Analysing Price, Quality and Lead Time Decisions with the Hybrid Solution Method of Fuzzy Logic and Genetic Algorithm

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Abstract

In this paper, the problem of determining the quality level, lead time for order delivery and price of a product produced by a manufacturer is considered. In this problem the demand for the product is influenced by all three decision variables: price, lead time and quality level. To formulate the demand function, a fuzzy rule base that estimates the demand value based on the three decision variables is developed. To do so, the linguistic knowledge of experts in the form of if-then rules is used to establish the fuzzy system. Moreover, in order to solve the problem, a genetic algorithm integrating the fuzzy rule base is proposed. Finally, to support the validity of the proposed solution, a numerical study is provided.

*Keyword:*Fuzzy Logic; Linguistic variable; Genetic algorithm; Pricing; Quality level; Lead time.

1. Introduction

One of the challenges in optimizing the profit and revenue of a manufacturer is demand forecasting. In some studies, demand is considered a deterministic function of price and other parameters, in which achieving an exact mathematical relation for demand is very difficult or impossible. Since in the real world there are many factors that influence demand ,to reach a more realistic demand function, some researchers consider the stochastic demand function. However, there is no way to guarantee that the derived function is the best one. In reality, the customers' evaluation of the product features and its price affects the demand. Formulating the customer behavior in determining the demand considering some input variables such as price, quality level and lead time to receive the order is a challenge for researchers. The relationship between the input variables and demand is too complex to be formulated with a mathematical function. Therefore, the approach of fuzzy rule base may be a more appropriate way to formulate the demand function.

Yao and Hin (2000) considered a fuzzy function containing a linear or demand function to formulate the cost and profit function. Chang (2000) considered a triangular fuzzy number for demand and then used developed rules for analyzing the revenue function. Yao and Shing (2002) investigated the revenue maximization problem in a fuzzy manner by considering interval fuzzy sets using a quadratic revenue function. Aliev et al. (2007) studied aggregate production-distribution planning in a supply chain with uncertain information about market demand, manufacturing capacities and process times. They used fuzzy functions for modeling. Haji and Assadi (2009) proposed a fuzzy expert system for pricing a new product. Ozdemir et al. (2010) developed a preferential model for house sale price prediction including a fuzzy system considering urban plans, culture, hygienic and education centers, transportation facilities, and other environmental factors to build the fuzzy system. Oderanti and Wilde (2010) used a fuzzy logic and game theory for strategy modeling in competitive organizations. In their model, competitive decisions are made based on market demand, production costs, marketing and other business variables that are uncertain information and they are modeled using the fuzzy logic. Shavandi and Alizadeh (2010) developed a hybrid forecasting model which combines artificial intelligence and fuzzy inference system to predict a short-term stock price index. Lin et al. (2011) considered an agent-based price negotiation for on-line auctions based on the fuzzy expert system. Wei and Zhao (2011) studied a fuzzy closed-loop supply chain with retail competition and optimized the profit of the chain using the game theory and fuzzy theory.

In the present study, we develop a fuzzy system using linguistic terms to formulate customer behaviors based on the three factors of price, quality level, and lead time. The victory of our researches in this field is that, to find a deterministic

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function for the demand, decision maker is encountered with difficulty. Therefore, linguistic terms can be more applicable and reliable in this regard. Moreover, this paper proposes a hybrid algorithm of a fuzzy system and genetic algorithm. In this hybrid algorithm the values of price, quality level, and lead time are generated by a genetic algorithm (GA) and these values enter a fuzzy rule base to estimate the demand value and after that the profit is calculated by the GA. This process is done iteratively to determine the appropriate values of price, quality level and lead time. The problem formulation with considering these facts is considered the main contribution of this paper. These features differentiate our research from similar works in the literature.

The rest of the paper is structured as follows: Section 2 presents the methodology of analysis. Numerical analysis is presented in Section 3 and finally Section 4 provides the conclusion of the paper.

2. Methodology

In this part, a manufacturer whose product demand is dependent on the parameters of price, quality level and lead time is considered. A fuzzy system is used to forecast the amount of demand based on three input variables including price, quality level, and lead time. First, the customers' demands have been collected using experts' opinions in terms of linguistic variables and if-then logic sentences, and then these sentences are converted to a fuzzy rule base and a fuzzy system is developed to control the demand.

