A parameter-tuned Genetic Algorithm for Vendor Managed Inventory Model for a Case Single-vendor Single-retailer with Multi-product and Multi-constraint

Javad Sadeghi [a,*] Ahmad Sadeghi [b], Mohammad Saidi-Mehrabad [c]

[a] *MSc, Faculty of Industrial and Mechanical Engineering, Qazvin Branch, Islamic Azad University, Qazvin, Iran*

[b] *Assistant Professor, Faculty of Industrial and Mechanical Engineering, Qazvin Branch, Islamic Azad University, Qazvin, Iran*

[c] *Professor, Department of Industrial Engineering, Iran University of Science and Technology, P.C. 16844, Narmak, Tehran, Iran*

Abstract

This paper develops a single-vendor single-retailer supply chain for multi-product. The proposed model is based on Vendor Managed Inventory (VMI) approach and vendor uses the retailer's data for better decision making. Number of orders and available capital are the constraints of the model. In this system, shortages are backordered; therefore, the vendor's warehouse capacity is another limitation of the problem. After the model formulation, an Integer Nonlinear Programming problem will be provided; hence, a genetic algorithm has been used to solve the model. Consequently, order quantities, number of shipments received by a retailer and maximum backorder levels for products have been determined with regard to cost consideration. Finally, a numerical example is presented to describe the sufficiency of the proposed strategy with respect to parameter-tuned by response surface methodology (RSM).

Keywords: Vendor managed inventory; Genetic algorithm; Multi-constraint; Multi-product; Parameter tuning.

1. Introduction

In recent years, according to the investigations conducted by researchers, inventory costs have been more important in supply chain management (SCM). Supply chain management is a set of approaches used to efficiently integrate suppliers, manufacturers, warehouses, and stores so that merchandise is produced and distributed in the right quantities, to the right locations, and at the right time in order to minimize total costs while satisfying service-level requirements. The supply chain consists of suppliers, warehouses, manufacturing centers, distribution centers, and retail outlets, as well as raw materials, work-in-process inventory, and finished products that flow between the facilities (Simchi-Levi&Kaminsky, 2004). Multi-echelon inventory management in supply chains has attracted some researchers; accordingly, these researches lead to Vendor Managed Inventory (VMI) models. The VMI model is a cooperative communication innovation where suppliers are permuted to manage the retailer's inventory. Vendors can manage retailer's orders and total inventory data between retailers by utilizing of information technologies such as Electronic Data Interchange (EDI) on a real time basis (Yao et al., 2007). The VMI is a business model in which the vendor is a responder to control the retailer's inventory levels and then determines the retailer's order quantity and time

(Disney&Towill, 2002). The possible advantages of the VMI models include a reduction of inventory costs for the supplier and the retailer and improvement of customer service levels (Achabal et al., 2000). Successful retailers and suppliers such as Kmart, Dillard Department Stores, JcPenney and Wal-Mart achieved these advantages (Dong&Xu, 2002). In other words, in these models, the supplier determines quantity of replenishment for a retailer in the specified time horizon, with regard to the minimum total inventory cost in the supply chain. According to the cost reduction, determination of the amount of orders is one of the important decisions that suppliers are involved in the supply chain. Two general models of economic order quantity (EOQ) and economic production quantity (EPQ) are frequently used. The order size, which minimizes the total inventory cost, is known as the EOQ. The EPQ model applies the logic of EOQ to parts that are made, as opposed to those purchased from an outside vendor. The EOQ is one of the most popular and successful optimization models in SCM, due to its simplicity of using, simplicity of concept, and robustness (Axsäter, 2010).

All of the mentioned models have been developed based on some basic assumptions, with regard to their applications in the real situations. In this paper, research is concentrated on this scenario: there is a single vendor who supplies multi products for a single retailer and the model has been completed by multi-constraint. These

---

[*] Corresponding author E-mail address: Keeyarash@gmail.com

constraints have an important influence on the conformity of model with inventory systems in the real world. In this model, shortages are backordered. In addition, the vendor's warehouse is limited by an upper bound for available inventory or maximum inventory. Constraints like number of orders, available capital, and average inventory have an important role in the inventory systems, so that they have been considered in the proposed model. The objective is determination of order quantities, number of shipments received by a retailer and maximum backorder levels for each product at a cycle time, with respect to minimization of total inventory cost in the desired supply chain. Under these conditions, the developed model is an integer nonlinear programming (INLP), therefore a proposed genetic algorithm (GA) via parameter-tuned with response surface methodology (RSM) is presented to find optimum values for the decision variables. Finally, results are illustrated via a numerical example.

This paper is structured as follows: in next Section, a review of the literatures and contributions is presented. Section 3 provides the notations and assumptions utilized for the problem description. The mathematical model is developed in Section 4 and a genetic algorithm has been presented in Section 5. Subsequently RSM parameter-tuned is described in Section 6 and numerical example is presented in Section 7. Finally, conclusions and recommendations for future research are mentioned in Section 8.

2. Literature Review

Determination of when and how much to order is the aim of an inventory control system. The most well-known inventory control model is the classical EOQ formula. The first EOQ formula was presented by Harris (1913), but Wilson (1934) is also recognized in connection with this model (Axsäter, 2010, Tersine, 1993). Then, the model was extended to EPQ in which production rate was considered in the model. The EOQ and EPQ inventory systems have been used in many practical applications, because these inventory models are simple and easy to implement in organizations, but the EOQ and EPQ inventory models have several assumptions that are very restrictive (Cárdenas&Leopoldo, 2009). Since the EOQ and the EPQ are obtained with some assumptions and conditions that their applications are limited in real issues, some researchers such as Goyal (Goyal, 1985), Chung (1998) have tried to develop formulated inventory models for more real issues.

