

# A New Hybrid Algorithm to Optimize Stochastic-fuzzy Capacitated Multi-Facility Location-allocation Problem

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## Abstract

Facility location-allocation models are used in a widespread variety of applications to determine the number of required facility along with the relevant allocation process. In this paper, a new mathematical model for the capacitated multi-facility location-allocation problem with probabilistic customer's locations and fuzzy customer's demands under the Hurwicz criterion is proposed. This model is formulated as  $\alpha$ -cost minimization model according to different criteria. Since our problem is strictly Np-hard, a new hybrid intelligent algorithm is presented to solve the stochastic-fuzzy model. The proposed algorithm is based on a vibration damping optimization (VDO) algorithm which is combined with the simplex algorithm and fuzzy simulation (SFVDO). Finally, a numerical example is presented to illustrate the capability of the proposed solving methodologies.

*Keywords:* Fuzzy simulation, Location-allocation problem, Vibration damping optimizatio.

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## 1. Introduction

Location-allocation (LA) problem aims to locate a set of new facilities such that the transportation cost from facilities to customers is minimized. This problem, initially introduced by Cooper [9], has been studied by many researchers in the past years. LA problem was studied in detail in Gen and Cheng [13, 14] where not only all types of cases were discussed but also many models were developed.

Logendran and Terrell [22] firstly introduced the stochastic uncapacitated facility Location-allocation (FLA) model. Zhou [32] proposed the expected value model, chance-constrained programming and dependent-chance programming for uncapacitated LA problem with stochastic demands. To solve LA models, numerous solving methodologies have been presented. It is worth mentioning that Sherali and Nordai [29] showed that capacitated LA problem is NP-hard even if all the customers are located on a straight line. The capacitated continuous location-allocation problem is also called capacitated multi source Weber problem. Kuenne and Soland [18] used a branch-and-bound algorithm to obtain the outputs of their proposed model. Aras et al. [2]

proposed a new heuristic method for the capacitated multi-facility Weber problem. The capacitated multi-facility Weber problem is concerned with locating  $m$  facilities in the Euclidean plane and allocating their capacities to  $n$  customers at minimum total cost. Durmas et al. [11] presented the discrete approximation heuristics for the capacitated continuous location-allocation problem with probabilistic customer locations. Altinel et al. [3] presented the location-allocation heuristic for the capacitated multi-facility Weber problem with probabilistic customer locations. Furthermore, many meta-heuristic algorithms involving simulated annealing (SA) were designed by Murray and Church [26]; tabu search by Ohlemuller [27], and so on. Some hybrid algorithms have also been developed such as combination of simulated annealing and random descent method by Ernst and Krishnamoorthy [12] and combination of the Lagrange relaxation method and genetic algorithm (GA) by Gong et al. [15]. Zhou and Liu [34] proposed the expected value model, chance-constrained programming and dependent-chance programming for capacitated LA problem with stochastic demands.

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To make the models more realistic, many researches assumed that customer's demand is stochastic, while in a vast range of situations it is not applicable. The reason is that due to the lack of data in such situations, the estimations of probability distributions for demands of customers are complicated. In this regard, expert viewpoints are analyzed to estimate the corresponding parameters. To do so, the incorporation of fuzzy set theory into LA problem has been carried out in recent researches. In the past decades, there have been many researchers bringing fuzzy theory into facility location problem. Bhattacharya et al. [4, 5] considered new facilities to be located under multiple fuzzy criteria. They also proposed a fuzzy goal programming approach to deal with such problems. Canos et al. [6] categorized quantitative fuzzy models. They specifically discussed the classical p-median problem as a fuzzy model. Chen and Wei [7], Darzentas [10], Rao and Saraswati [28] discussed various facility location problems in fuzzy environment. However, all the parameters in these problems are deterministic, and fuzzy theory is only used to solve the classical mathematical programming effectively. Zhou and Liu [35] proposed three types of programming models with fuzzy demands-fuzzy expected cost minimization model, fuzzy  $\alpha$ -cost minimization model, and credibility maximization model based on different decision criteria. To solve these fuzzy models efficiently, the network simplex algorithm, fuzzy simulation and GA are hybridized to produce a hybrid intelligent algorithm.

One of the most familiar criteria which are more useful in uncertainty environments is the optimistic and pessimistic criterion. In this regard, various criteria have been proposed as a list of properties of rationality and consistency [8, 25]. The most well-known criteria is the Hurwicz criterion presented by Hurwicz [16, 17]. This criterion attempts to find a middle ground between the extremes posed by the optimistic and pessimistic criteria. Instead of assuming total optimism or pessimism, the Hurwicz criterion incorporates a measure of both by assigning a certain percentage weight  $\lambda$  to optimism and  $1-\lambda$  to pessimism  $\lambda \in [0,1]$ . Many researchers use the optimistic criterion or pessimistic criterion to model the FLA problem, which are both extreme cases. In order to compromise these two models, we employed the Hurwicz criterion to model the FLA problem.

