Complete / Incomplete Hierarchical Hub Center Single Assignment Network Problem

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Abstract

In this paper we present the problem of designing a three level hub center network. In our network, the top level consists of a complete network where a direct link is between all central hubs. The second and third levels consist of star networks that connect the hubs to central hubs and the demand nodes to hubs and thus to central hubs, respectively. We model this problem in an incomplete network environment. In this case, the top level is an incomplete network where the direct link between all central hubs is not necessary and may lead to lower transportation costs. We propose mixed integer programming model for these problems and conduct a computational study for these two developed models by using the CAB data.
Keywords: Hub Location, Hub Center, Hierarchical, Complete and Incomplete Network.

1. Introduction

Hubs are facilities used to consolidate and disseminate flow and serve as points for switching, transshipment and sorting flows in many-to-many distribution systems. In practice, the use of hubs can result in lower network costs, but it can be shifting to determine where hubs should be located or how demands should be allocated to them.
In a particular hub location problem the objective is to determine locations of hubs and also assigning other nodes to these hubs with minimum distribution costs. Hub location problems have many applications, including telecommunications, airlines, delivery services, postal, emergency services and many others. The hub location problem deals with finding the location of hub facilities and the allocation of the non-hub nodes to them.
Consolidation is a major privilege of using hubs since flows with same source and different destinations can be combined on their route to hub nodes and also flows with different sources and same destination can be combined from hub nodes to their destination which yields a significant reduction of transportation costs.
There are two types of hub networks problems. Single allocation is the first type in which every demand node is connected to only one hub and all the incoming or outgoing flow is routed through that single hub. Multi allocation is the second type which allows demand nodes to be connected to a set of hub nodes and send or receive traffic flows from this set.

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The hub location problems have been introduced by O'Kelly (1986, 1987). The hub problems discussed in the literature are typically p-hub median and p-hub center and p-hub covering problems. The p-hub center problem is to locate p hubs in a network and to allocate non-hub nodes to hub nodes so that the maximum travel distance (or time) between any source–destination pair is minimized. P-hub center problem with single assignment was introduced by Campbell (1994). Campbell defined three different types of p-hub center problems. The first type : the maximum cost for any source–destination pair is minimized. The second type: the maximum cost of movement between a hub and an origin-destination is minimized
The third type: the maximum cost for move on any single link (source to hub, hub to hub and hub to destination) is minimized.
The p-hub center problem is important for guaranteed time or time-sensitive distribution systems, such as emergency services and express mail services. Applications of the three types are as follows:
The first type of hub center problem is significant for a hub system involving perishable or time sensitive items in which cost refers to time. The second and third types are significant for the vehicle drivers that are subject to a time limit on continuous service or a hub system requires some preserving-processing such as cooling or heating which is available at the hub locations.

The p-hub center location problem is NP-complete, for this reason many algorithms for the p-hub center location problem iteratively select hubs, and then solve the resulting allocation problem. Thus, beneficial methods for solving the allocation problem can be useful as part of solving the p-hub center location problem. Since Campbell's pioneering work, a lot of researchers developed the idea to many other structures and applications. Kara and Tansel (2000) developed various linear formulations for the single allocation p-hub center problem. They also provided a combinatorial formulation of the single allocation p-hub center problem. Ernst et al. (2002) considered a new formulation for the single allocation p-hub center problem. Baumgartner (2003) inquired the polyhedral properties of the formulation and identified some facet-defining inequalities and defined separation procedures and finally proposed a branch-and-cut algorithm. Hamacher and Meyer (2006) proposed solving hub covering problems with binary search for the solution of the p-hub center problem. Pamuk and Sepil (2001) proposed first heuristic for the single allocation p-hub center problem. Ernst et al. (2002) studied the allocation sub problem of the single allocation p-hub center problem when hub locations are fixed. Campbell et al. (2007) presented various complexity results and provided integer programming formulations for both incapacitated and capacitated cases. Gavriliouk and Hamacher (2006) applied aggregation to various hub location models and proposed some error measurements and developed error bounds for these models.
Additional information was introduced by Alumur et al. (2008).

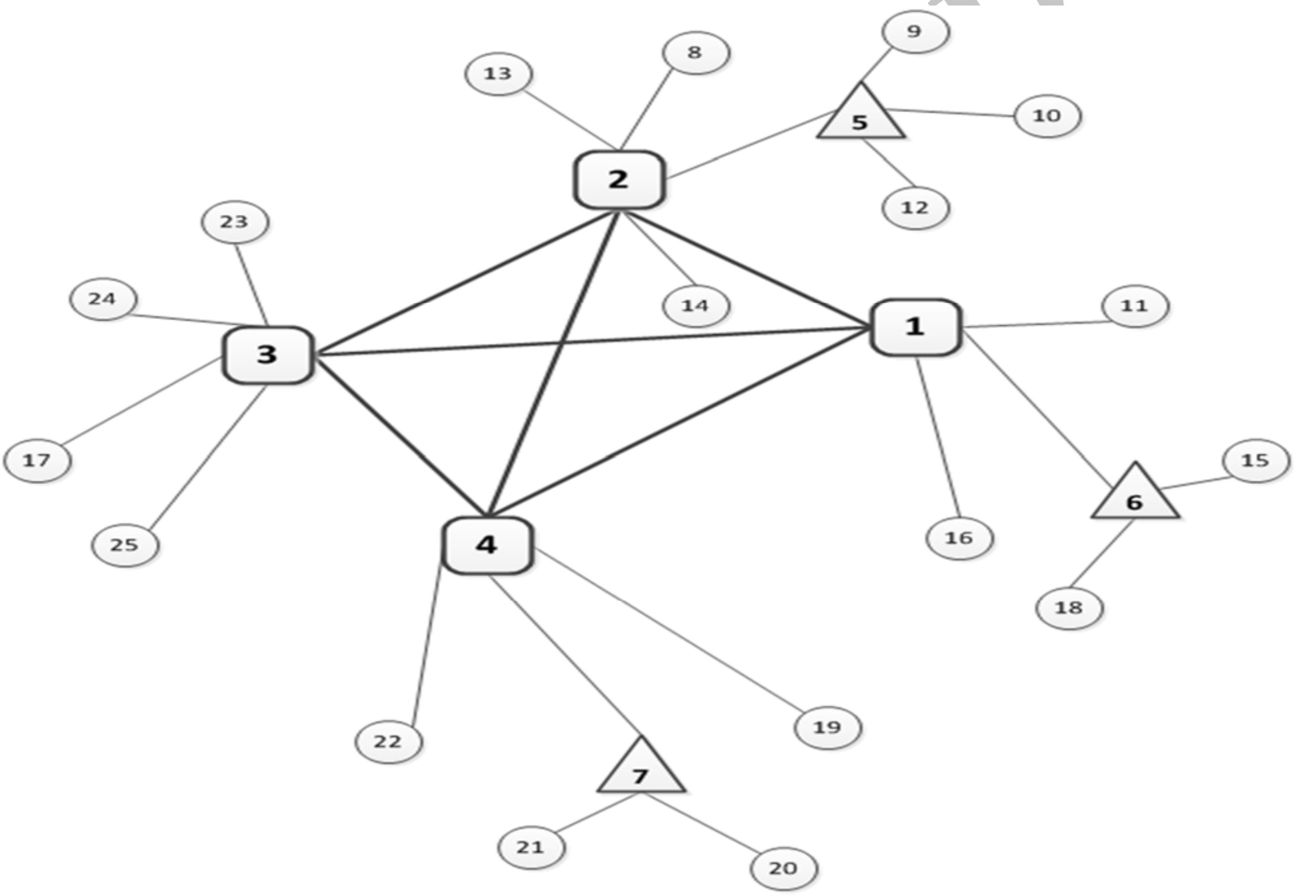

Fig. 1. A three level complete network on 25 nodes with 7 hubs and 4 central hubs