The retailer's inventory system can be described by an EOQ policy based on deterministic demand and deterministic lead-times (Dong&Xu, 2002). Different companies work together to improve the coordination of the total material flow; an example is the implementation of so-called VMI systems (Axsäter, 2010). Next, Hill (1997) researched to minimize the total cost per year of the buyer–vendor system.

The basic hypothesis is that the vendor only knows the buyer's demand and his order frequency. Some researchers have begun to investigate the usefulness of implementing VMI in the inventory systems for supply chain and have showed cost reduction under the assumed conditions. By raising the VMI policy, companies utilize impact of the VMI policy as a tool to reduce costs in the inventory system for the supply chain (Disney&Towill, 2003). For instance, the usefulness of the VMI implications is achieved with coordination between retailers and suppliers, such as Kmart, Dillard Department Stores, JcPenney, and Wal-Mart (Simchi-Levi & Kaminsky, 2004).

The single-vendor single-buyer supply chain was investigated for a product just in case of the EPQ environment by Hill (1997). Next, Disney et al. (2003) investigated the effect of a VMI policy on transport cost savings in a supply chain. Sarmah et al. (2006) provided a literature review about trading with buyer and vendor coordination models in case of EOQ assumptions. Huang et al. (2010) extend Goyal model, as a cost reduction for the single product for single-vendor and single-buyer supply chain situation under conditions of order-processing cost. Sajadieh et al. (2009) extended a coordinated vendor–buyer model for stochastic lead-times in which shortage is allowed. Duan et al. (2010) presented inventory model for a single-vendor single-buyer supply chain for fixed lifetime product in which quantity discount policy is applied.

Meantime, the research used an evolutionary approach like GA in the supply chain problems. Pasandideh and Niaki (2008) extended the EPQ model under the condition that discrete delivery for orders was considered in the formation with multiple pallets; because an INLP model was developed, a GA was presented to solve it. Nachiappan and Jawahar (2007) considered optimum sales quantity for each buyer, under the VMI mode and provided a GA based heuristic model. Michaelraj and Shahabudeen (2009) considered two objectives including maximizing the distributors' sell and minimizing the distributors' balance payment in the VMI distribution system and used a GA to solve them. Pasandideh et al. (2011) formulated an INLP model and proposed a GA to determine optimal order quantities and backorder levels for reducing cost in a VMI system with storage space and number of order constraints. In addition, Rezaei and Davoodi (2011) combined the lot-sizing problem with supplier selection and present two multi-objective models with regard to shortages, and then proposed a GA to solve it. Therefore, according to these studies, the researchers obtained an INLP model and proposed GA to solve it.

3. Notation and Assumptions

The following set of notations will be used in this research:

A_i	vendor's order cost for the ith product
a	vendor's order cost of the ith product (per unit) for retailer
D_i	demand rate for the ith product
p	annual holding cost as a fraction of unit cost for retailer
p'	annual holding cost as a fraction of unit cost for vendor
u_i	purchase cost for ith product
$\hat{\pi}$	Fixed backorder cost per unit in each period
π	Fixed backorder cost per unit
V_i	space required to store one unit of ith product
W	maximum utilization of vendor's warehouse space
X	total number of vendor's order
O	maximum available capital for vendor
Z	upper limit for vendor's average inventory level
n	number of shipments received by a retailer
q_i	quantity of ith product dispatched to retailer
b_i	maximum backorder level of ith product for vendor
Q_i	order quantity of ith product for vendor
T	vendor cycle time
T_i	cycle time for the ith product
T_R	common cycle time for products
TC_R	total retailer's cost in case of VMI system
TC_V	total vendor's cost in case of VMI system
TC_{vmi}	total cost incurred by VMI system

Consider a single-vendor single-retailer supply chain consist of m products according to pervious mentioned assumptions for the proposed model. It is assumed that, inventory level is available inventory with respect to three constraints: available capital, vendor's warehouse capacity and average inventory, i.e. (Q_i - b_i). An important point must be considered if the model is the EPQ, or encounters with shortages as backordered, the maximum inventory must use for applying warehouse capacity and available capital constraints (Axsäter, 2010, Tersine, 1993). Under VMI system policy, the vendor manages the holding and ordering costs and forwards cost to the retailer. When the retailer's inventory level goes down to reorder point R, a batch quantity of size q is ordered. Moreover, exceeding demand will be repaid and any surplus shipment is not allowed. Furthermore, it is assumed that retailer sells all of products received from the vendor. Thus annual demand for the vendor and retailer is the same and is deterministic.

Under the VMI strategy, a retailer's order cost is smaller than the retailer's order cost in case of no-VMI strategy (Yao et al., 2007). As it is assumed, the vendor dispatches products at the same time, i.e. $T_i = T_j = T_R$. It is logical in VMI policy because the vendor selects the best alternative for relationships between the time and volume of replenishment (Darwish&Odah, 2010). Therefore, the vendor a lot of size Q_i to a retailer transferred that takes n shipments each of size q_i. Following equations represent the relationship between the delivered products to the retailer:

$$\frac{q_i}{D_i} = \frac{q_j}{D_j} \tag{1}$$

So,

$$\frac{q_1}{D_1} = \frac{q_i}{D_i} \tag{2}$$

And

$$Q_i = nq_i \tag{3}$$

Finally, the specifications of the supply chain in which vendor and retailer cooperate are defined as follows:
Finally, the specifications of the supply chain in which vendor and retailer work together are defined as follows:

1. Vendor decides for the timings and the quantities of production considering inventory cost that is the total cost of the VMI system.
2. Shortages are allowed and backordered.
3. Lead-time is zero and inventory system follows immediately replenishment.
4. All of costs are fixed.
5. The rate of production for all products is infinite (EOQ model).
6. Vendor's warehouse capacity is fixed and pre-determined.
7. Available inventory has an upper bound.
8. Number of vendor's order is limited.
9. Available capital is finite.