As a state-of-the-art study, Wen and Iwamura [31] proposed a new model, namely,  $\alpha$ -cost model under the Hurwicz criterion with fuzzy demands. In order to solve the model, the simplex algorithm, fuzzy simulations, and GA have been integrated to propose a hybrid intelligent algorithm. In the solving methodologies area, Mehdizadeh and Tavakkoli-Moghaddam [23] introduced a new meta-heuristic algorithm namely vibration damping optimization (VDO) which was used to solve parallel machine scheduling problem. This stochastic search

method is inspired by SA algorithm and is created based on the concept of the vibration damping in mechanical vibration.

The goal of this paper is to propose a new mathematical model for the capacitated multi-facility location allocation problem with probabilistic customer's locations and fuzzy customer's demands under the Hurwicz criterion. To this end, the proposed LA model is formulated as  $\alpha$ -cost minimization model under different criteria. To solve this stochastic-fuzzy model, a new hybrid intelligent algorithm is presented. The proposed algorithm is based on VDO algorithm which is combined with the simplex algorithm and fuzzy simulation (SFVDO).

The paper is organized as follows: In Section 2, first, we review the concept of possibility space and credibility of fuzzy variable. Afterwards, the proposed LA problem as fuzzy  $\alpha$ -cost minimization model under the Hurwicz criterion is modeled. The evaluation of the expected distance is carried out in Section 3. The proposed hybrid intelligent algorithm is represented in Section 4. Finally, Section 5 provides a numerical example to illustrate the capability of the proposed solving methodologies.

## 2. The Proposed Fuzzy Location-Allocation Model

First, we briefly review the concepts of possibility space and credibility of fuzzy event and then we describe the proposed model in details.

Let  $\delta$  be a nonempty set,  $P(\delta)$  the power set of  $\delta$ , and  $Pos$  a possibility measure. Then, the triplet  $(\delta, P(\delta), Pos)$  is called a possibility space. A fuzzy variable is defined as a function from a possibility space  $(\delta, P(\delta), Pos)$  to the set of real numbers.

Suppose that  $v$  is a fuzzy variable with membership function  $\mu$ . Then, the possibility, necessity, and credibility of a fuzzy event  $\{v \geq r\}$  can be respectively defined as Eq. (1).

$$Pos\{v \geq r\} = \sup_{u \geq r} \mu(u) \quad (1)$$

$$Nec\{v \geq r\} = 1 - \sup_{u < r} \mu(u)$$

$$Cr\{v \geq r\} = \frac{1}{2} (Pos\{v \geq r\} + Nec\{v \geq r\})$$

Note that a fuzzy event may fail even though its possibility achieves one, and hold even though its necessity is zero. However, the fuzzy event must hold if its credibility is one, and must fail if it is zero.

It should be mentioned that our proposed model is formulated using two well-known models which are the capacitated location allocation problem with fuzzy demands under the Hurwicz criterion (proposed by Wen and Iwamura [31]) and the capacitated multi-facility

Weber problem with probabilistic customer locations (proposed by Altinel et al. [3]). To do that, the following indices, parameters, and decision variables are used:

- $i$ : Index of facilities;  $i = 1, 2, \dots, n$
- $j$ : Index of customers;  $j = 1, 2, \dots, m$
- $a_j = (a_{j1}, a_{j2})$ : Location of customer  $j$ ;  $1 \leq j \leq m$
- $v_j$ : Fuzzy demand of customer  $j$ ;  $1 \leq j \leq m$
- $S_i$ : Capacity of facility  $i$ ;  $1 \leq i \leq n$
- $x_i = (x_{i1}, x_{i2})$ : Location of facility  $i$  as a decision variable;  $1 \leq i \leq n$
- $z_{ij}$ : Quantity supplied to customer  $j$  by facility  $i$  after the fuzzy demands are realized,  $1 \leq i \leq n$ ;  $1 \leq j \leq m$ .

In order to formulate the model, we assumed that the path between any customer and facility is connected and unit transportation cost is proportionate of the quantity supplied and the travel distance and facility  $i$  which located within a certain region as Eq. (2):

$$R_i = \{(x_{i1}, x_{i2}) \mid g_i(x_{i1}, x_{i2}) \leq 0\} \quad (2)$$

For the sake of convenience, the following notation is denoted as Eq. (3).

$$v = (v_1, v_2, \dots, v_m)$$

$$X = \begin{pmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \\ \dots & \dots \\ x_{n1} & x_{n2} \end{pmatrix}$$

$$Z = \begin{pmatrix} z_{11} & z_{12} & \dots & z_{1m} \\ z_{21} & z_{22} & \dots & z_{2m} \\ \dots & \dots & \dots & \dots \\ z_{n1} & z_{n2} & \dots & z_{nm} \end{pmatrix} \quad (3)$$