Elmastas (2006) considered a three-level network. The top level that connects hub airports is a star, the second level that connects hubs among themselves and to hub airports has a mesh structure and the third level connecting demand points to hubs is composed of star networks. Yaman (2009) presented formulation for the hierarchical hub median problem with single assignment. She introduced a three-level network (hierarchical network) and added central hub nodes to classical models in order to relax the complete connections between hubs. In hierarchical networks, the traffic between two nodes may pass four hubs or less in its path. If two nodes are assigned to hubs which are assigned to two different central hubs then the traffic passes all the four hubs. In any other combinations of assignment the number of passed hubs may be less than four. Fig 1 shows a hierarchical network with 25 demand nodes, 7 hubs and four central hubs. Alumur et al. (2009) introduced incomplete hub networks. In incomplete hub network a direct route between two hubs is not necessary but in the hub network every hub is accessible from another through the network. They use a parameter called hub links to control the number of routes between hubs.

The incomplete hub network concept is more realistic than the previous studies. Our model's hub network is based on incomplete networks in the hierarchical structure. Since establishing links between every central hub is expensive, the complete network may lead to non-optimal solutions. By introducing incomplete network between central hubs we design a hierarchical network in which a direct link between central hubs is not necessary. Therefore, the model can decide which links to be established. The selection of links can lead to a network with total costs lower than a complete central hub network.

Figure 2 shows an incomplete hierarchical network with 25 demand nodes, 7 hubs, four central hubs and four links. Contreras et al. (2010) presented the tree of hubs location problem that the hubs are connected through a tree. Yaman (2011) presented allocation strategies and their effects on total routing costs in hub networks. This problem has two versions in single allocation problems and multiple allocation problems. Yaman and Elloumi (2012) considered Star p-hub center problem and star p-hub median problem with bounded path lengths. Alumur et al. (2012) introduced the multimodal hub location and hub network design problem. They also studied on how the hub networks with different possible transportation modes must be designed.

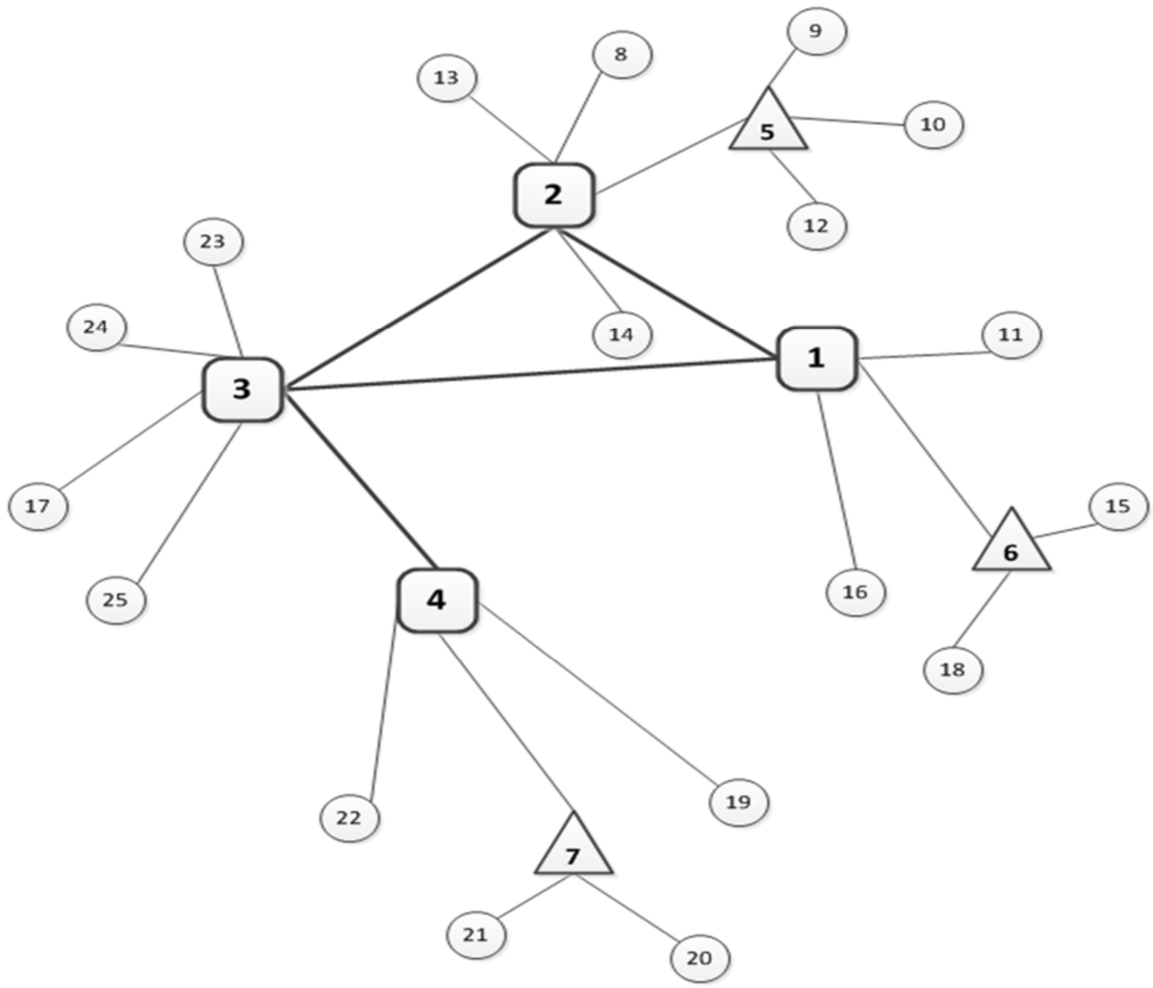

Fig. 2. A three level incomplete network on 25 nodes with 7 hubs, 4 central hubs and 4 links

Our model determines which hub and central hub must be opened and finalize their links; it also assigns nodes to both hub types which is similar to classical hub network problem, so we name this design a hierarchical hub center network problem with single assignment as SA-HHCN and an incomplete hierarchical hub center network problem with single assignment as SA-IHHCN.