2.1.Definitions and assumptions

The linguistic variables used in this article include very low (VL), low (L), medium (M), high (H), and very high (VH). Like Becher (2009), the linguistic variables Low, medium and high are applied as triangular fuzzy numbers and the linguistic variables very low and very high are formulated as trapezoidal fuzzy numbers(see Figures 1 and2).

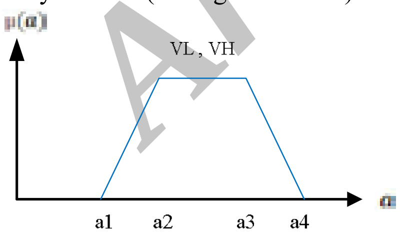

Fig. 1. Trapezoidal fuzzy numbers related to very low and very high variables

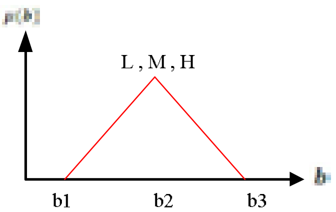

Fig. 2. Triangular fuzzy numbers related to low, medium and high linguistic variables

The designed fuzzy system includes three input variables and one output. Therefore, it is a multiple input single output fuzzy system (MISO). The input parameters are price (P), quality level (Q), and lead time (L), and the output variable is demand (D).

*Difinition1:*The universe set of P contains very low, low, medium, high and very high linguistic variables. The linguistic variables of price in the range of price domain are presented in Figure 3.

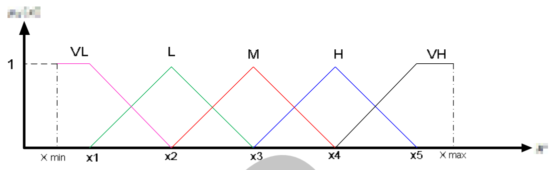

Fig. 3.Price membership functions

Difinition2: The universe set of *Q* contains low, medium, and high linguistic variables. The linguistic variables of Quality in the range of zero (the lowest quality level) and one (the highest quality level) are depicted in Figure 4.

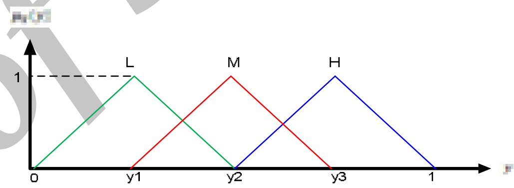

Fig. 4.Quality level membership functions

*Difinition3:*The universe set of *L* contains very low, low, medium, high and very high linguistic variables. The membership functions of the linguistic variables related to the lead time in the range of lower bound and upper bound are presented in Figure 5.

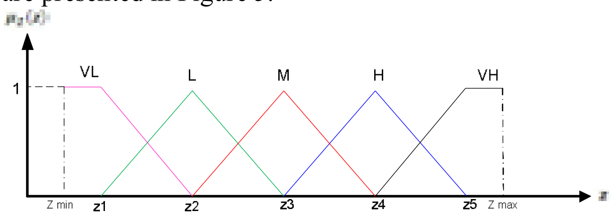

Fig. 5.Lead time membership functions

*Difinition4:*The universe set of *D* contains very low, low, medium, high and very high linguistic variables. The membership functions related to the linguistic variables of demand are presented in Figure 6.

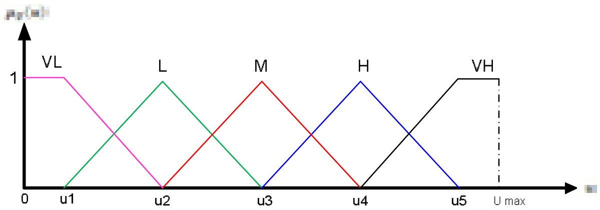

Fig. 6.Demand membership functions

2.2.The fuzzy rule base

The fuzzy logic introduced by Zadeh for the first time has been used for formulating knowledge and experience of human being (Zadeh, 1965, 1996 and 1999).A fuzzy rule base consists of a set of fuzzy if-then rules that are collected in the form of linguistic variables by asking experts'opinions and then converting them into fuzzy sets. In this study, the authors of the paper play the role of experts and design if-then rules. In order to justify the accuracy of the designed fuzzy rule base, the demand behavior towards the input parameters is shown in Figures 14 and 15.The general form of the fuzzy rule used to forecast the demand is as follows:

IF price <Linguistic variable> and quality level<linguistic variable>and lead time<linguistic variable> THEN demand <linguistic variable>.(1)

Since the price variable consists of five linguistic terms, the quality level variable consists of three linguistic terms and the lead time variable consists of five linguistic terms, there are 75If-then fuzzy rules in the proposed fuzzy system. As an example, one of the fuzzy rules is as follows:

$$If\ P\ is\ VL\ \ and\ \ Q\ is\ M\ \ and\ \ L\ is\ VH\ \ Then\ D\ is\ L \quad (2)$$

This fuzzy rule means that if price is very low and quality level is medium and lead time is very high, then demand is low. Table 1 shows all the fuzzy rules extracted from the expert's opinions.