In a deterministic LA problem,  $X$  and  $Z$  are decision variables, and  $Z$  is decided along with  $X$ . However, in a LA problem with fuzzy demands, the decision  $Z$  will be made every period after the fuzzy demands are realized. For each  $\theta \in \delta$ , the value  $v_j(\theta)$  is a realization of  $v_j$  for each  $j$ . It should be mentioned that an allocation  $z$  is feasible if  $z$  is in the feasible allocation set as follows

$$Z(\theta) = \left\{ \begin{array}{l} z_{ij} \geq 0, i=1, 2, \dots, n, j=1, 2, \dots, m \\ z \mid \sum_{i=1}^n z_{ij} = v_j(\theta), j=1, 2, \dots, m \\ \sum_{j=1}^m z_{ij} \leq s_i, i=1, 2, \dots, n \end{array} \right. \quad (4)$$

Note that  $Z(\theta)$  is to be an empty set for some  $\theta$ . For each fixed  $(x_{i1}, x_{i2})$ , we should determine the optimal allocation  $z^*$  for each  $\theta \in \delta$  in order to minimize the transportation cost  $C(X|\theta)$  where

$$C(X|\theta) = \min_{z \in Z(\theta)} \sum_{i=1}^n \sum_{j=1}^m z_{ij} E(d(x_i, a_j)) \quad (5)$$

If  $Z(\theta)$  is an empty set, Eq. (6) can be defined

$$C(X|\theta) = \sum_{j=1}^m v_j(\theta) \max_{1 \leq i \leq n} E(d(x_i, a_j)) \quad (6)$$

As mentioned earlier, since the demands of customers and locations of customers are fuzzy and probabilistic, respectively, the transportation cost  $C(X|\theta)$  will be stochastic-fuzzy. It is meaningless to minimize the transportation cost directly; therefore we should attempt to utilize other methods to model it.

### 2.1. Hurwicz Criterion

In the early 1950s, investigating various criteria for decision making attracted many scholars. In particular, the decision-theoretic view of statistics advanced by Wald [30] had an obvious interpretation in terms of decision-making under complete ignorance, in which the maximin strategy was shown to be a best response against nature's minimax strategy. Wald's criterion is extremely conservative even in a context of complete ignorance, though ultra-conservatism may sometimes make good sense. Several other criteria were proposed in which a list of properties of rationality and consistency were set forth as a set of axioms to be obeyed by a rational criterion [8, 25]. The most well-known criterion is the Hurwicz criterion, developed by Leonid Hurwicz [16] in 1951, which selects the minimum and maximum payoff to each given action  $x$ , and then associates to each action as the following index:

$$\lambda \max(x) + (1 - \lambda) \min(x).$$

Applying Hurwicz criterion to this study, we get the Hurwicz criterion under fuzzy environment:

$$\lambda f_{\min}(\alpha) + (1 - \lambda) f_{\max}(\alpha) \quad (7)$$

where  $f_{\min}(\alpha)$  and  $f_{\max}(\alpha)$  are the  $\alpha$ -optimistic and  $\alpha$ -pessimistic values defined as following equations:

$$f_{\min}(\alpha) = \min_f \{f \mid Cr\{C(x, y|\theta) \leq f\} \geq \alpha\} \quad (8)$$

$$f_{\max}(\alpha) = \max_f \{ f \mid Cr\{C(x, y \mid \theta) \geq f\} \geq \alpha \} \quad (9)$$

The parameter  $\alpha \in (0,1]$  reflects the level of satisfying the event  $C(x,y|\theta) \leq f$  or  $C(x,y|\theta) \geq f$ . This means that the transportation cost will be blow the  $\alpha$ -optimistic value  $f_{\min}(\alpha)$  with credibility  $\alpha$  and will reach upwards of  $\alpha$ -pessimistic value  $f_{\max}(\alpha)$  with credibility  $\alpha$ . According to Hurwicz criterion, the parameter  $\lambda \in [0,1]$  which reflects the degree of the decision maker's optimism, must be determined by the decision maker. Generally speaking, it is difficult to determine the appropriate  $\lambda$  for decision makers, since it varies from person to person. By varying the parameter  $\lambda$ , the Hurwicz criterion changes into various criteria, e.g., when  $\lambda=1$ , the criterion is the optimistic criterion; when  $\lambda=0$ , it degenerate to a pessimistic criterion. This implies that the Hurwicz criterion is fairly flexible.