4. Defining the Mathematical Model

In this section, the mathematical model is defined with respect to the aforementioned assumptions for the VMI system where intents to minimize the total inventory cost in a supply chain per unit time T. The model involves costs such as holding, ordering and shortage cost as well as the purchase cost. According to the assumptions, total retailer's cost is calculated as follows:

$$TC_R = \sum_{i=1}^{m} \frac{pu_i q_i}{2} \tag{4}$$

Although the order cost involved TC_R, but it belongs to the vendor's cost in the VMI system. Based on Eqs. (5) and (3) the following equations will be provided:

$$Q_i = (D_i q_1 n / D_1) \tag{5}$$

$$q_i = (D_i q_1 / D_1) \tag{6}$$

As a result, the total vendor's cost has been calculated here:

$$TC_V = \sum_{i=1}^{m} \left(\frac{A_i D_i}{Q_i} + \frac{a'_i D_i}{q_i} + p' u_i \frac{(Q_i - b_i)^2}{2Q_i} \right.$$
$$\left. \frac{\hat{\pi} b_i^2}{2Q_i} + + \frac{\pi b_i D_i}{Q_i} + D_i u_i \right) \qquad (7)$$

Therefore, the total cost for the whole system is:

$$TC_{VMI} = TC_R + TC_V \qquad (8)$$

Total cost incurred by the VMI system, with regard to Eqs. (5) and (6) can be determined as follows.

$$TC_{VMI}(b, q_1, n) =$$

$$\sum_{i=1}^{m} \left(\frac{D_i p u_i q_1}{2D_1} + \frac{A_i D_1}{nq_1} + \frac{a'_i D_i D_1}{D_i q_1} \right) +$$

$$\sum_{i=1}^{m} \left(p' u_i \frac{((D_i n q_1 / D_1) - b_i)^2}{2(D_i n q_1 / D_1)} \right) +$$

$$\sum_{i=1}^{m} \left(\frac{D_1 \hat{\pi} b_i^2}{2D_i n q_1} + \frac{D_i D_1 \pi b_i}{D_i n q_1} + D_i u_i \right); \qquad (9)$$

According to the previous statements, the aim of this research is to calculate synchronous order quantities, number of shipments received by a retailer and maximum backorder levels for each product in a cycle time with respect to the vendor's warehouse capacity; *W* as follows:

$$\sum_{i=1}^{m} V_i (Q_i - b_i) \leq W \qquad (10)$$

In addition, the amount of available capital is *O*,

$$\sum_{i=1}^{m} u_i (Q_i - b_i) \leq O \qquad (11)$$

and *Z* is upper bound of vendor's available inventory,

$$\sum_{i=1}^{m} \frac{(Q_i - b_i)^2}{2Q_i} \leq Z \qquad (12)$$

Finally, number of vendor's order is bounded to *X*,

$$\sum_{i=1}^{m} \frac{D_i}{Q_i} \leq X \qquad (13)$$

Hence, the mathematical model can be set out as follows:

$$\text{Min } TC_{VMI} =$$

$$\sum_{i=1}^{m} \left(\frac{D_i p u_i q_1}{2D_1} + \frac{A_i D_1}{nq_1} + \frac{a'_i D_i D_1}{D_i q_1} \right) +$$
$$\sum_{i=1}^{m} \left(p' u_i \frac{((D_i n q_1 / D_1) - b_i)^2}{2(D_i n q_1 / D_1)} \right) +$$

$$\sum_{i=1}^{m} \left(\frac{D_1 \hat{\pi} b_i^2}{2D_i n q_1} + \frac{D_i D_1 \pi b_i}{D_i n q_1} + D_i u_i \right);$$
s.t,
$$\sum_{i=1}^{m} V_i ((D_i n q_1 / D_1) - b_i) \leq W$$
$$\sum_{i=1}^{m} u_i ((D_i n q_1 / D_1) - b_i) \leq O$$
$$\sum_{i=1}^{m} \frac{((D_i n q_1 / D_1) - b_i)^2}{2(D_i n q_1 / D_1)} \leq Z$$
$$\sum_{i=1}^{m} \frac{D_i}{(D_i n q_1 / D_1)} \leq X$$
$$\sum_{i=1}^{m} b_i \leq Q_i$$
$$b_i, q_1, n > 0 : integer$$
$$i = 1, 2, 3, \ldots, m. \qquad (14)$$

Where $b_i \leq Q_i$ means that backorder levels cannot bigger than order quantity. In next section, a proposed GA will be presented to solve the obtained model in Eq. (14).