Table 1
The fuzzy rule base from the experts' opinions

Rule number	***If part***			***Then part***	***Rule number***	***If part***			***Then part***
	P	***AND Q***	***AND L***	***D***		***P***	***AND Q***	***AND L***	***D***
1	VL	L	VL	M	41	M	H	VL	VH
2	VL	L	L	M	42	M	H	L	VH
3	VL	L	M	L	43	M	H	M	H
4	VL	L	H	L	44	M	H	H	M
5	VL	L	VH	VL	45	M	H	VH	L
6	VL	M	VL	VH	46	H	L	VL	L
7	VL	M	L	H	47	H	L	L	L
8	VL	M	M	M	48	H	L	M	VL
10	VL	M	VH	L	49	H	L	H	VL
11	VL	H	VL	VH	50	H	L	VH	VL
12	VL	H	L	H	51	H	M	VL	M
13	VL	H	M	H	52	H	M	L	M
14	VL	H	H	H	53	H	M	M	L
15	VL	H	VH	M	54	H	M	H	VL
16	L	L	VL	M	55	H	M	VH	VL
17	L	L	L	M	56	H	H	VL	H
18	L	L	M	L	57	H	H	L	M
19	L	L	H	L	58	H	H	M	M
20	L	L	VH	L	59	H	H	H	L
21	L	M	VL	H	60	H	H	VH	VL
22	L	M	L	H	61	VH	L	VL	L
23	L	M	M	M	62	VH	L	L	L
24	L	M	H	M	63	VH	L	M	VL
25	L	M	VH	L	64	VH	L	H	VL
26	L	H	VL	VH	65	VH	L	VH	VL
27	L	H	L	H	66	VH	M	VL	L
28	L	H	M	H	67	VH	M	L	L
29	L	H	H	H	68	VH	M	M	L
30	L	H	VH	M	69	VH	M	H	VL
31	M	L	VL	M	70	VH	M	VH	VL
32	M	L	L	L	71	VH	H	VL	M
33	M	L	M	L	72	VH	H	L	L
34	M	L	H	VL	73	VH	H	M	L
35	M	L	VH	VL	74	VH	H	H	VL
36	M	M	VL	M	75	VH	H	VH	VL
37	M	M	L	M					
38	M	M	M	M					
39	M	M	H	L					
40	M	M	VH	L					

2.3.The fuzzy system for demand estimation

To design the fuzzy system for estimating demand, the singleton fuzzifier, Mamdani implication engine and center Average defuzzifier are used (Klir and Yuan, 1995; Zimmermann, 1995).

First, we adjust the singleton fuzzifier of the input parameters of price, quality level and lead time with their universe sets. Then, the fuzzy sets that include input singleton fuzzifier in every parameter are considered. After that, possible rules from the composition of these fuzzy sets are established. We name these rules that are parts of all the existed rules in the fuzzy rule base (Table 1) active rules. Then, the membership degree of every input parameter (price (α_1), quality level (α_2) and lead time (α_3)) in every active rule is calculated. The matching degree (h_L) is also obtained by choosing the minimum values of α_1, α_2 and α_3 in every active rule:

$$h_L = \min(\alpha_1,\ \alpha_2,\ \alpha_3) \quad ; \ \mathrm{L}=1,2,\ldots,\mathrm{M} \tag{3}$$

Therefore, the max-min operator is used among the matching degree and the fuzzy set of the output variable (demand), and through this the output of every active rule is obtained. This cycle will be completed for all active rules until every rule's output is established.