### 2.2. $\alpha$ -cost Minimization Model

Chance-constrained programming (CCP) is also applied to solve the practical optimization problems with the requirement that the chance constraints should hold with at least some given confidence levels provided as an appropriate safety margin by the decision-maker. In [27, 28], CCP was applied into stochastic un capacitated and capacitated LA problem, respectively, to meet such a type of requirement. A framework of fuzzy CCP has been presented in Liu and Iwamura [20, 21] and Liu [19]. In a stochastic-fuzzy LA problem, the decision-maker may just want to obtain the optimization goals with some stochastic and fuzzy constraints holding at least some confidence levels. In this article, a novel  $\alpha$ -cost minimization model is proposed based on a new concept of  $\alpha$ -cost. The  $\alpha$ -cost of a stochastic-fuzzy LA problem is defined as  $\min Cr\{f \mid C(X \mid \theta) \leq f\} \geq \alpha$  where  $\alpha$  is the predetermined confidence level. In order to minimize the  $\alpha$ -cost of a stochastic-fuzzy LA problem, the following mathematical  $\alpha$ -cost minimization model is presented:

$$\underset{x}{\text{Minimize}} f$$

Subject to:

$$Cr\{\theta \in \delta \mid C(X \mid \theta) \leq f\} \geq \alpha \quad (10)$$

$$g_k(x_{i1}, x_{i2}) \leq 0, i = 1, \dots, n, k = 1, 2, \dots, p$$

where  $f$  is called  $\alpha$ -optimistic transportation cost, and  $C(X|\theta)$  is defined by Eq.(5) and Eq.(6). Therefore, we provided the FLA model to minimize the  $\alpha$ -pessimistic transportation cost as Eq. (11).

$$\underset{x}{\text{Minimize}} \quad \text{Max } f$$

Subject to:

$$Cr\{\theta \in \delta \mid C(X \mid \theta) \geq f\} \geq \alpha \quad (11)$$

$$g_k(x_{i1}, x_{i2}) \leq 0, i = 1, \dots, n, k = 1, 2, \dots, p$$

### 2.3. $\alpha$ -cost Model Under the Hurwicz Criterion

The  $\alpha$ -cost model, as one of the most frequently used models in FLA problem, follows to minimize the optimistic value or pessimistic value [33, 35]. In order to compromise these two extreme models, we utilized the Hurwicz criterion to model the FLA problem, named  $\alpha$ -cost model under the Hurwicz criterion.

$$\left\{ \begin{array}{l} \underset{x}{\text{Minimize}} \quad (\lambda \min_{f_1} f_1 + (1-\lambda) \max_{f_2} f_2) \\ \text{subject to} \quad Cr\{\theta \in \delta \mid C(X \mid \theta) \leq f_1\} \geq \alpha, \\ \quad \quad \quad Cr\{\theta \in \delta \mid C(X \mid \theta) \geq f_2\} \geq \alpha, \\ \quad \quad \quad g_k(x_{i1}, x_{i2}) \leq 0, k = 1, 2, \dots, p, i = 1, 2, \dots, n \end{array} \right. \quad (12)$$

where  $\lambda \in [0,1]$ . This model is different from the traditional stochastic-fuzzy programming models because there is a sub-optimal problem in the model, i.e.

$$\underset{z \in Z(\theta)}{\text{Minimize}} \sum_{i=1}^n \sum_{j=1}^m z_{ij} E(d(x_i, a_j))$$

Subject to:

$$\sum_{i=1}^n z_{ij} = v_j(\theta) \quad , \quad j = 1, \dots, m$$

$$\sum_{j=1}^m z_{ij} \leq s_i \quad , \quad i = 1, \dots, n \quad (13)$$

$$Z_{ij} \geq 0 \quad , i = 1, \dots, n \quad j = 1, \dots, m$$

The objective of the above model is to get total cost  $C(X|\theta)$ . In this model, the parameter  $\lambda$  should be decided in advance. The process of decision making  $\lambda$  may be different, e.g. it can be made completely by decision makers, investigation and analysis, and even history data. In short, the parameter  $\lambda$  is dissimilar in various cases with different methods. This fact means that the  $\alpha$ -cost minimization model under the Hurwicz criterion is fairly flexible by varying the value  $\lambda$ , e.g., when  $\lambda=1$ , the model degenerates to the  $\alpha$ -cost minimization model (11).

### 3. Expected Distance Evaluation

In this paper, to evaluate expected distances, the forms proposed by Aly and White [1] are presented in two different states as follows:

3.1. State 1

In this state, distance is Euclidean and probability distribution is bi-variate symmetric normal. Therefore,  $E(d(x_i, a_j))$  is evaluated as follow:

$$a_{j1} \sim N(\mu_{j1}, \sigma_j) \quad , \quad a_{j2} \sim N(\mu_{j2}, \sigma_j)$$

$$E(d(x_i, a_j)) = \sqrt{\frac{\pi}{2}} \sigma_j H\left(-\frac{1}{2}, 1, -\frac{\lambda_{ij}^2}{2\sigma_j^2}\right) \quad (14)$$

Where

$$\lambda_{ij} = \sqrt{(x_{i1} - \mu_{j1})^2 + (x_{i2} - \mu_{j2})^2},$$

$$H(u, v, t) = \sum_{k=0}^{\infty} \frac{\Gamma(u+k)}{\Gamma(u)} \frac{\Gamma(v)}{\Gamma(v+k)} \frac{t^k}{k!}$$

where  $\Gamma(\cdot)$  is the gamma function and  $H(\cdot, \cdot, \cdot)$  is the confluent hyper geometric function.

