A Bi-objective Optimization for Vendor Managed Inventory Model

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Abstract

Vendor managed inventory is a continuous replenishment program that is designed to provide major cost saving benefits for both vendors and retailers. Previous research on this area mainly included single objective optimization models where the objective is to minimize the total supply chain costs or to maximize the total supply chain benefits. This paper presents a bi-objective mathematical model for single-manufacture multi-retailer with multi-product in order to maximize their benefits. It is assumed that demand is a decreasing and convex function of the retail price. In this paper, common replenishment cycle is considered for the manufacturer and its retailers. Then, the proposed model converts to the single-objective optimization problem using a weighted sum method. A genetic algorithm (GA) is applied to solve it and response surface methodology is employed to tune the GA parameters. Finally, several numerical examples are investigated to demonstrate the applicability of the proposed model and solution approach.
Keywords: Bi-objective optimization; Vendor managed inventory; Genetic algorithm.

1. Introduction

Supply chain management is a set of approaches used to efficiently integrate suppliers, manufacturers, warehouses, and stores so that merchandise is produced and distributed in the right quantities, to the right locations, and at the right time in order to minimize total costs while satisfying service-level requirements (Pasandideh et al., 2011). One of the well-known concepts utilized in supply chain is the vendor managed inventory (VMI) models (see Disney & Towill, 2002; Cheung & Lee, 2002).

VMI is a program that has been recognized as one of the most successful practices that enhances supply chain integration. VMI is a industry practice for supply chain collaboration, in which the manufacturer manages inventory at hand the retailer and decides when and how much to replenish. Under VMI program, the vendor is able to determine the timing and quantity of replenishment and access to the retailer's inventory and demand data. Consequently, the vendor can coordinate his long-term plans and control the day-to-day flow of goods and material. On the other hand, retailers incur no ordering cost and are guarded against excessive inventory cost by contractual agreements (Guan & Zhao, 2010).

The benefits of VMI for retailers included reduction of overhead costs when consignent stock is adopted, and

transfer of inventory costs to the manufacture; while the benefits of VMI for manufacture are not very straightforward (Lee & Ren, 2011). There are many researches on the VMI-driven supply chains. A part of these researches focus on measuring the benefits of adopting this inventory cooperative strategy, and another part look into the optimal decisions made by the members of the supply chain by adopting a VMI contract (Almehdawe & Mantin, 2010).

Yao et al. (2007) introduced a model to explore the effects of cooperative supply chain initiatives such as vendor managed inventory (VMI). This model was developed by Vlist et al. (2007) where the shipment sizes from supplier to buyer increase, inventory at the supplier goes down and inventory at the buyer goes up. Zhang et al. (2007) presented an integrated VMI model for a single-vendor and multi-buyer, where the vendor purchases and processes raw materials and then delivers finished items to multi-buyer. Investment decision and constant production are considered where the buyers' ordering cycles are different. In their research, buyers can replenish more than once in a production cycle time.

The impact of the consignment inventory (CI) and vendor-managed inventory was studied by Gümüş et al. (2008). In this paper, analyzing the CI is the goal in a two-

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party supply chain under deterministic demand. This research provided some general conditions under CI benefits for the vendor, customer, and two parties together. Sari (2008) presented a comprehensive simulation model representing two popular supply chain initiatives, which is collaborative planning, forecasting and replenishment (CPFR) along with VMI. The aim is selection of an appropriate collaboration mode in business conditions. Their results showed that benefits of CPFR are always higher than VMI. An integrated production-inventory model is developed by Zavanella and Zanoni (2009). They studied a particular VMI policy, which is known as consignment stock (CS) for both the buyer and supplier. Yu et al. (2009a) illustrated how the vendor can take into account the advantage of the information for increasing his own profit by using a Stackelberg game in a VMI system. Yu et al. (2009b) studied how to analyze the intrinsic evolutionary mechanism of the VMI supply chains by applying the evolutionary game theories. Darwish and Odah (2010) developed a model for a supply chain with single vendor and multiple retailers based on VMI, considering capacity constraints by selecting high penalty cost. Almehdawe and Mantin (2010) proposed supply chain included a single capacitated manufacturer and multiple retailers. They formulated a Stackelberg game vendor managed inventory framework under two scenarios: in the first, the manufacturer is the leader; in the second, one of the retailers acts as the dominant player of the supply chain. In addition, market demand is a function of the retail price. Also, This model was developed by Yu et al. (2009c), considering advertising investment and pricing.

Quaternary policy system towards integrated logistics and inventory aspect of the supply chain are presented by Arora et al. (2010). They considered a supply chain with multi-retailer and distributors, in which all distributors follow a unique policy and VMI system is used for updating the inventory of the retailers. Yang et al. (2010) studied the effects of the distribution centre (DC) in a VMI system comprising one manufacturer, one DC and multi- retailer, where the system aims to maximize the overall system profit. Lee and Ren (2011) examined the total supply chain cost decreases under VMI, and the reduction of the supply chain total cost is more when there is exchange rate uncertainty, as compared to the case of exchange rate without uncertainty. They considered a state-dependent (s, S) policy for the supplier. Pasandideh et al. (2011) presented an economic order quantity model for vendor managed inventory control system with multi-product and multi-constraint and developed a genetic algorithm to find the best order quantities and the maximum backorder levels so that the total inventory cost of the supply chain is minimized.