Finally, we get the aggregation of fuzzy outputs from all active rules to calculate the final fuzzy set of demand. Moreover, the center average operator is used to defuzzify the fuzzy set of demand (equation 4).

$$D = \frac{\sum_{L=1}^{M} h_L \times \bar{D}^L}{\sum_{L=1}^{M} h_L} \tag{4}$$

For instance, suppose there is just one rule, the operation of Mamdani implication is presented in Figure 7. The equation to derive the crisp value of demand is calculated by equations (4) and (5).

$$\begin{cases} e1 = h_L(u3 - u2) + u2 \\ e2 = u4 - h_L(u4 - u3) \\ \bar{D}^L = (e1 + e2)/2 \end{cases} \tag{5}$$

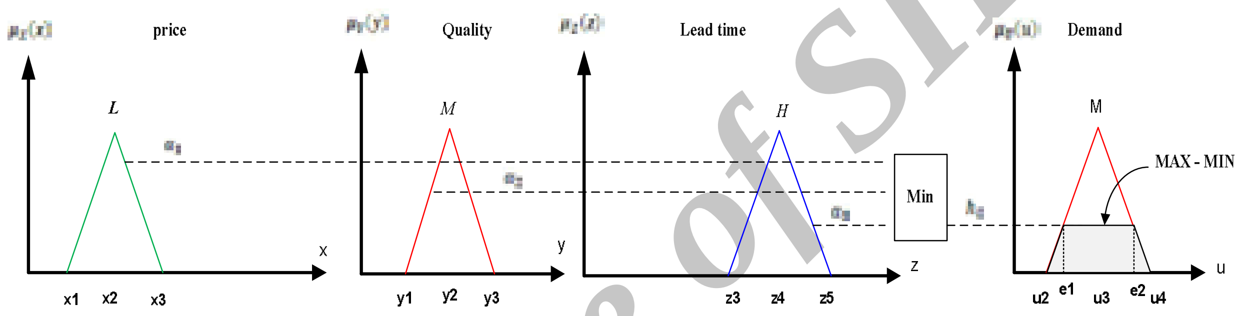

Fig.7.The Mamdani implication for a rule

2.4.Manufacturer's profit function

Notations:

π_M = Manufacturer's profit function
C_M =Production costs for a unit of product.
C_q =Quality degree costs for every unit of Q.
C_L = Maximum lead time cost per unit of product per unit of time.
p = The unit price of product.
Q =The quality level of product
L =The lead time of delivering order to customers
A = The lead time cost.

$$\begin{aligned} &Max:\ \pi_M = (p - C_q Q - A - C_M)D \\ &s.t \\ &p_{\min} \le p \le p_{\max} \\ &L_{\min} \le L \le L_{\max} \\ &0 \le Q \le 1 \end{aligned} \tag{6}$$

where $A = {C_L}/{L}$ is the lead time cost in which by decreasing the L, the Ain creases. D is also the demand value which is calculated by the fuzzy system in the previous section.

2.5. The genetic algorithm

In this sub-section the genetic algorithm is used in the hybrid solution method because victories of the GA are shown in various areas of operational and industrial applications to optimize the problem (Pasandideh et al. (2011)).GA is one of the best heuristic optimization methods which was originally developed by Holland (1975). The GA optimization procedure is started with randomly creating an initial set of solutions (the initial population). Each solution in the population is named as chromosome. Each chromosome includes many genes and each gene indicates a decision variable of the problem. Then, with using the crossover and mutation operator, the next generation is produced. During each generation, the chromosomes are evaluated with calculating the cost function value. Therefore, to obtain the appropriate solution, this procedure is continued. The structure and elements of the proposed GA and the hybrid solution are presented below.

2.5.1. Chromosomes

The evaluation function of our GA is the manufacturer's profit function in which the decision variables (p, Q and L) are chromosomes of the algorithm (see Figure 8).

p	***Q***	***L***

Fig. 8. Chromosomes of the genetic algorithm

2.5.2. Initial population

To generate the initial population, m chromosomes are generated randomly subject to the constraints on the variables.

2.5.3. Parents selection

The rank based proportional selection and Roulette wheel method are used to select the parents for producing the new generation.

2.5.4. Cross over operator

The one-cut uniform crossover is applied to each pair of parents selected to produce two children.

2.5.5. Mutation operator

To mute a selected parent, a gene is chosen randomly and its value is changed randomly.

2.5.6. Replacement

The tournament replacement approach is used to form the new generation by replacing the produced chromosomes in the population.

2.5.7. The hybrid solution method

The price, quality level and lead time decisions are analyzed with the hybrid solution method of fuzzy logic and the genetic algorithm as shown in the flowchart (see Figure 9).