The rest of the paper is organized as follows: in section 2, we present a mixed integer programming formulation for SA-HHCN and SA-IHHCN problem. In section 3, we present our computational results for cab data test problems and section 4 includes our conclusion as well as ideas for future developments.

2. MIP formulation for SA-HHCN and SA-IHHCN problem

In this section, we first review the formulations for the classical p-hub center problem with single assignment. Campbell (1994) presented formulations for both single and multiple allocation versions for all three types of p-hub center problem. Kara and Tansel (2000) provided various linear formulations for the single allocation p-hub center problem.

Ernst et al. (2002) defined a new variable r_k as the maximum collection-distribution cost between hub k and the nodes that are allocated to hub k and developed a new formulation for the single allocation p-hub center problem.

We propose two mixed integer programming models for hierarchical hub center network problem with single assignment in complete and incomplete network environment. In our first model, between all central hubs should have a direct connection; we used the idea developed in Yaman (2009) for our model structure. In our second model, it is allowed to have no direct connection between some central hubs; we used the idea developed in Alumur et al. (2009) for our model's structure. The set of nodes is denoted by I, $H \subseteq I$ is the set of possible alternatives for locations of hubs, and $C \subseteq H$ is the set of possible alternatives for locations of central hubs. We denote the number of hubs by p and the number of central hubs to be opened by p_0. Let d_{ij} be the cost of routing a unit traffic from node $i \in I$ to node $j \in I$. We also assume that $d_{ij} = d_{ji}$ for all pair of nodes i and j and $d_{ii} = 0$ for all i. Let α_H denote the discount factor in routing costs between hubs and central hubs and Let α_C denote the discount factor in routing cost among central hubs.

The variable y_{ijl} is 1, if node $i \in I$ is assigned to hub $j \in H$ and hub j is assigned to central hub $l \in C$ and is 0 otherwise. Let Z denote the maximum travel distance between any origin–destination pair and r_l denote the radius of central hub $l \in C$, we require that $d_{ij} + d_{jk} \geq d_{ik}$ for all nodes $i; j; k$ in I.

We propose the following model for SA-HHCN.

(Note that , $j \in H \setminus \{ i \}$ means $j \in H$ and $j \neq i$)

$$\text{MIN} \quad Z \qquad (1)$$

$$\text{s.t.} \quad \sum_{j\in H} \sum_{l\in C} y_{ijl} = 1 \qquad \forall\, i \in I \qquad (2)$$

$$y_{ijl} \leq y_{jjl} \qquad \forall\, i \in I, j \in H \setminus \{ i \}, L \in C \qquad (3)$$

$$\sum_{m\in H} y_{jml} \leq y_{lll} \qquad \forall\, j \in H, l \in C \setminus \{ j \} \qquad (4)$$

$$\sum_{j\in H} \sum_{L\in C} y_{jjl} = P \qquad (5)$$

$$\sum_{L\in C} y_{lll} = p_0 \qquad (6)$$

$$r_l + r_k + \alpha_c d_{lk} \leq Z \qquad \forall\, k \in C, l \in C \setminus \{ k \} \qquad (7)$$

$$\sum_{j\in H} (d_{ij} + \alpha_H d_{jl}) y_{ijl} + \sum_{h\in H} (d_{mh} + \alpha_H d_{hl}) y_{mhl} \leq Z$$
$$\forall\, l \in C, i \in I, m \in I \setminus \{ i \} \qquad (8)$$

$$r_l \geq (d_{ij} + \alpha_H d_{jl}) y_{ijl} \qquad \forall\, i \in I, j \in H, l \in C \qquad (9)$$

$$\sum_{j\in H} \sum_{l\in c \setminus \{j\}} y_{ljl} = 0 \qquad (10)$$

$$r_l \geq 0 \qquad \forall\, l \in C \qquad (11)$$

$$Z \geq 0 \qquad (12)$$

$$y_{ijl} \in \{0, 1\} \qquad \forall\, i \in I, j \in H, l \in C \qquad (13)$$

The objective function (1) minimizes the value of Z. Constraint (2); assign each demand node to a hub and ultimately a central hub.

If a node i is assigned to hub j and central hub l, then hub j should be assigned to central hub l and this is obtained via constraint (3).Constraint (4) ensures that if node j is assigned to central hub l, then l must be a central hub. The number of hubs and central hubs to be opened is fixed to p and p_0, respectively, with constraints (5) and (6).

Due to constraint (7), the minimum value of Z is the maximum distance between any two nodes if the two nodes belongs to two different central hubs .Constraint (8) ensures that the minimum value of Z is the Maximum distance between any two nodes that belongs to a central hub. Constraint (9) determines radius of central hub values for any central hub. Constraint (10) is helpful to cut non-feasible solutions. The rest of the constraints of the model (11) – (13) represent non-negativity and binary requirements of variables.

Now, we present a mixed integer programming formulation for SA-IHHCN. We used the ideas developed in Alumur et al. (2009) for our model structure. We need to know which central hub links are used on the path from any source to destination to calculate the travel distance. For each established central hub, we would like to find a spanning tree rooted at this central hub that visits every other central hub in the central hub network using only the established hub links. We use these spanning trees to calculate the travel distance between all pairs of central hubs. Let q denote the number of central hub links to be established and in addition to the previously defined decision variables y, r and z, the new decision variables of the mathematical model are: x_{ij}, it is 1 if a central hub link is established between central hubs $i \in C$ and $j \in C$ and is 0 otherwise. Let V_{ijl} denote 1 if the spanning tree rooted at central hub $l \in C$ uses the central hub link $i; j$ from central hub $i \in C$ to central hub $j \in C$; otherwise, zero. Let b_{ij} denote travel distance from central hub $i \in C$ to central hub $j \in C$ in the central hub network. In general, in our model by changing the parameters we have both incomplete and complete versions. We propose the following model for SA-IHHCN.