3.2. State 2

In this step, distance is squared Euclidean and probability distribution is bi-variate symmetric normal. Therefore,  $E(d(x_i, a_j))$  is evaluated as follow:

$$a_{j1} \sim N(\mu_{j1}, \sigma_j) \quad , \quad a_{j2} \sim N(\mu_{j2}, \sigma_j)$$

$$E(d(x_i, a_j)) = 2\sigma_j^2 + \lambda_{ij}^2 \quad (15)$$

where  $\lambda_{ij} = \sqrt{(x_{i1} - \mu_{j1})^2 + (x_{i2} - \mu_{j2})^2}$

4. Novel Hybrid Intelligence Algorithm

Generally speaking, uncertain programming models of LA problem are difficult to solve by traditional methods. As mentioned earlier, our problem is overly Np-hard [29]. To do so, a new hybrid intelligent algorithm is presented to solve the stochastic-fuzzy model. In this situation, an appropriate matter is to design efficient hybrid intelligent algorithms for solving them. In this section, we integrated the simplex algorithm, fuzzy simulations, and a new meta-heuristic algorithm namely VDO to produce a hybrid intelligent algorithm for solving  $\alpha$ -cost model under the Hurwicz criterion.

4.1. Computing Uncertain Functions

Uncertain functions consist of probabilistic and fuzzy parameters. Due to its high complexity, we designed some fuzzy simulations to estimate the uncertain functions. First, an uncertain function is presented as the following:

$$U_1(X) \rightarrow \min\{f \mid Cr\{\theta \in \delta \mid C(X \mid \theta) \leq f\} \geq \alpha\} \quad (16)$$

This type of function can be estimated through the following procedure:

**Step 1:** Generate  $\theta_k$  from  $\delta$  such that  $Pos\{\theta_k\} \geq \varepsilon$  for  $k = 1, 2, \dots, M$ , where  $\varepsilon$  is a sufficiently small number and  $M$  is a large number.

**Step 2:** For  $v(\theta_k)$ , solve the linear programming (13) by the simplex algorithm and denote the optimal objective value by  $c_k, k = 1, 2, \dots, M$ .

**Step 3:** Set  $v_k = Pos\{\theta_k\}$  for  $k = 1, 2, \dots, M$ .

**Step 4:** Find the maximal value  $r$  such that  $L(r) \geq \alpha$  holds, where

$$L(r) = \frac{1}{2} (\max_{1 \leq k \leq M} \{v_k \mid c_k \geq r\} + \min_{1 \leq k \leq M} \{1 - v_k \mid c_k > r\}).$$

**Step 5:** Return  $r$ .

Similarly, the second uncertain function can be estimated.

$$U_2(X) \rightarrow \min\{f \mid Cr\{\theta \in \delta \mid C(X \mid \theta) \geq f\} \geq \alpha\} \quad (17)$$

Finally, the total cost is defined as follow:

$$U(X) = \lambda U_1(X) + (1 - \lambda) U_2(X) \quad (18)$$

4.2. SFVDO Hybrid Intelligent Algorithm

Recently, a new heuristic optimization technique based on the concept of the vibration damping in mechanical vibration was introduced by Mehdizadeh and Tavakkoli-Moghaddam [23] named vibration damping optimization (VDO) algorithm. They already utilized the algorithm to solve parallel machine scheduling problem.

Our proposed algorithm is based on a vibration damping optimization (VDO) algorithm which is combined with the simplex algorithm and fuzzy simulation (SFVDO). The proposed SFVDO algorithm is illustrated in the following steps:

**Step 1:** Generating feasible initial solution.

In this paper, the initial solution is uniformly generated from potential region below

$$\{(x, y) \mid g_k(x_i, y_i) \leq 0, i = 1, 2, \dots, n, k = 1, 2, \dots, p\}$$

Solution representation is also provided as follows:



$$v_0 = (x, y) = (x_1, x_2, \dots, x_n, y_1, y_2, \dots, y_n)$$

**Step 2:** Initializing the algorithm parameters which consist of: initial amplitude ( $A_0$ ), max of iteration at each amplitude ( $l_{max}$ ), damping coefficient ( $\gamma$ ), max of iteration of outer loop of algorithm ( $t_{max}$ ), and standard deviation ( $\sigma$ ). Finally, parameter  $t$  is set in one ( $t=1$ )

**Step 3:** Calculating the objective value  $U_0$  for initial solution  $v_0$  by fuzzy simulations in which the simplex algorithm is used to solve the linear programming (13) in one step of simulation process.

**Step 4:** Initializing the internal loop

In this step, the internal loop is carried out for  $l = 1$  and repeat while  $l < l_{max}$ .

**Step 5:** Neighbourhood generation

In this step, neighbourhood structure  $v$  are uniformly generated from potential region below

$$\{(x, y) \mid g_k(x_i, y_i) \leq 0, i = 1, 2, \dots, n, k = 1, 2, \dots, p\}$$

It should be mentioned that the objective value  $U$  for solution  $v$  is calculated similar to step 3.