3. The Numerical Study

In this part, a numerical example is provided to estimate the customers' demand for the fuzzy rule base (Table 1). Assume the partition of the universe sets of p, Q, L and D as depicted in Figure 10. The input values of price, quality level and lead time are 2000, 0.4, and 16 respectively.

As you see in Figure 10, it includes the price parameter singleton fuzzifier with the two fuzzy sets of low (L) and very low (VL). The quality level singleton fuzzifier consists of the two fuzzy sets of medium (M) and low (L), and the lead time singleton fuzzifier includes the two fuzzy sets of high (H) and very high (VH). Thus, there are $2 \times 2 \times 2 = 8$ active rules that are presented in Table 2.

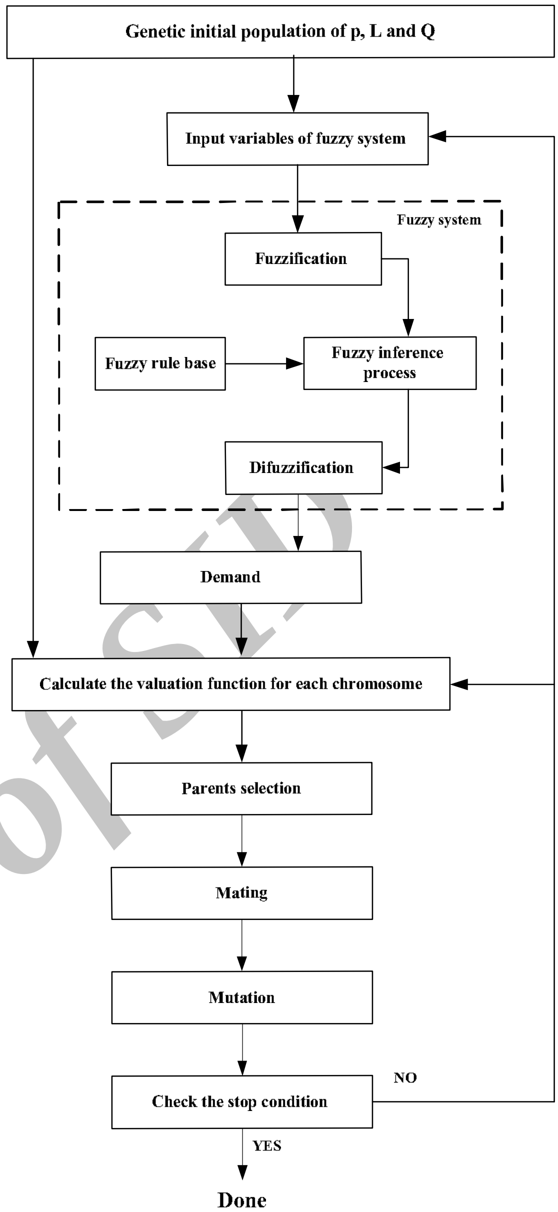

Fig. 9.The hybrid solution method of fuzzy logic and genetic

Every parameter's membership degree $(\alpha_{ij}\ ; i = 1,2,3 \text{ and } j = 1,\dots,8)$ for every active rule is obtained. Then the matching degree is calculated by $h_j = \min(\{\alpha_{1j}, \alpha_{2j}, \alpha_{3j}\}\ ; j = 1,\dots,8)$ for every active rule. Finally, the output parameter fuzzy set is achieved by using the operator $\max\min(\{h_j\ , \mu_{Uj}(u)\}\ ; j = 1,\dots,8)$. Figure 11 shows the Mamdani implication for the first active rule.

The demand output parameter results for other active rules are shown in Figure 12. After every individual rule implication, the final result of the demand fuzzy set is obtained by aggregating the individual results illustrated in Figure 13. After calculating the fuzzy set of demand, we convert its output parameter to a real number by using the center average defuzzifier.

