.574	0.354	0.203	----	0.364	10

Table 6
Pairwise comparison matrix of criterion relative to the objective

Criteria	a	b	c	Priority before normalization	Priority after normalization
a	*	3	2	2.001	0.423
b	1/3	*	1/2	1.236	0.261
c	1/2	2	*	1.499	0.316

5. Conclusion

In this paper, we pointed out the rank reversal is caused by alternation and variance of local priorities under some or all criteria before and after an alternative is added or removed. Using geometric mean will reduce the variance ratings substantially; hence, yielding lower inconsistency ratings.

Accordingly, we worked out an approach to avoid the rank reversal in AHP. We compared the REMBRANDT system with the proposed approach for avoiding rank reversal. Our proposed approach requires no changes in the weights or number of criteria when an alternative is added or removed. The examination of the data confirmed the validity and practicability of the REMBRANDT system and the proposed approach in rank preservation. This example also shows that Wang and Elhag approach still suffers from rank reversal.

Rescaled eigenvector and rescaled geometric mean ranking indicated that alternative B has 4.19 and 4.13 times the value alternative J, respectively. The REMBRANDT scores can be interpreted as indicating that overall, alternative B is 7.76 times as valuable as alternative J. However, REMBRANDT uses a longer scale than the proposed approach. There was also a slightly more divergent scoring of the alternatives in REMBRANDT system, which can be explained by the use of longer scales.

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