5. Solution Algorithm

The provided model by Eq. (14) is an INLP problem; solving the INLP problems are hard with exact methods because the INLP is an NP-complete problem (Kuk, 2004). Exact methods are complex and not very affective for solving the INLP models. In the past decades, applying genetic algorithms (GAs) was developed to solve the INLP problems that have been a developing attempt. Researchers have obtained many convenient alternatives of GAs for different nonlinear problems. GAs is one of the important tools to find feasible solutions in these kinds of problems (Mitsuo Gen, 2000). GAs are strong tools for solving the INLP models (Yokota et al., 1996). Therefore, in this section, a proposed GA has been presented to solve the mathematical model.

5.1. Genetic Algorithm

GAs pertains to the larger class of evolutionary algorithms (EA) which generate solutions for optimization problems using techniques derived by natural evolution. In a GA, every unknown parameter of the problem called a gene and the chromosome is set of genes; in brief, the initial population is generated randomly which included candidate solutions (are called

individuals or chromosomes) according to the GA operators as crossover and mutation operators to an optimization problem. Traditionally, the evolution usually starts from a population of randomly generated individuals that occurs in the generations. In each generation, the fitness of every individual is evaluated in the population; multiple individuals are selected stochastically from the current population with respect to their fitness. Then new population (new chromosomes), called offspring can be created by modifying previous individuals and will be used in the next iteration of the algorithm. As a general rule, the algorithm terminates when either a maximum number of generations has been produced, or a satisfactory fitness level has been reached for the population (Pasandideh&Niaki, 2008).

GAs were first presented by John Holland in 1960 (Holland, 1992), but the custom form of the GA was explained by his student Goldberg in 1989 (Michaelraj & Shahabudeen, 2009). The GA is a heuristic search that mimics the process of natural evolution as regards Darwin's theory of evolution (Talbi, 2009). However, apart from the effective of definition, the chromosome on the qualification solution, the GA is known as a problem-independent approach (Pasandideh et al., 2011).

5.1.1. GA Algorithm in Initial and General Conditions

The required primary pieces of information for starting GA is:
 1. Population size (N_{pop}): A group of interbreeding individuals
 2. Crossover rate (Pc): Crossover probability
 3. Mutation rate (Pm): Mutation probability
 The general stages in genetic algorithm are as follows:
1. Initialization.
 1.1 Set the parameters (N_{pop}, P_c, P_m, stopping criteria, selection strategy, crossover operation, mutation operation, and number of generation)
 1.2 Generate an initial population randomly
2. Compute and save the fitness for each individual in the current population
3. Define selection probabilities for each individual based on fitness criteria
4. Generate the next population by selecting individuals from current population randomly to produce offspring via GA operators such as crossover and mutation operators
5. Repeat step 2 until stopping criteria is satisfied.
 According to what follows, the proposed GA is described in details.

5.2. Chromosome Representation

Designing a suitable chromosome is the most important stage in applying the GA in the solution process of the problem. The chromosome in which represents the number of shipments received by a retailer (n), the quantity of the first product dispatched to the retailer (q_1), and the maximum backorder level of the products for the vendor (b_i), is provided by a $1*$ ($m+2$) matrix. The first element of the matrix is n and from the second element of matrix to the penult is b_i and last element presents q_1. Fig. 1 represents the general form of a chromosome.

$$[n \qquad b_1 \ \ldots \ b_m \qquad q_1]$$

Fig. 1. The chromosome presentation

5.3. Evaluation and Initial Population

In this step, an initial population (or collection of chromosomes) is generated randomly. After the new chromosomes generation, there are some chromosomes do not satisfy model constraints in Eq. (14); so, the generation of the chromosomes is controlled via Death Penalty method to generate feasible chromosomes. The penalty method changes a constrained optimization problem to an unconstrained optimization problem via associating a penalty or cost with all constraint violations. This penalty is included in the objective function evaluation. Thus, chromosomes will be generated without any penalty for mating pool.

5.4. Genetic Operators

Genetic operators such as crossover and mutation operators generate the next population.

5.4.1. Crossover

The main reproduction operator used is the crossover. Two strings are used as parents and new individuals are formed by swapping a sub-sequence between the two strings. Crossover creates offspring via mating pairs with respect to selection of a pair of chromosomes from the random generation with crossover rate P_c. Many crossover techniques exist for organisms such as One-point, Two-points, Multiple-points and uniform. In this research, a Two-point crossover operator is selected that works as follows:
 i.   Two crossover points are chosen randomly
 ii. The contents between these points are exchanged between two mated parents
Fig. 2 shows a graphical representation of the crossover operation for the proposed chromosome with four products.
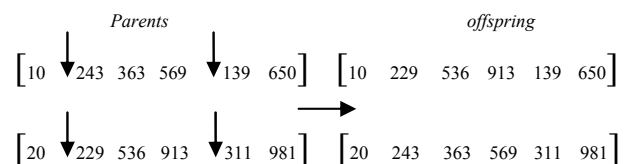

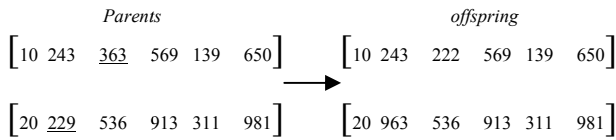Fig. 2. An example of the Two-point crossover operation

Parents   offspring

$\begin{bmatrix} 10 & 243 & \underline{363} & 569 & 139 & 650 \end{bmatrix}$ → $\begin{bmatrix} 10 & 243 & 222 & 569 & 139 & 650 \end{bmatrix}$

$\begin{bmatrix} 20 & \underline{229} & 536 & 913 & 311 & 981 \end{bmatrix}$ $\begin{bmatrix} 20 & 963 & 536 & 913 & 311 & 981 \end{bmatrix}$

Fig. 3. An example of the mutation operation

5.4.2. Mutation

After crossover, each parent in the mating pool is mutated with the mutation rate. Mutation prevents trapping the algorithm in a local minimum. It is applied to a single chromosome. In this stage, the mutation calls Random Mutation; because, Random Mutation selects a chromosome from the population, and changes one of the selected random genes with a gene that is generated randomly. Fig. 3 shows the mutation operation for the proposed chromosome with four products.