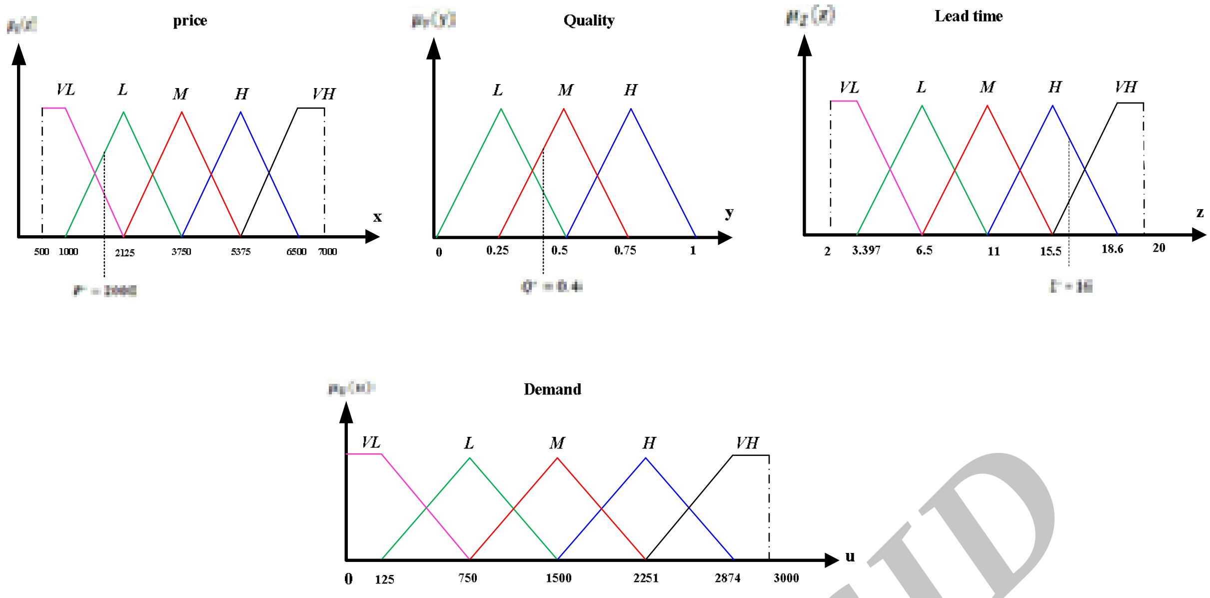

Fig. 10.The numerical example study forth fuzzy sets of output and input parameters

Table 2
Active rules in the numeric example

Rule base number	*Active rule number*	*If part*			*Then part*
		p	*AND Q*	*AND L*	*D*
4	*1*	*VL*	*L*	*H*	*L*
5	*2*	*VL*	*L*	*VH*	*VL*
9	*3*	*VL*	*M*	*H*	*L*
10	*4*	*VL*	*M*	*VH*	*L*
19	*5*	*L*	*L*	*H*	*L*
20	*6*	*L*	*L*	*VH*	*L*
24	*7*	*L*	*M*	*H*	*M*
25	*8*	*L*	*M*	*VH*	*L*

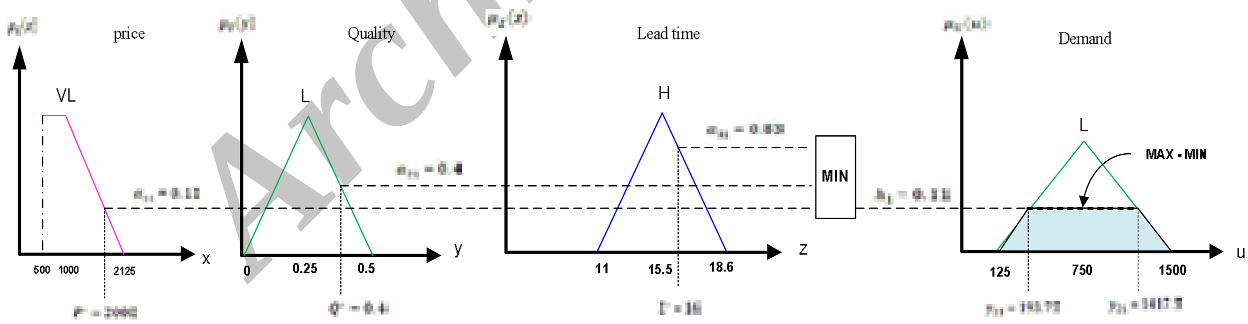

Fig. 11.The Mamdani implication for the first active rule of the numerical example