$$\text{MIN} \quad Z \qquad (14)$$

s.t. (2) – (6), (8) – (13)

$$r_l + r_k + \alpha_c b_{lk} \leq Z$$
$$\forall\, k \in C, l \in C \setminus \{ k \} \qquad (15)$$

$$x_{ij} \leq y_{iii} \qquad \forall\, i, j \in C: \; i < j \qquad (16)$$

$$x_{ij} \leq y_{jjj} \qquad \forall\, i, j \in C: \; i < j \qquad (17)$$

$\sum_{i\in C\setminus\{j\}} v_{ijl} \geq y_{lll} + y_{jjj} - 1$
$\forall\, j, l\in C:\; j\setminus\{l\}$ (19)
$\sum_{i\in C\setminus\{j\}} v_{ijl} \leq y_{lll}$ $\forall\, j, l\in C:\; j\setminus\{l\}$ (20)
$v_{ijl} + v_{jil} \leq x_{ij}$
$\forall\, i, j, l\in C:\; i < j$ (21)
$b_{lj} \geq b_{li} + d_{ij} * v_{ijl} - M*(1- v_{ijl})$
$\forall i, j, l\in C:\; i\setminus\{j\}$ and $j\setminus\{l\}$ (22)
$b_{ij} = b_{ji}$ $\forall\, i, j \in C:\; i\setminus\{j\}$ (23)
$b_{ii} = 0$ $\forall\, i \in C$ (24)
$v_{iji} + v_{jij} \geq 2 * x_{ij}$ $\forall\, i, j\in C:\; i < j$ (25)
$\alpha_c\, d_{ij} \leq Z$ $\forall\, i, j \in I$ (26)
$v_{ijl}\in \{0, 1\}$ $\forall\, i, j, l\in C:\; i\setminus\{j\}$ and $j\setminus\{l\}$ (27)
$b_{ij} \geq 0$ $\forall\, i, j \in C:\; i\setminus\{j\}$ (28)
$x_{ij}\in \{0, 1\}$ $\forall\, i, j \in C:\; i < j$ (29)

The objective function (14) minimizes the value of *Z*. Constraint (15) determine that the minimum value for *Z* that is Maximum distance between any two nodes if two nodes belong to two different central hubs. Constraints (16) and (17) ensure that central hub links are established between nodes that are central hubs. We defined x_{ij} variables only for $i < j$. Due to constraint (18), the number of central hub links to be established is fixed to *q*.

Constraint (19) ensures that the degree for each central hub node is at least one; so every central hub node is an end node for at least one central hub link. Through this constraint, the model guarantees that the tree rooted at central hub *l* will have an entering arc into every other central hub *j*.

Constraint (20) determines that each spanning tree rooted at central hub *l* can have at most one entering arc into another central hub node *j* and forces the spanning tree arcs associated with a non-central hub node to take zero values. Constraint (21) causes the spanning tree arcs to be central hub arcs. Constraint (22) calculates the distance travel from one central hub node to another using the established spanning tree arcs in the central hub network. Constraint (23), ensures that *b* variable will be symmetric and Constraint (24) ensures that the distance from a node to itself will be zero. Constraint (25) is a Conceptual Constraint that reduces solving time. This Constraint ensures that when a central hub link is established between central hubs $i\in C$ and $j\in C$, *V* variables must be one. Constraint (26) ensures that value of *Z* is greater than Maximum distance between any pair of nodes *i* and *j*.

The rest of the constraints of the model (27)–(29) represent binary and non-negativity requirements of variables.

3. Run the Proposed Models

For evaluation the performance of our proposed models, we use CAB data set. The Civil Aeronautics Board (CAB) data set introduced by O'Kelly (1997) is based on the airline passenger traffic between 25 US cities. The data contains the traffic demands and distances. We take all 25 cities as candidates for hubs and central hubs, H = C = I.

All instances are solved using optimization software GAMS version 23.4 and CPLEX version 12.0.0. We took our runs on a system with a 2.40 GHz Intel Core™2 Quad Processor and 2GB of RAM.

3.1. SA-HHCN problem

We tested the performance of our SA-HHCN model on CAB data with 25 cities.

For all states, p_0 and p are varying from 2 to 9 and 3 to 9, respectively. As mentioned in the previous works, we assumed α_C and α_H values 0.9, 0.8, and 0.7, respectively to evaluate the effect of some parameters on the transportation cost and the locations of central hubs and to see the computation times.

Now we consider the effect of the number of central hubs and discount factors on the transportation cost. In our first experiment, we investigate how the transportation cost is affected by changing the number of central hubs. To see the effect of the number of central hubs on the transportation cost, we use instances from the CAB data with n=25 and p=7, 8, 9.

In Figs. 3, 4 and 5, we plot the transportation cost for different values of p_0 and discount factors for the CAB data.

In Fig. 3 and Fig. 4, when (α_C,α_H) is equal to (0.7, 0.8) and (0.8, 0.9), the transportation cost decrease as we increase p_0, respectively. We observe that in all cases, for a fixed choice of (α_C,α_H), the transportation cost does not increase as we increase p_0. We see that substantial cost improvements are possible when we move from a star hub network ($p_0 = 1$) towards a complete hub network ($p_0 = p$).We report our results on the CAB data set with 25 cities in Table 1. For each instance,

Table reports the required CPU time in seconds, transportation costs and increasing (%) in transportation costs. Research has addressed how the computation times are affected by the parameters of the problem.

For instances with (α_C, α_H) equal to (0.8, 0.9), when *p* values is 6, percent increase in transportation cost for p_0=5 was 2.8%. When *p* value is 7, increasing percent in transportation costs for p_0=5, 6 and 7 was 2.3%, 0% and 0%, respectively.

For instances with (α_C, α_H) equal to (0.7, 0.8), when *p* values is 6, percent increase in transportation cost for p_0=5 was 2.6%. When *p* values is 7, percent increase in transportation costs for p_0=5, 6 and 7 was 3%, 1.2% and 0%, respectively. By considering results of our several numerical examples (Table 1), we can conclude that the percentage of increase in the transportation costs is higher for instances with lower values of discount factors