5.5. Chromosomes Selection and Search Termination

Feasible chromosomes have to compete to candidacy in the next generation. The selection is one of GA operators that select chromosomes from the current generation for inclusion in the next generation. The selection operator impresses on the generation performance with respect to their fitness. There are several types of the selection, such as the elitist, proportional, ranking, tournament and roulette wheel introduced by Michalewicz (1996). Since chromosomes with higher fitness should have a greater chance of selection than those with lower fitness, the roulette wheel selection is used to select the chromosomes of this research. In this method, the selection operates proportional to relative fitness of the chromosomes. Consequently, N_{pop} chromosomes are selected through the parents and offspring according to the best fitness values.

In the GAs, employment of the termination condition is the last step. The generation process is continued until a termination condition is satisfied. In this research, the determined number of generations is used for termination condition via tuning parameters by using design of experiments methods.

6. Parameter Tuning

The parameters of GA impress the solution qualification. There are several ways to tune parameters: choosing suggested amounts by other researchers or trustful on a trial and error procedure. These ways cannot certify qualification of the solution, so that another method maybe used to find the best set of the GA parameters is applying design of experiments (DOE) technique in which works based on of statistical and mathematical methods. In this research, response surface methodology approach (RSM) using systematic experiments was utilized for tuning the GA parameters. RSM is a set of useful statistical and mathematical

techniques for optimization, specially exploration of relationships between several explanatory variables and one or more performance measures or quality characteristics; called responses. In Fig. 4 one surface plot is presented. Here the aim is to find the amounts of the GA parameters as input variables to obtain an optimal response (Y). Thus, k factors that affect the response are: the population size (N_{pop}); the maximum number of generations; the crossover probability (P_c); the mutation probability (P_m) and the problem size (P_s). Table 1 presents the levels of the input variables. The values of N_{pop}, number of generations, P_c, P_m, P_s are coded as X_1, X_2, X_3, X_4, and X_5 and their low, middle, and high measure for each given variable is −1, 0, and 1 respectively.

Table 1
Experimental range and level of each factor

Factors	Symbol	Range		level	
N_{pop}	X_1	90-110	90	100	110
Generation	X_2	200-800	200	500	800
P_c	X_3	0.7-0.8	0.7	0.75	0.8
P_m	X_4	0.2-0.3	0.2	0.25	0.3
n	X_5	4-10	4	7	10

The main idea of the RSM is using a sequence of experiments, designed to obtain an optimal response. The first step is usually to fit a first-order model (linear model) with performing the lack of fit test. If the test indicates the first-order model is inadequate, then a second-order model can be used. Second-order models are very flexible. Eq. (15) represents a second order (Raymond H. Myers, 2009):

$$Y = \beta_0 + \sum_{i=1}^{k} \beta_i X_i + \sum_{i=1}^{k} \beta_i X_i^2 + \sum_{i<j}^{k} \sum_{i<j}^{k} \beta_{ij} X_i X_j \qquad (15)$$

Y is the response variable and β_0, β_i, β_{ij} are the regression coefficients must be calculated. It is important to check the adequacy of the fitted model, because an incorrect or under-specified model can lead to misleading conclusions. The small p-value (p = 0.001) for the lack of fit test indicates the first-order model does not fit the response surface adequately. So that it is needed to fit a second-order model. In the other words, it is needed to the second-order model because there is curvature in the first-order model (*P-value*=0.001<0.05).
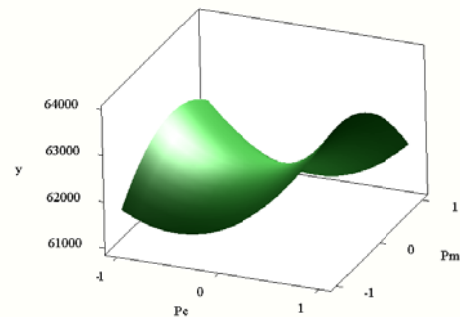
Fig. 4. Surface plot of Y vs. P_m and P_c.

Note that for fitting the first-order model, matrix without axial points represented by Table 3 is used.

Table 2
The example information

Product(i)	D_i	A_i	a'	u_i	V_i
1	420	4	3	13	3
2	360	3	2	30	2
3	540	4	1	23	3
4	390	5	4	6	1
5	480	3	2	13	4
6	510	3	2	20	3
7	530	4	3	16	2
8	380	2	1	10	1
9	430	5	4	6	3
10	580	3	2	26	4

O=130000, W=18000, Z=250, X=8, π=0, $\hat{\pi}=3$, p=0.3, p'=0.4.