First, the center of every rule's output variable is obtained. Assume that y_{1j} and y_{2j} are two maximum points of the output parameter's fuzzy set in the jth active rule. Consequently, these sets' centers ($\bar{D}^1$)are calculated through equation (4). As an example, $\bar{D}^L$ point's calculation for the first active rule is as follows:

$$y_{11} = 0.11(750-125)+125 = 193.75$$

$$y_{21} = 1500-0.11(1500-750) = 1417.5$$

$$\bar{D}^1 = {(193.74+1417.5)}\big/_{2} = 805.625$$

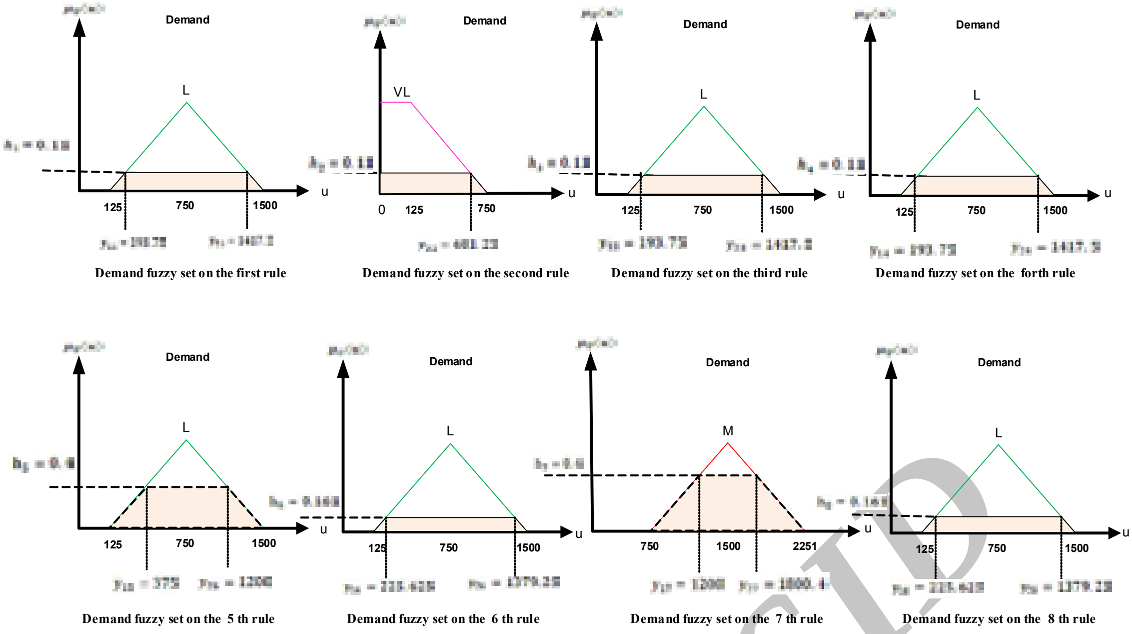

Fig.12.The demand parameter implication for all active rules

After calculating $\bar{D}^L$ and h_L for all active rules, the amount of defuzzifier is obtained by using equation (4) which is as follows:

$$D = \frac{\sum_{L=1}^{8} h_L \times \bar{D}^L}{\sum_{L=1}^{8} h_L} = 1008$$

The demand quantity regarding price, quality level and lead time of the fuzzy control system input is 1008. Figures 14 and 15 show the behavior of demand with respect to price, quality level, and lead time.

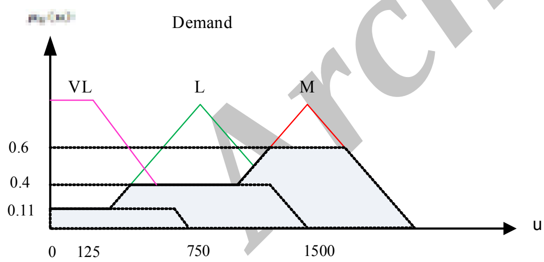

Fig.13.The demand total result of fuzzy inference

Now, assuming that Cq =800, CL =1000 and CM =400,the manufacturer's profit is calculated as follows:

$$\pi_M = (2000 - 800 \times 0.4 - {}^{1000}\!/_{16} - 400) \times 1008 = 1227240$$