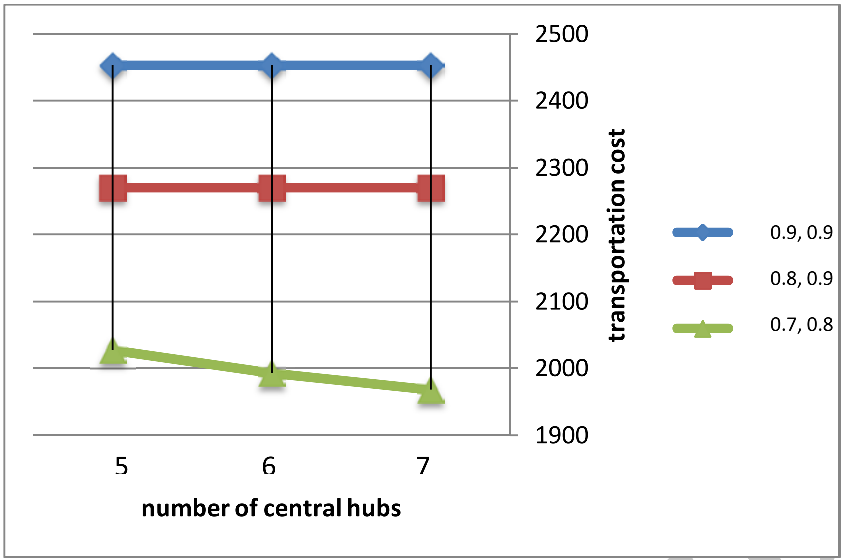

Fig. 3. The transportation cost for the CAB data with 25 nodes and 7 hubs

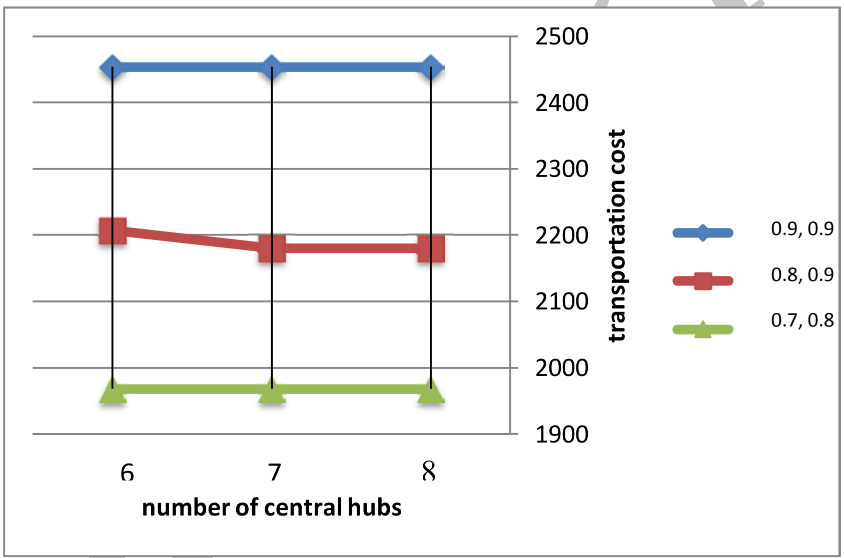

Fig. 4. The transportation cost for the CAB data with 25 nodes and 8 hubs

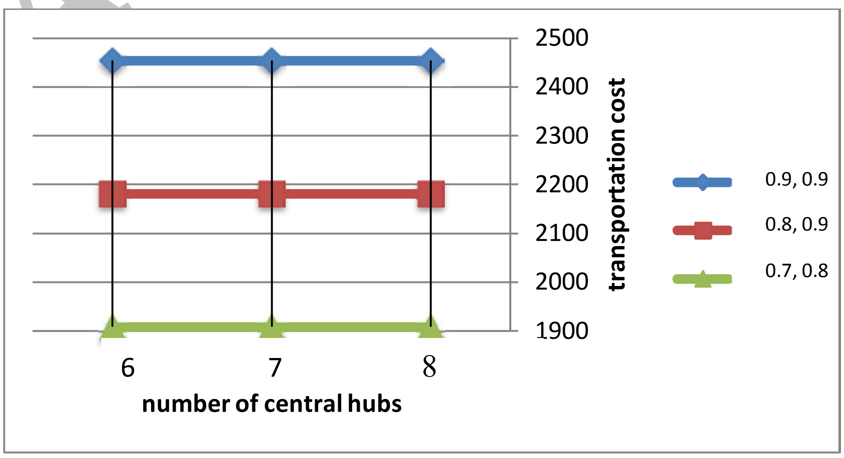

Fig. 5. The transportation cost for the CAB data with 25 nodes and 9 hubs

Table 1
The results on the CAB data set with 25 cities for SA-HHCN problem

(α_C,α_H)	p	P_0	CPU Time (s)	Transportation Costs	% Increase in Transportation Costs
(0.9,0.9)	3	2	45	2703.231	1
(0.9,0.9)	3	3	1	2675.309	0
(0.9,0.9)	4	3	29	2606.545	0
(0.9,0.9)	4	4	1	2606.545	0
(0.9,0.9)	5	4	62	2543.677	0
(0.9,0.9)	5	5	1	2543.677	0
(0.9,0.9)	6	5	30	2453.211	0
(0.9,0.9)	6	6	1	2453.211	0
(0.9,0.9)	7	5	41	2453.211	0
(0.9,0.9)	7	6	39	2453.211	0
(0.9,0.9)	7	7	1	2453.211	0
(0.9,0.9)	8	6	19	2453.211	0
(0.9,0.9)	8	7	21	2453.211	0
(0.9,0.9)	8	8	1	2453.211	0
(0.9,0.9)	9	7	32	2453.211	0
(0.9,0.9)	9	8	86	2453.211	0
(0.9,0.9)	9	9	1	2453.211	0
(0.8, 0.9)	3	2	177	2563.374	0. 4
(0.8, 0.9)	3	3	1	2554.131	0
(0.8, 0.9)	4	3	29	2456.123	0. 1
(0.8, 0.9)	4	4	1	2454.349	0
(0.8, 0.9)	5	4	29	2389.787	0. 8
(0.8, 0.9)	5	5	1	2371.189	0
(0.8, 0.9)	6	5	35	2317.734	2.8
(0.8, 0.9)	6	6	1	2253.700	0
(0.8, 0.9)	7	5	50	2270.583	2.3
(0.8, 0.9)	7	6	57	2220.585	0
(0.8, 0.9)	7	7	1	2220.585	0
(0.8, 0.9)	8	6	20	2206.872	1.2
(0.8, 0.9)	8	7	25	2180.632	0
(0.8, 0.9)	8	8	1	2180.632	0
(0.8, 0.9)	9	7	17	2180.632	0
(0.8, 0.9)	9	8	29	2180.632	0
(0.8, 0.9)	9	9	1	2180.632	0
(0.7, 0.8)	4	4	1	2305.478	0
(0.7, 0.8)	5	4	53	2263.969	3.3
(0.7, 0.8)	5	5	1	2190.960	0
(0.7, 0.8)	6	5	61	2103.055	2.6
(0.7, 0.8)	6	6	1	2049.187	0
(0.7, 0.8)	7	5	45	2026.969	3
(0.7, 0.8)	7	6	125	1992.173	1.2
(0.7, 0.8)	7	7	1	1967.977	0
(0.7, 0.8)	8	6	27	1967.737	0
(0.7, 0.8)	8	7	37	1967.737	0
(0.7, 0.8)	8	8	1	1967.737	0
(0.7, 0.8)	9	7	17	1908.053	0
(0.7, 0.8)	9	8	20	1908.053	0
(0.7, 0.8)	9	9	1	1908.053	0

According to the triangle inequality theorem, traveling directly cannot be higher than traveling between these two hubs or nodes by passing through a central hub. Also, the distances between two hubs and a central hub are reduced by the factor α_H and the distances between two central hubs are reduced by the factor α_C.

Now we consider effect of the number of central hubs and discount factors on the locations of central hubs and the locations of hubs .We use the CAB data with n = 25; p = {3,4,5,6,7,8,9} ; p_0 = {2,3,4,5,6,7,8,9} and different discount factors.

In Table 2, we report the locations of hubs and central hubs in the optimal solutions for these instances. Looking at the locations of the hub nodes in Table 2, we observe that San Francisco (22) and Seattle (23) are usually selected as a central hub node or hub node. To see the effect of decreasing the value of the discount factor for the transportation cost among central hubs, we compare the results for the instances with (α_C ,α_H) equal to (0.9, 0.9), (0.8, 0.9), and (0.7, 0.8).