Table 3
Central composite design matrix

		Input variables					Response
Run	PTYPE	X_1	X_2	X_3	X_4	X_5	Y
1	1	1	-1	-1	-1	-1	33104.8
2	1	-1	1	-1	-1	-1	33038.7
3	1	-1	-1	1	-1	-1	34190.8
4	1	1	1	1	-1	-1	33075.5
5	1	-1	-1	-1	1	-1	35028.1
6	1	1	1	-1	1	-1	32968.5
7	1	1	-1	1	1	-1	33182.1
8	1	-1	1	1	1	-1	33376.9
9	1	-1	-1	-1	-1	1	86935.6
10	1	1	1	-1	-1	1	84667.4
11	1	1	-1	1	-1	1	93434.7
12	1	-1	1	1	-1	1	84918.3
13	1	1	-1	-1	1	1	86385.9
14	1	-1	1	-1	1	1	85218.4
15	1	-1	-1	1	1	1	86992.8
16	1	1	1	1	1	1	84635.7
17	0	0	0	0	0	0	62914.0
18	0	0	0	0	0	0	61670.7
19	0	0	0	0	0	0	62914.0
20	0	0	0	0	0	0	62914.0
21	-1	1	0	0	0	0	61017.7
22	-1	-1	0	0	0	0	62017.7
23	-1	0	1	0	0	0	60752.6
24	-1	0	-1	0	0	0	62913.3
25	-1	0	0	1	0	0	63510.6
26	-1	0	0	-1	0	0	61889.0
27	-1	0	0	0	1	0	60725.4
28	-1	0	0	0	-1	0	61000.6
29	-1	0	0	0	0	1	84912.2
30	-1	0	0	0	0	-1	32977.4
31	0	0	0	0	0	0	62914.0
32	0	0	0	0	0	0	62914.0

Central composite design (CCD) is one of the designs that fit the second-order model. CCD consists of 2^{k-1} factorial points, where k is the number of factors, $2k$ axial points $(\pm 1, 0, 0, 0, 0)$, $(0, \pm 1, 0, 0, 0)$, $(0, 0, \pm 1, 0, 0)$, $(0, 0, 0, \pm 1, 0)$, $(0, 0, 0, 0, \pm 1)$ and n_c center points $(0, 0, 0, 0, 0)$ that was used in random orders. Table 3 summarizes the obtained response; in other words, the last column displays the best fitness value for each obtained problem with respect to proposed points. For details in Table 3, the PTYPE column displays the type of the design points (-1, 0, 1 are the axial points, the central points and the factorial points, respectively).
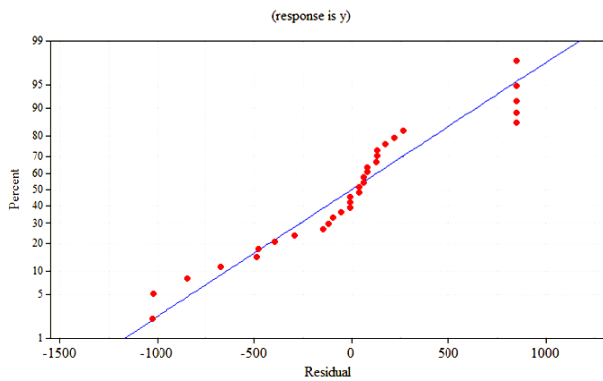

Fig. 5. Residual normal probability plot

6.1. Results

Three variant problem instances of size 4, 7 and 10 products are generated to evaluate the GA parameters in Table 2. According to the procedure of DOE, to use RSM, a second-order model is needed because there is curvature in the model regression. Experiments have been analyzed with Minitab software (Version Minitab® 15.1.30.0) to fit the data by a regression model.

In the model adequacy checking, the second order model adequately fits the response surface. With respect to the lack of fit test in the Table 4 and graphical analysis of residuals represented by Fig. 5 that shows the residuals

are structureless; hence, the model is adequate. The analysis of variance (ANOVA) and the coefficients of determination (R^2) are utilized to consider the goodness of fit. Table 4 shows ANOVA for the model regression. Because value of F_0 (894.74) is large, we would conclude that at least one variable has a nonzero effect, and the model is significant. R^2 statistic measures the proportion of total variability explained by the model. R^2_{adj} is a statistic that is adjusted for the size of the model. R^2 and R^2_{adj} ($0 < R^2 < 1$) statistics are evidence for fitness of the regression model. Since they are big enough, the proposed regression model has been fitted well (Montgomery, 2001). The analysis was performed for 5% significance level. Since the lack of fitness is not significant at the 0.05 level (*P-value* > 0.05), it confirms the good predictability of the model has been verified. The insignificant terms were eliminated regarding to their *P-values*. Deletion of the nonsignificant coefficients from the full model concludes the final model in which works better as a predictor for new data.

6.2. Discussion

Table 5 presents the results were used to estimate the second-order model for the response. In Eq. (16) the fitted response, a regression model obtained from only 32 observations with 20 variables, has been shown. Since the aim is to find the best GA parameters so that fitness value optimized, it is necessary to solve Eq. (16). Results are gathered in Table 6 and indicate optimum values of the GA parameters for four products.

$$
\begin{aligned}
Y = {}& 62064.1 + 42X_1 - 1084.2X_2 + 448.9X_3 - 325.2X_4 + \\
& 26508.8X_5 - 64.4X_1^2 + 250.9\,X_2^2 + 1117.7\,X_3^2 - \\
& 719.1X_4^2 - 2637.3X_5^2 - 260.3X_1X_2 + 496.5X_1X_3 - \\
& 540.2X_1X_4 + 522.6X_1X_5 - 389.5X_2X_3 + 411X_2X_4 - \\
& 703.9X_2X_5 - 580.4X_3X_4 + 443.1X_3X_5 - 491.8X_4X_5.
\end{aligned}
$$

$$(16)$$

Table 4
Analysis of variance for fitness

Source	DF	Seq SS	Adj SS	Adj MS	*F*	*P-value*
Regression	20	12757463293	12757463293	637873165	894.74	0.000
Linear	5	12675614821	12675614821	2535122964	3556.00	0.000
Square	5	42320830	42320830	8464166	11.87	0.000
Interaction	10	39527641	39527641	3952764	5.54	0.005
Residual Error	11	7842058	7842058	712914		
Lack-of-Fit	6	6553921	6553921	1092320	4.24	0.067
Pure Error	5	1288137	1288137	257627		
Total	31	12765305351				