**Step 6:** Accepting the new solution

Set  $\Delta = U - U_0$ . Now, if  $\Delta < 0$ , accept the new solution, else if  $\Delta > 0$  generate a random number  $r$  between (0, 1);

If  $r < 1 - \exp(-\frac{A^2}{2\sigma^2})$ , then accept a new solution; otherwise, reject the new solution and accept the previous solution.

If  $l > l_{max}$ , then  $t+1 \rightarrow t$ ; otherwise  $l+1 \rightarrow l$  and go back to step 5.

**Step 7:** Adjusting the amplitude

In this step,  $A_t = A_0 \exp(-\frac{\gamma t}{2})$  is used for reducing amplitude at each iteration. If  $t > t_{max}$  return to step 8; otherwise, go back to step 4.

**Step 8:** Stopping criteria

In this step, the proposed algorithm will be stopped after predetermined number of iteration. At the end, best solution is obtained.

## 5. Numerical Example

In order to represent the capability of the proposed hybrid intelligent algorithm to solve the location-allocation problem, a numerical example has been illustrated. Suppose that there are 20 customers whose locations and demands are given in Table 1, and there are

4 facilities to be located whose capacities  $S_i$  ( $i = 1, 2, 3, 4$ ) are 80, 90, 100 and 100, respectively, where all demands are assumed to be trapezoidal fuzzy numbers. In practical problems, the parameter  $\lambda$  can be decided completely by decision makers, investigation and analysis, and even historical data. In this example, we set  $\lambda = 0.5$ .

The proposed algorithms were coded in MATLAB and ran on a PC with Intel® core™ 2Duo CPU 2.00GHz, 4GB of RAM qualifications. The Linprog function of MATLAB was used to solve the linear programming (13) during running program. To generate a suitable test for the presented models, we set the mean of customer coordinates equal to customer's given locations. Furthermore, the standard deviations along both dimensions of customer coordinates were taken equal to 10% of the largest value of the range in which the mean coordinates vary. In all problems for locating each facility, the feasible region is assumed as a rectangle area described by

$$\{0 \leq x_{i1} \leq 100, 0 \leq x_{i2} \leq 100 \quad i = 1, 2, 3, 4\}$$

To represent the capability of the presented algorithm, some problems with different parameters were generated and each problem was run for an equal determined period. This period is the mean of enough time for algorithm to converge to the final output. Moreover, we computed a relative efficiency measure called relative deviation index (RDI).

The relative deviation index (RDI) is computed for each problem as  $|Alg_{sol} - min_{sol}| / |max_{sol} - min_{sol}|$ , where  $Alg_{sol}$  is the performance value of a problem, and  $min_{sol}$  and  $max_{sol}$  are the minimum value and the maximum value, respectively, among the performance values of the same problem. We computed the average, the average RDI ( $RDI_{avg}$ ) and the standard deviation RDI ( $RDI_{std}$ ) for the performance values of all problems.

When  $RDI_{avg} = 0$  it means the best state and  $RDI_{avg} = 1$  indicates the worst one. The experimental results are summarized in Table 4. In the following given tables,  $n$  denotes the number of facility and  $m$  denotes the number of customer.

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Table 1  
Location and demand of 20 customers

Customer No.	(a <sub>j</sub> , b <sub>j</sub> )	v <sub>j</sub>
1	(28,42)	(14,15,16,17)
2	(18,50)	(13,14,16,18)
3	(74,34)	(12,14,15,16)
4	(74,6)	(17,18,19,20)
5	(70,18)	(21,23,24,26)
6	(72,98)	(24,25,26,28)
7	(60,50)	(13,14,15,16)
8	(36,40)	(12,14,16,17)
9	(12,4)	(13,15,16,17)
10	(18,20)	(22,24,26,28)
11	(14,78)	(13,15,16,17)
12	(90,36)	(11,14,15,17)
13	(78,20)	(13,14,15,19)
14	(24,52)	(11,13,14,16)
15	(54,6)	(20,24,25,26)
16	(62,60)	(16,18,19,23)
17	(98,14)	(18,19,21,22)
18	(36,58)	(13,14,16,17)
19	(38,88)	(16,17,19,20)
20	(32,54)	(19,21,24,25)

We compute the average, the average RDI ( $RDI_{avg}$ ), and the standard deviation RDI ( $RDI_{std}$ ) for the performance values of all problems. Value  $RDI_{avg} = 0$  means the best state and  $RDI_{avg} = 1$  indicates the worst one. The experimental results are summarized in Table 4. In the following given tables,  $n$  denotes the number of facility and  $m$  denotes the number of customer.