When p=6 and more, for (α_C ,α_H) equal to (0.9, 0.9), Miami (14) is always selected as a central hub node. For (α_C ,α_H) equal to (0.8, 0.9), Boston (3) is always selected as a central hub node.

In Table 2, when (α_C ,α_H) equal to (0.9, 0.9) and p_0=7, for located eight and nine hub nodes, Boston (3), Chicago (4), Kansas City (11), Los Angeles (12), Miami (14), San Francisco (22), and Seattle (23) are always selected as central hub nodes. For located seven hub nodes, Phoenix (19) and St. Louis (21) instead of Chicago (4) and Kansas

City (11) selected as central hub nodes, respectively. In Fig. 6, we give the United States map with the 25 cities and illustrate a sample of solutions on the CAB data set. In order to analyze the flow behavior of the designed network links, we use green color to represent the central hubs and yellow color to represent the hubs. We explored the flow data with (α_C ,α_H) equal to (0.9, 0.9), p=5 and p_0=4 corresponding to instances (a) in Fig. 6.

Table 2
The results on the CAB data set with 25 cities for SA-HHCN problem

(α_C,α_H)	p	P_0	Hub locations	Central Hub locations
(0.9,0.9)	3	2	8,11,23	8,11
(0.9,0.9)	3	3	8,9,16	8,9,16
(0.9,0.9)	4	3	8,9,16,23	8,9,16
(0.9,0.9)	4	4	8,9,16,23	8,9,16,23
(0.9,0.9)	5	4	8,13,20,22,23	8,13,20,23
(0.9,0.9)	5	5	4,8,13,22,23	4,8,13,22,23
(0.9,0.9)	6	5	3,8,14,21,22,23	3,8,14,21,23
(0.9,0.9)	6	6	3,14,19,21,22,23	3,14,19,21,22,23
(0.9,0.9)	7	5	3,4,8,14,21,22,23	3,8,14,21,23
(0.9,0.9)	7	6	3,4,11,12,14,22,23	4,11,12,14,22,23
(0.9,0.9)	7	7	3,12,14,19,21,22,23	3,12,14,19,21,22,23
(0.9,0.9)	8	6	3,4,7,14,19,21,22,23	4,7,14,19,22,23
(0.9,0.9)	8	7	3,4,11,12,14,17,22,23	3,4,11,12,14,22,23
(0.9,0.9)	8	8	3,12,14,19,21,22,23,24	3,12,14,19,21,22,23,24
(0.9,0.9)	9	7	3,4,5,11,12,14,19,22,23	3,4,11,12,14,22,23
(0.9,0.9)	9	8	3,4,7,11,14,16,19,22,23	3,4,11,14,16,19,22,23
(0.9,0.9)	9	9	3,12,14,17,18,19,21,22,23	3,12,14,17,18,19,21,22,23
(0.8,0.9)	3	2	11,22,23	11,23
(0.8,0.9)	3	3	6,8,16	6,8,16
(0.8,0.9)	4	3	11,12,22,23	11,22,23
(0.8,0.9)	4	4	19,21,22,23	19,21,22,23
(0.8,0.9)	5	4	6,8,16,22,23	6,8,16,23
(0.8,0.9)	5	5	6,8,16,22,23	6,8,16,22,23
(0.8,0.9)	6	5	3,11,12,14,22,23	3,11,12,22,23
(0.8,0.9)	6	6	11,12,17,22,23,24	11,12,17,22,23,24
(0.8,0.9)	7	5	8,12,13,14,17,22,23	8,13,17,22,23
(0.8,0.9)	7	6	3,17,19,21,22,23,24	3,19,21,22,23,24
(0.8,0.9)	7	7	3,11,18,19,22,23,24	3,11,18,19,22,23,24
(0.8,0.9)	8	6	3,14,17,19,21,22,23,24	3,19,21,22,23,24
(0.8,0.9)	8	7	3,6,8,12,13,14,22,23	3,6,8,13,14,22,23
(0.8,0.9)	8	8	3,8,12,13,14,20,22,23	3,8,12,13,14,20,22,23
(0.8,0.9)	9	7	3,12,14,19,21,22,23,24,25	3,14,19,21,22,23,25
(0.8,0.9)	9	8	3,12,14,17,19,21,22,23,24	3,12,14,17,19,21,22,23
(0.8,0.9)	9	9	3,12,14,17,19,21,22,23,24	3,12,14,17,19,21,22,23,24
(0.7,0.8)	3	2	11,22,23	11,23
(0.7,0.8)	3	3	11,22,23	11,22,23
(0.7,0.8)	4	4	11,12,22,23	11,12,22,23
(0.7,0.8)	5	4	8,16,20,22,23	8,16,20,22
(0.7,0.8)	5	5	9,13,19,22,23	9,13,19,22,23
(0.7,0.8)	6	5	11,12,17,22,23,24	11,12,17,22,23
(0.7,0.8)	6	6	17,19,21,22,23,24	17,19,21,22,23,24
(0.7,0.8)	7	5	8,12,13,14,17,22,23	8,13,17,22,23
(0.7,0.8)	7	6	11,12,14,17,22,23,24	11,12,17,22,23,24
(0.7,0.8)	7	7	2,3,11,19,22,23,24	2,3,11,19,22,23,24
(0.7,0.8)	8	6	3,6,11,12,19,22,23,24	6,11,12,22,23,24
(0.7,0.8)	8	7	3,6,8,11,12,22,23,24	3,6,8,11,22,23,24
(0.7,0.8)	8	8	3,8,12,20,21,22,23,24	3,8,12,20,21,22,23,24
(0.7,0.8)	9	7	2,3,11,12,14,19,22,23,24	2,3,11,14,19,22,23
(0.7,0.8)	9	8	2,3,8,11,12,14,22,23,24	2,3,8,11,12,14,22,23
(0.7,0.8)	9	9	3,8,12,14,20,21,22,23,24	3,8,12,14,20,21,22,23,24