Table 5
Estimated Regression Coefficients for *Y*

Term	Coef	SE Coef	*T*	*P*
Constant	62064.1	241.3	257.206	0.000
X_1	42.0	199.0	0.211	0.837
X_2	-1084.2	199.0	-5.448	0.000
X_3	448.9	199.0	2.256	0.045
X_4	-325.2	199.0	-1.634	0.131
X_5	26508.8	199.0	133.201	0.000
$X_1 X_1$	-64.4	538.2	-0.120	0.907
$X_2 X_2$	250.9	538.2	0.466	0.650
$X_3 X_3$	1117.7	538.2	2.077	0.062
$X_4 X_4$	-719.1	538.2	-1.336	0.209
$X_5 X_5$	-2637.3	538.2	-4.900	0.000
$X_1 X_2$	-260.3	211.1	-1.233	0.243
$X_1 X_3$	496.5	211.1	2.352	0.038
$X_1 X_4$	-540.2	211.1	-2.559	0.027
$X_1 X_5$	522.6	211.1	2.476	0.031
$X_2 X_3$	-389.5	211.1	-1.845	0.092
$X_2 X_4$	411.0	211.1	1.947	0.077
$X_2 X_5$	-703.9	211.1	-3.335	0.007
$X_3 X_4$	-580.4	211.1	-2.750	0.019
$X_3 X_5$	443.1	211.1	2.099	0.060
$X_4 X_5$	-491.8	211.1	-2.330	0.040

S = 844.343; *R-Sq* = 99.94%; R-*Sq* (*pred*) = 98.31%; R-*Sq* (*adj*) = 99.83%.

Table 6
Optimum value of the input variables

Variable	Value
Npop	110
Generation	800
Pc	0.725
Pm	0.2

In the next section, a numerical example will be presented to describe the sufficiency of the proposed strategy with respect to parameter-tuned by response surface methodology (RSM).

7. A Numerical Example

The GA is coded by a Matlab computer program (Version 7.11.0.584, R2010b) and numerical example is solved by a PC CPU Duo T6600 2.20 GHz and 4GB RAM under the windows 7 operating system. The single-vendor single-retailer problem with ten products has been investigated according to given data in Table 2. In this example optimum value of the GA parameters that is presented in Table 6 is applied. The best fitness values of this problem with regard to the proposed algorithm and parameter-tuned is as follows:

Table 7
Best fitness values

Product(*i*)	1	2	3	4	5	6	7	8	9	10
b_i	370	392	542	227	473	505	455	315	333	577

n=25; q_1=21; Q_1=525

TC_{VMI} = 84341.5;

Additionally, the convergence path graph for finding the best fitness values is presented in Fig. 6.
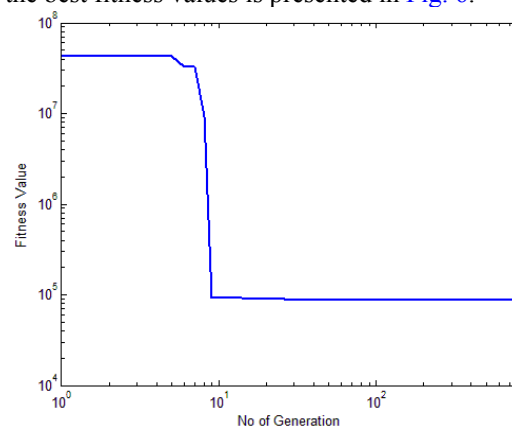

Fig. 6. The convergence path

8. Conclusions and Recommendation for Future Research

In this paper, we have developed an inventory model under conditions multi-product and multi-constraint for the vendor managed inventory (VMI) system in a two-echelon supply chain. Moreover, it was assumed that shortages are backordered, so the vendor's warehouse capacity was limited by an upper bound for available inventory or maximum inventory. Since in inventory models, constraints like number of orders, available capital, and average inventory have an important role, these constraints were added to the model, too. The obtained model was an Integer Nonlinear Programming (INLP) problem; thus, the genetic algorithm (GA) was proposed to solve it. Furthermore, GA parameters were tuned by Response Surface Methodology (RSM) method. RSM method ensures to obtain the best fitness values of this problem and dedicated reasonable amounts for GA parameters. Finally, a numerical example was presented to describe the sufficiency of the proposed strategy.

For future work extensions, the followings are recommended for other researchers:

1. The lead-time effects can be considered.
2. Alternative meta-heuristic search algorithms such as Tabu search (TS) or simulated annealing (SA) can be used.
3. Other situations like variable costs and discounts can be considered.
4. Non-deterministic parameters such as fuzzy or stochastic demand can be considered.
5. Other cases of VMI system like the single-vendor multi-retailer, multi-vendor single-retailer and multi-vendor multi-retailer systems can be modeled.

9. References

[1] Achabal, D.D., Mcintyre, S.H., Smith, S.A., and Kalyanam, K. (2000). A decision support system for vendor managed inventory. Journal of retailing, 76(4), 430-454.