If we want to minimize the 0.9-cost, we have the following 0.9-cost minimization model under the Hurwicz criterion,

$$\left\{ \begin{array}{l} \min_x \quad (0.5 \min_{f_1} f_1 + 0.5 \max_{f_2} f_2) \\ \text{subject to} \quad Cr\{\theta \in \delta \mid C(X \mid \theta) \leq f_1\} \geq 0.9, \\ \quad \quad \quad Cr\{\theta \in \delta \mid C(X \mid \theta) \geq f_2\} \geq 0.9, \\ \quad \quad \quad 0 \leq x_{i1} \leq 100, \quad i = 1, 2, 3, 4, \\ \quad \quad \quad 0 \leq x_{i2} \leq 100, \quad i = 1, 2, 3, 4, \end{array} \right. \quad (19)$$

Where

$$C(X \mid \theta) = \left\{ \begin{array}{l} \min_{z \in z(\theta)} \sum_{i=1}^4 \sum_{j=1}^{20} z_{ij} E(d(x_i, a_j)), \text{ if } z(\theta) \neq \emptyset \\ \sum_{j=1}^{20} v_j(\theta) \max_{1 \leq i \leq 4} E(d(x_i, a_j)), \text{ otherwise} \end{array} \right. \quad (20)$$

And

$$z(\theta) = \left\{ \begin{array}{l} \sum_{i=1}^4 z_{ij} = v_j(\theta), \quad j = 1, 2, \dots, 20 \\ z_{ij} \leq s_i, \quad i = 1, 2, 3, 4 \\ z_{ij} \geq 0, \quad i = 1, 2, 3, 4, \quad j = 1, 2, \dots, 20 \end{array} \right. \quad (21)$$

with regards to Section 5, for each  $i$  and  $j$  we have:

$$E(d(x_i, a_j)) = \left\{ \begin{array}{l} \sqrt{\frac{\pi}{2}} \sigma_j H\left(-\frac{1}{2}, 1, -\frac{\lambda_{ij}^2}{2\sigma_j^2}\right), \quad \text{Euclidean distance} \\ 2\sigma_j^2 + \lambda_{ij}^2, \quad \text{Squared Euclidean distance} \end{array} \right. \quad (22)$$

In order to solve model (19), the proposed hybrid intelligent SFVDO algorithm was run to obtain the computational results shown in Tables 2 and 3 where  $A_0$  is the initial amplitude,  $l_{max}$  is the maximum of iteration in each amplitude,  $\gamma$  is the damping coefficient,  $\sigma$  is the standard deviation, and also ‘‘cost’’ is the minimal cost which is set based on the values of Table 2, 3, and 4. These numbers are found based on implemented design of the experiment by Mehdizadeh [24]. In this paper, the parameter values are fitted on current values by trial and error approach using several runs. Fig. 1 shows that the proposed SFVDO consistently converges to the near optimal solution.

## 6. Conclusion and Future Works

In this paper, a mathematical model for capacitated location allocation problem with fuzzy demands and probabilistic locations under the Hurwicz criterion has been proposed. To formulate the problem, a stochastic-fuzzy programming called  $\alpha$ -cost model with fuzzy demands and probabilistic locations of customers was presented. As the main assumption, the distance between customers and facilities are Euclidean and squared Euclidean. Since the proposed model is Np-Hard, for solving the two stochastic-fuzzy mentioned models, optimal solutions of the problem are not available. Therefore, this paper has developed a new hybrid intelligent algorithm based on VDO algorithm which is combined with simplex algorithm and fuzzy simulation. To illustrate the paper more explicitly, a numerical example is given to show the capability of the proposed hybrid algorithm. Finally, with regard to the obtained results based on RDI, we concluded that SFVDO algorithm is effective in solving the  $\alpha$ -cost model under the Hurwicz criterion especially for Euclidean distance. As a direction for future research, it could be interesting to take both locations and demands in fuzzy

environments. Furthermore, the proposed model can be investigated within queuing framework.

Table 2  
The obtained results from SFVDO algorithm based on squared Euclidean distance

Problem No.	$A_0$	$L_{max}$	$\sigma$	$\gamma$	Optimal Location	Cost
1	6	20	1.5	0.005	(11.47,16.81), (41.54,49.28), (71.61,89.49), (79.25,8.88)	216220
2	6	30	1.5	0.005	(80.58,20.20), (47.77,87.56), (26.74,43.10), (65.53,36.11)	207480
3	6	40	2.0	0.005	(85.97,19.74), (62.29,95.93), (36.31,55.14), (24.91,23.13)	201610
4	8	40	2.0	0.005	(28.28,14.60), (40.49,53.41), (57.41,74.68), (66.74,12.55)	200150
5	8	20	1.5	0.050	(44.90,64.36), (17.06,32.12), (80.77,14.60), (70.75,80.22)	199970
6	8	40	1.0	0.050	(56.27,71.53), (84.98,22.04), (17.07,22.41), (39.08,54.76)	199790
7	8	30	2.0	0.050	(79.24,70.35), (27.58,67.17), (80.20,30.47), (31.97,18.18)	194710
8	8	40	1.0	0.005	(21.94,33.25), (27.00,53.73), (77.84,24.51), (50.71,73.78)	191880
9	6	20	1.0	0.005	(35.02,15.01), (61.53,98.53), (21.82,52.02), (78.08,13.60)	189720
10	6	30	1.0	0.050	(25.79,70.69), (60.43,73.47), (79.40,26.02), (26.96,28.19)	185790