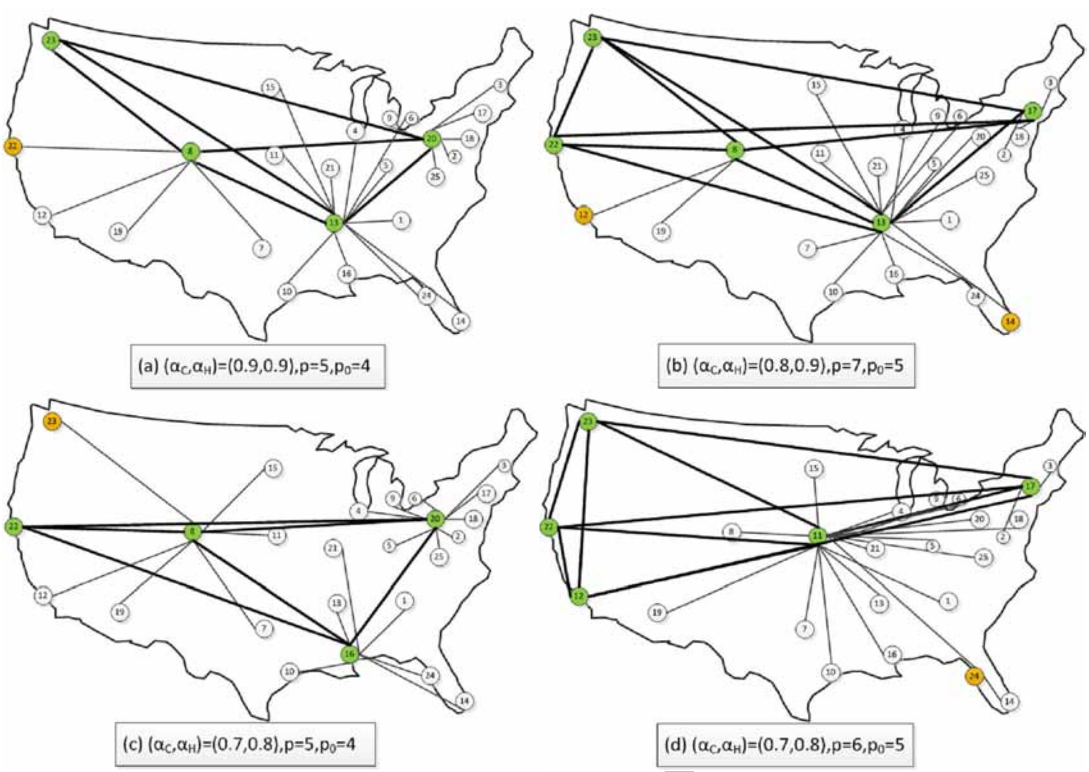

Fig. 6. CAB data set results with 25 cities for SA-HHMN problem

As we observe in Fig. 6, the cities Denver (8), Memphis (13), New York (17), Pittsburgh (20), San Francisco (22) and Seattle (23) are good locations for central hubs.

3.2. SA-IHHCN problem

We tested the performance of our **SA-IHHCN** model on CAB data with 20 cities.

For the CAB data set with 20 cities, *p* varies from 5 to 8 and p_0 increases from 3 to 5 .We tested differing *q* values for our incomplete hierarchical p-hub center network design formulation. We took α_C and α_H values to be 0.9, 0.8 and 0.7.

In all the instances of tables, if the number of established central hub links is equal to p_0 (p_0 -1)/2, then these instances are complete network. Also if the number of established central hub links is less than p_0 (p_0 -1)/2, then these instances are incomplete network.

We report our results on the CAB data set with 20 cities in Table 3. For each instance, Table 3 reports the required CPU time in seconds, the locations of the hub nodes, the locations of the central hub nodes, gap, transportation cost and increase in transportation costs. The time was limited to 2000 sec (about 33min).

In Table 3, we observe that at the instances where we located four central hub nodes, for instance with (α_C ,α_H) equal to (0.9, 0.9), two cities Cincinnati (5) and Los Angeles (12) are always selected as hub nodes or central hub nodes. For instance, with (α_C ,α_H) equal to (0.8, 0.9), two cities Cincinnati (5) and Denver (8) are always selected as hub nodes or central hub nodes. For instance with (α_C ,α_H) equal to (0.7, 0.8), two cities Detroit (9) and Los Angeles (12) are always selected as hub nodes or central hub nodes. In all instances where we located seven hub nodes and five central hub nodes, for instance with (α_C ,α_H) equal to (0.9, 0.9), two cities Dallas (7) and Pittsburgh (20) are always selected as central hub nodes. For instance with (α_C ,α_H) equal to (0.7, 0.8), Denver (٨) are always selected as hub nodes or central hub nodes.

The percentage of increasing in transportation costs is reported as zero for the instances with complete central hub networks. We also observed from Table 3 that the percentage of increase in the transportation costs is higher for instances with lowest number of established central hub links (*q*).

In Fig. 7 and Fig. 8, we observe the transportation costs with respect to the number of established central hub links; we decided to draw three trades off curve.

We analyzed the instance with different values of discount factors, *p* = 7, 8, p_0=5 and different values of central hub links. Fig. 7 and Fig. 8 depict the trades off curve. In Fig. 7, when we forced the model to establish with seven central hub links the transportation costs was about 2188 - 2518 and when the model is established with six central hub links the transportation costs was about 2250 – 2636. In Fig. 8, when we forced the model to establish with nine central hub links the transportation costs was about 2152 - 2410 and when the model to establish with eight central hub links the transportation costs was about 2176 – 2654.

Table 3
The results on the CAB data set with 20 cities for SA-IHHCN problem

(α_C,α_H)	p	P_0	q	CPU Time (s)	Hub Locations	Central Hub Locations	GAP	Transportation Costs	% Increase in Transportation Costs
(0.9,0.9)	6	4	5	2000	3,5,11,12,14,20	3,5,11,12	0.00	2340.0720	0.00
(0.9,0.9)	7	5	6	>2000	6,7,11,12,13,14,20	6,7,11,13,20	7.00	2636.9090	4.69
(0.9,0.9)	7	5	7	>2000	2,3,6,7,11,17,20	2,3,7,17,20	11.25	2518.7420	0.00
(0.9,0.9)	8	5	8	>2000	1,4,5,10,12,13,15,19	5,10,12,15,19	11.80	2654.5260	10.11
(0.9,0.9)	8	5	9	>2000	3,7,8,10,11,12,14,16	3,7,10,11,14	2.90	2410.7730	0.00
(0.8,0.9)	5	3	2	>2000	3,11,12,14,17	11,14,17	8.40	2271.0450	0.00
(0.8,0.9)	6	4	5	>2000	3,5,7,8,10,12	3,5,7,10	11.77	2357.8050	0.00
(0.8,0.9)	7	5	6	>2000	9,12,13,15,17,18,19	9,13,15,17,19	7.50	2250.4970	2.83
(0.8,0.9)	7	5	7	>2000	1,3,4,7,11,12,16	3,4,11,12,16	4.70	2188.5590	0.00
(0.8,0.9)	8	5	8	>2000	1,5,8,9,10,14,19,20	1,5,9,19,20	17.30	2515.4740	7.73
(0.8,0.9)	8	5	9	>2000	2,5,6,7,11,12,14,19	2,5,6,7,11	10.91	2334.9650	0.00
(0.7,0.8)	8	5	9	>2000	4,8,11,12,14,15,17,19	8,12,14,15,19	11.35	2152.4140	0.00