[2] Axsäter, S. (2010). Inventory Control. 2nd ed., Springer, New York, NY, USA.

[3] Cárdenas, B. and Leopoldo, E. (2009). Economic production quantity with rework process at a single-stage manufacturing system with planned backorders. Computers & Industrial Engineering, 57(3), 1105-1113.

[4] Chung, K.-J. (1998). A theorem on the determination of economic order quantity under conditions of permissible delay in payments. Computers & Operations Research, 25(1), 49-52.

[5] Darwish, M.A. and Odah, O.M. (2010). Vendor managed inventory model for single-vendor multi-retailer supply chains. European Journal of Operational Research, 204(3), 473-484.

[6] Montgomery, Dc. (2001). Design and analysis of experiments. 5 ed., John Wiley and Sons, New York, NY USA.

[7] Disney, S.M., Potter, A.T., and Gardner, B.M. (2003). The impact of vendor managed inventory on transport operations. Transportation research part E: logistics and transportation review, 39(5), 363-380.

[8] Disney, S.M. and Towill, D.R. (2003). The effect of vendor managed inventory (VMI) dynamics on the Bullwhip Effect in supply chains. International Journal of Production Economics, 85(2), 199-215.

[9] Disney, S.M. and Towill, D.R. (2002). A procedure for the optimization of the dynamic response of a vendor managed inventory system. Computers & Industrial Engineering, 43(1-2), 27-58.

[10] Dong, Y. and Xu, K. (2002). A supply chain model of vendor managed inventory. Transportation research part E: logistics and transportation review, 38(2), 75-95.

[11] Duan, Y., Luo, J., and Huo, J. (2010). Buyer-vendor inventory coordination with quantity discount incentive for fixed lifetime product. International Journal of Production Economics, 128(1), 351-357.

[12] Goyal, S.K. (1985). Economic Order Quantity Under Conditions of Permissible Delay in Payments. Operational Research Society, 36, 335-338.

[13] Hill, R.M. (1997). The single-vendor single-buyer integrated production-inventory model with a generalised policy. European Journal of Operational Research, 97(3), 493-499.

[14] Holland, J.H. (1992). Adaptation in Natural and Artificial Systems: An Introductory Analysis with Applications to Biology, Control, and Artificial Intelligence. MIT Press, Cambridge, USA.

[15] Huang, C.-K., Tsai, D.-M., Wu, J.-C., and Chung, K.-J. (2010). An integrated vendor-buyer inventory model with order-processing cost reduction and permissible delay in payments. European Journal of Operational Research, 202(2), 473-478.

[16] Kuk, G. (2004). Effectiveness of vendor-managed inventory in the electronics industry: determinants and outcomes. Information & Management, 41(5), 645-654.

[17] Michaelraj, L.A. and Shahabudeen, P. (2009). Replenishment policies for sustainable business development in a continuous credit based vendor managed inventory distribution system. Computers & Industrial Engineering, 56(1), 260-266.

[18] Michalewicz, Z. (1996). Genetic Algorithms + Data Structures = Evolution Programs. Springer-Verlag, Berlin, Germany.

[19] Mitsuo Gen, R.C. (2000). Genetic algorithms and engineering design. 1 ed., John Wiley & Sons New York, NY, USA.

[20] Nachiappan, S.P. and Jawahar, N. (2007). A genetic algorithm for optimal operating parameters of VMI system in a two-echelon supply chain. European Journal of Operational Research, 182(3), 1433-1452.

[21] Pasandideh, S.H.R. and Niaki, S.T.A. (2008). A genetic algorithm approach to optimize a multi-products EPQ model with discrete delivery orders and constrained space. Applied Mathematics and Computation, 195(2), 506-514.

[22] Pasandideh, S.H.R., Niaki, S.T.A., and Nia, A.R. (2011). A genetic algorithm for vendor managed inventory control system of multi-product multi-constraint economic order quantity model. Expert Systems with Applications, 38(3), 2708-2716.

[23] Raymond H. Myers, D.C.M., Christine M. Anderson-Cook. (2009). Response surface methodology: process and

product optimization using designed experiments. John Wiley & Sons New York, NY USA.

[24] Rezaei, J. and Davoodi, M. (2011). Multi-objective models for lot-sizing with supplier selection. International Journal of Production Economics, 130(1), 77-86.

[25] Sajadieh, M.S., Jokar, M.R.A., and Modarres, M. (2009). Developing a coordinated vendor-buyer model in two-stage supply chains with stochastic lead-times. Computers & Operations Research, 36(8), 2484-2489.

[26] Sarmah, S.P., Acharya, D., and Goyal, S.K. (2006). Buyer vendor coordination models in supply chain management. European Journal of Operational Research, 175(1), 1-15.

[27] Simchi-Levi, D. and Kaminsky, P. (2004). Managing the Supply Chain : The Definitive Guide for the Business Professional. McGraw-Hill, New York, NY, USA.

[28] Talbi, E.-G. (2009). Metaheuristics : from design to implementation. John Wiley & Sons, New York, NY, USA.

[29] Tersine, R.J. (1993). Principles of inventory and materials management. 4 ed., Prentice Hall, New Jersey.

[30] Yao, Y., Evers, P.T., and Dresner, M.E. (2007). Supply chain integration in vendor-managed inventory. Decision Support Systems, 43(2), 663-674.

[31] Yokota, T., Gen, M., and Li, Y.-X. (1996). Genetic algorithm for non-linear mixed integer programming problems and its applications. Computers & Industrial Engineering, 30(4), 905-917.