Table 3  
The obtained results from SFVDO algorithm based on Euclidean distance

Problem No.	$A_0$	$L_{max}$	$\sigma$	$\gamma$	Optimal Location	Cost
1	6	20	1.0	0.05	(11.47,16.81), (41.54,49.28), (71.61,89.49), (79.25,8.88)	8560
2	6	30	1.5	0.005	(80.58,20.20), (47.77,87.56), (26.74,43.10), (65.53,36.11)	8332
3	6	20	1.0	0.005	(85.97,19.74), (62.29,95.93), (36.31,55.14), (24.91,23.13)	8185
4	8	40	1.0	0.050	(28.28,14.60), (40.49,53.41), (57.41,74.68), (66.74,12.55)	8169
5	8	30	2	0.050	(44.90,64.36), (17.06,32.12), (80.77,14.60), (70.75,80.22)	7931
6	8	20	1.5	0.050	(56.27,71.53), (84.98,22.04), (17.07,22.41), (39.08,54.76)	7700
7	8	40	1.0	0.005	(79.24,70.35), (27.58,67.17), (80.20,30.47), (31.97,18.18)	7626
8	6	20	1.5	0.050	(21.94,33.25), (27.00,53.73), (77.84,24.51), (50.71,73.78)	7481
9	6	40	2.0	0.005	(35.02,15.01), (61.53,98.53), (21.82,52.02), (78.08,13.60)	7284
10	8	40	2.0	0.050	(25.79,70.69), (60.43,73.47), (79.40,26.02), (26.96,28.19)	7260

Table 4  
Comparison of squared Euclidean and Euclidean distance based on average relative deviation index ( $\overline{RDI}$ )

Problem No.	$A_0$	$L_{max}$	$\sigma$	$\gamma$	Squared Euclidean	Euclidean
1	6	20	1.5	0.005	0.54	0.45
2	6	30	1.5	0.005	0.49	0.45
3	6	40	2.0	0.005	0.68	0.25
4	8	40	2.0	0.050	0.56	0.42
5	8	20	1.5	0.050	0.59	0.57
6	8	40	1.0	0.050	0.50	0.34
7	8	30	2.0	0.050	0.52	0.32
8	8	40	1.0	0.005	0.36	0.34
9	6	20	1.0	0.005	0.38	0.25
10	6	30	1.0	0.050	0.48	0.31
Average					0.51	0.37



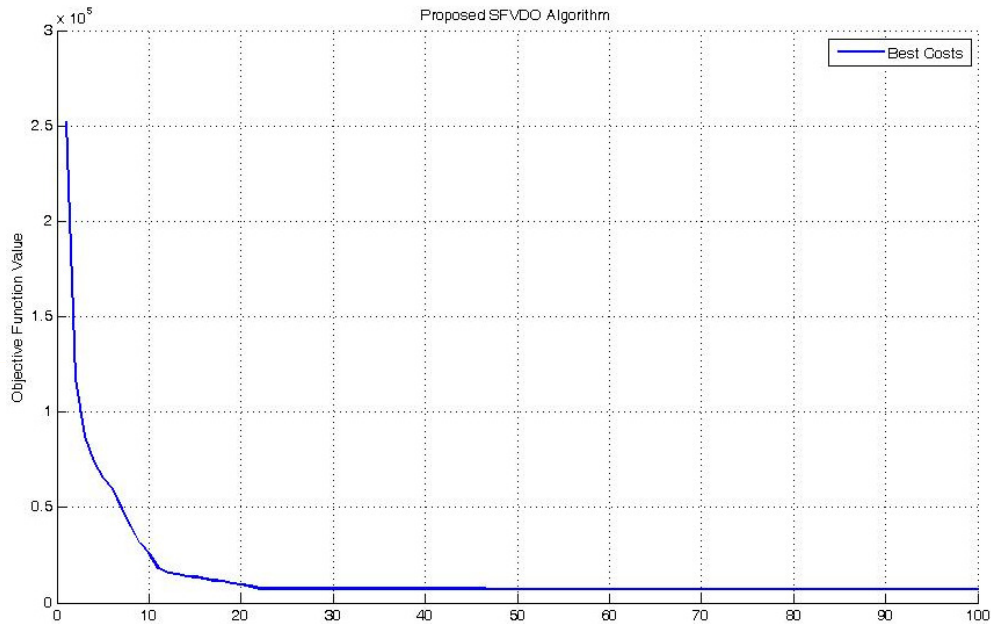


Fig. 1. The convergence graph of proposed SFVDO algorithm

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