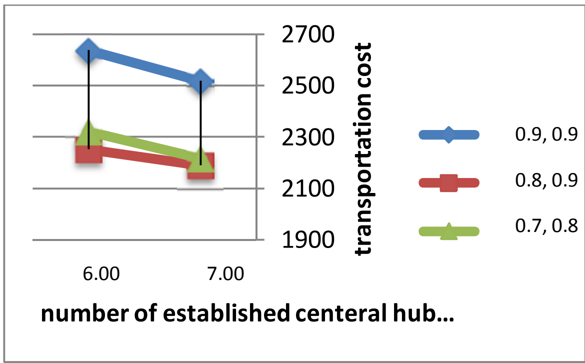

Fig. 7. The transportation costs for CAB data with 20 nodes, 7 hubs and 5 central hubs

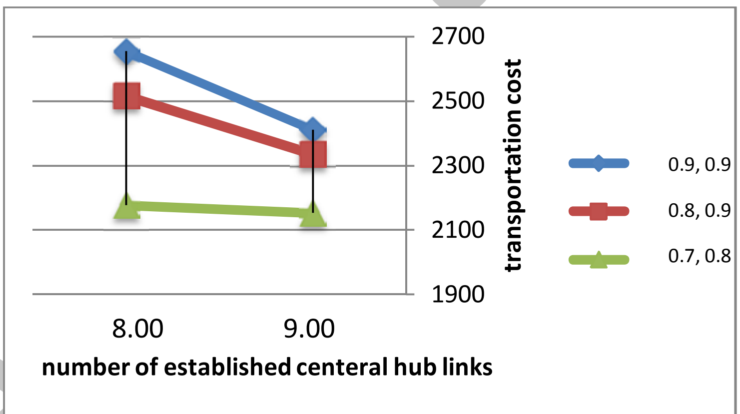

Fig. 8. The transportation costs for CAB data with 20 nodes, 8 hubs and 5 central hubs

In Fig. 9, we give the United States map with the 20 cities and illustrate a sample of solutions on the CAB data set in order to analyze the flow behavior of the designed network links.
We use green color to represent the central hubs and yellow ones to represent the hubs.

As we observe in Fig. 9, the allocations was for instances with gaps unequal zero. For this reason, allocation can be done better. The cities Cincinnati (5), Dallas (7), Los Angeles (12), Memphis (13) and Pittsburgh (20) are good locations for central hubs.

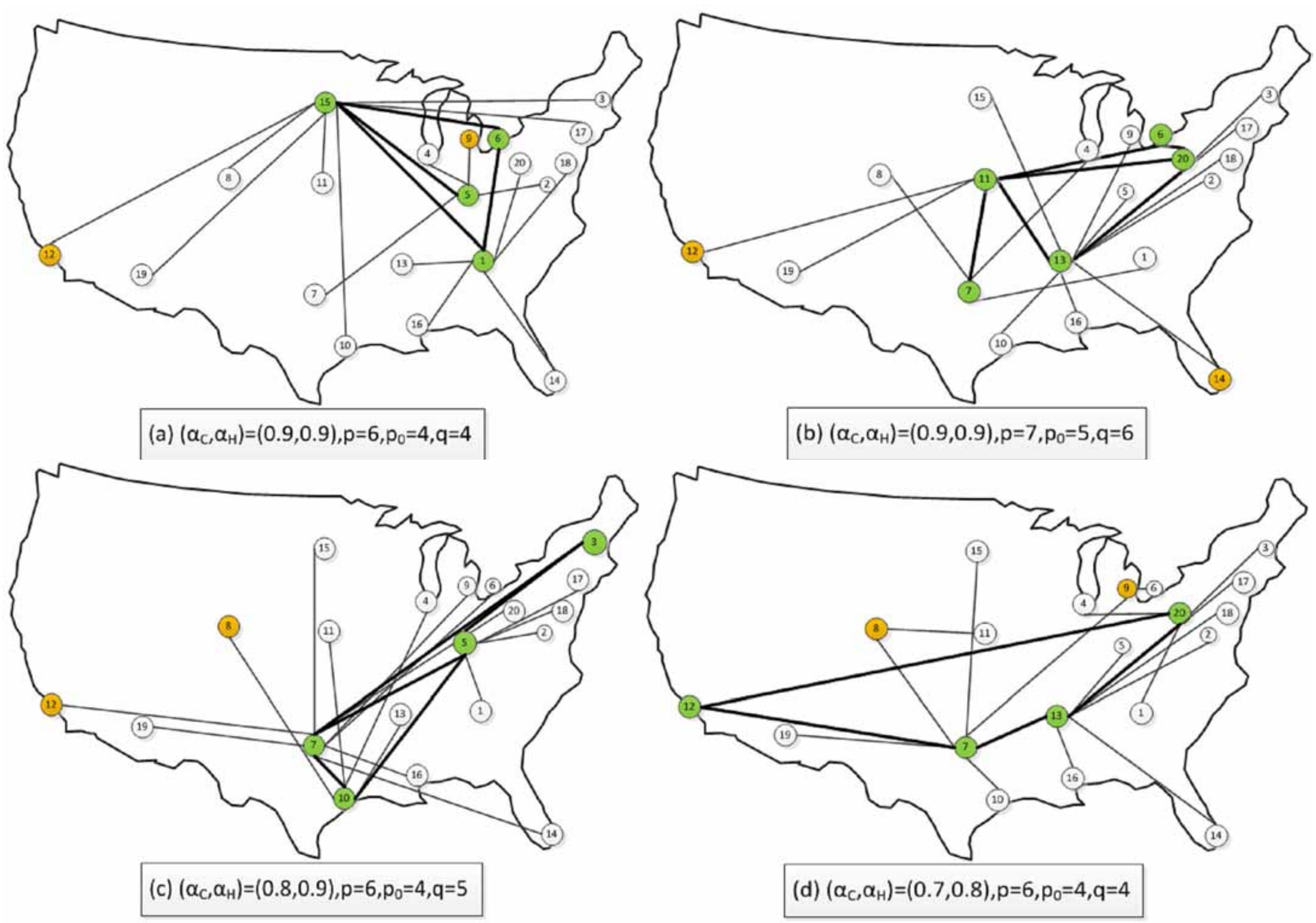

Fig. 9. CAB data set results with 20 cities for SA-IHHMN problem

4. Conclusion

In this paper, we introduced hierarchical hub center network problem with single assignment for complete network environment and presented a mixed integer programming model to solve it. Also we introduced this problem for incomplete network environment and presented a mixed integer programming model to solve it. We presented computational analyses with these formulations on the CAB data set. All of the proposed test instances have been solved with our proposed models considering a reasonable CPU time. The problems were motivated from real-life observations of many central hub networks. In this study, when a direct link between all central hubs is not necessary, we observed effects on the central hub location selections. In general, the decision makers have to choose among more cases when using an incomplete setting for the network instead of complete setting. In real world considering a complete hub network problem must be very expensive and as we have shown in this paper, definition of some incomplete hierarchical hubs will be more real and economic.

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