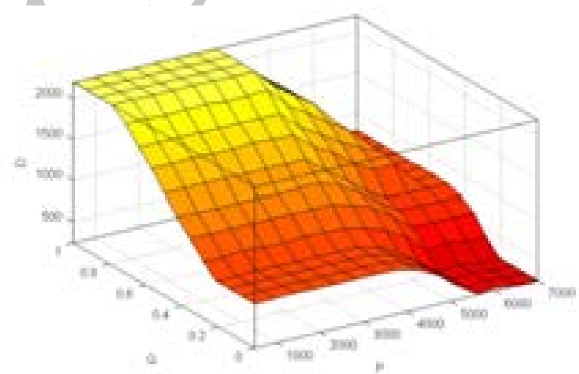

Fig.14.The demand behavior towards the price and quality level parameters

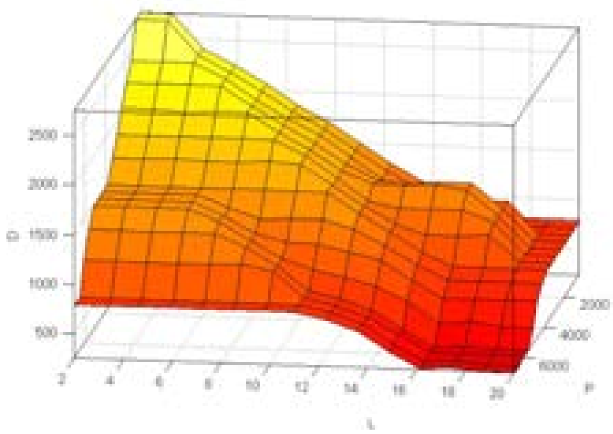

Fig.15.The demand behavior towards price and lead time parameters

We performed the genetic algorithm in Matlab to achieve the best answer for price, quality level and lead time to maximize the profit. The outputs of the genetic algorithm performance are reported in Table 3:

Table 3
The outputs of the genetic algorithm performance in Matlab

Run number	*price*	*Quality level*	*Lead time*	*Profit*
1	6320.76	0.96	3	8755205.22
2	5374.92	0.77	3	8883230.30
3	5385.43	0.75	3	8900037.38
4	5413.13	0.79	3	8775296.94
5	6463.2	0.99	3	8957124.23
6	5274.53	0.79	3	8673104.48
7	5432.35	0.78	3	8768466.49
8	6259.79	0.95	3	8669872.00
9	5361.87	0.84	2.61	8612828.81
10	5242.51	0.76	3.4	8747804.46

As Table 3 indicates, the run number of 5 has obtained the maximum profit and the best answer for the lead time, quality level and price parameters. The isotropy diagram of the genetic algorithm performance is shown in Figure16.

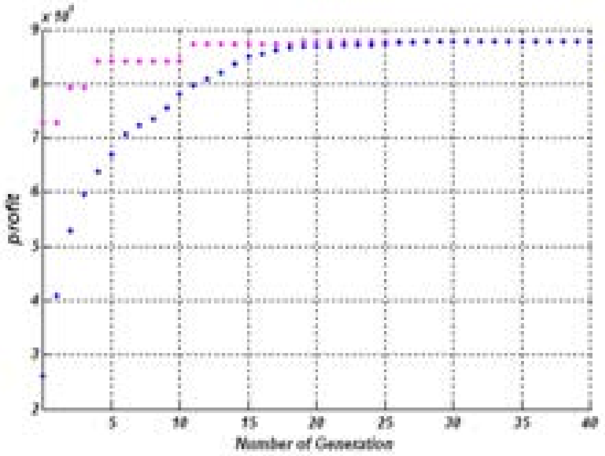

Fig.16.Isotropy diagram of the genetic algorithm performance

4. Conclusion

In this article, we consider an uncertain environment for demand which is dependent on the three parameters of price, quality level and lead time. Meanwhile, there is no mathematical relation for demand. Therefore, we use the fuzzy rule base to control demand. Then, we can predict the demand quantity for different price, quality level and lead time input parameters and use this quantity to decide about production quantity, necessary warehouse space, raw material ordering, necessary budget, and profit optimization. Next, we use the genetic optimization algorithm to obtain suitable amounts for price, quality level and lead time and through this, we achieve the maximum profit (Table 3). In the results of the numerical study, the variables are achieved between their domains (Figure 10) and this justifies the accuracy of the fuzzy rule base and the proposed hybrid solution. As in the real world generally there is not an appropriate mathematical relation for analysis and decision making, and we use experts' linguistic statements for designing a system, applying fuzzy systems to formulate human linguistic knowledge is very useful.

For future research, the hybridization of this fuzzy system with other Meta-heuristic methods such as simulated annealing (SA) may be studied and compared with our proposed method. In addition, the hybrid solution proposed in this paper can be used for other concepts of revenue management and SCM science and queuing theory.

5. Acknowledgement

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