A logistics network design problem under VMI by considering location, transportation, pricing, and warehouse-retailer echelon inventory replenishment are presented by Shu et al. (2012). Zanoni et al. (2012) provided a two-level supply chain system with a single-vendor and a single-buyer at each level. They studied different policies that the vendor may adopt to exploit the advantages offered by the VMI with consignment agreement when the vendor's production process is subject to learning effects.

In many researches related to this issue, the relationship between the manufacturer and retailers is a non-cooperative as Stackelberg game where the manufacturer is the leader and retailers are the followers. In addition, a few of them are related to the concept of pricing. Also, previous research on this area mainly included a single objective optimization model where the objective is to minimize the total supply chain costs or to maximize the total supply chain benefits. In this sense, some current approaches are based on the development of design-planning models (Yu et al., 2009a; Almehdawe and Mantin 2010).

This paper presents a two-level supply chain by assuming a single capacitated manufacture at the first level and multiple retailers at the second level. The manufacturer (vendor) produces multiple products, sells to retailers, and manages the retailers' inventories under VMI. The demand rate for each product in each retailer is assumed to be considered a decreasing function of the retail price. This is called Cobb–Douglas demand function. This paper formulates a non-linear mathematical model with two-objective that is called manufacture-retailers model, in order to maximize benefits for manufacturer and its retailers. The manufacturer and its retailers determine wholesale prices, retail prices, replenishment cycles and backorder quantity to maximize their profits. Finally, this model was converted to the single-objective optimization problem using a weighted sum method and genetic algorithm (GA) is applied for solving the proposed model. Response surface methodology is employed to tune the GA parameters.

The paper is organized as follows. Section 2 contains defining the problem. Section 3 discusses Mathematical model of the problem. In Section 4, a genetic algorithm is developed to solve the problem. RSM parameter-tuned is described in Section 5. In order to demonstrate the application of the proposed methodology, we provide several numerical examples in Section 6. Finally, conclusion is provided in Section 7.

2. Defining the Problem

Let us consider a two-echelon supply chain that consists of a single manufacturer and multiple retailers. The manufacturer's capacity is finite and produces different products with a fixed production rate, and sells the products to its retailers with a common replenishment cycle. Thus, a common replenishment cycle eliminates the influence of the variation of the common replenishment cycle and backorder rate of every retailer. The manufacturer must sell the different products at the different wholesale prices to its retailers. Manufacturers and retailers are operating in distinctive markets and there is not any competition against

each other. In order to facilitate the VMI contract, the retailers allow the manufacturer to access their inventory data. For having their managed inventory by the manufacturer, each retailer pays to the manufacturer a cost of ξ_{ic} per unit consumed per time unit. The manufacturer's decisions are included in its replenishment cycle of the finished products, wholesale prices and fraction of backlogging. On the other hand, the retailers' decisions are included their retail prices. The following basic assumptions are made for the proposed models:

1. The demand function for every retailer and every product is constant over time and convex function with respect to its retail price.
2. Lead-time of the each product at each level of the supply chain is zero.
3. The production setup cost occurs at every beginning of the common replenishment cycle.
4. The whole production process is continuous without any production setup cost.
5. Planning horizon is infinite.

3. Mathematical Model of the Problem

In order to develop the mathematical model of the problem, let us introduce the notations.

3.1. Indices

c Index for retailers ($c = 1,2, ... , n$)
i Index for product types ($i = 1,2, ... , I$)

3.2. Input Parameters

ξ_{ic} Inventory management cost of the product i for retailer c (\$/unit/time)
cm Production cost per unit for finished product (\$/unit)
Φ Transportation cost per unit for finished product shipped from the manufacturer to retailer (\$/unit)
r Production rate of the finished product
S_i Setup cost for a common cycle time for product i (\$)
SR_C Fixed order cost paid by the manufacturer to retailer c (\$)
π_{ic} Backorder cost paid by the manufacturer to retailer c for product i (\$/unit/time)
H_i Holding cost at the manufacturer's side (\$/unit/time)
h_{ic} Holding cost paid by the manufacturer at retailer c's side for product i (\$/unit/time)

3.3. Decision Variables

w_{ic} Wholesale price of the finished product i for retailer c (\$/unit)
p_{ic} Retail price charged by retailer c for product i (\$/unit)
b_{ic} Fraction of backlogging rate of product i in a cycle for retailer c (\$/time)

C_i Common replenishment cycle time for the finished product i

The demand faced by each retailer for the each product, which is controlled through the VMI setting, is assumed a Cobb–Douglas demand function that characterized by a constant elasticity demand function of the following

$$D_{ic} = k_c p_{ic}^{-e_c} \quad \forall i = 1, ... , I, c = 1, ... , n \qquad (1)$$

In which k_c and $e_c > 1$ represents the market scale of retailer c and the demand elasticity of retailer c with respect to its retail price respectively.

3.4. Mathematical Model

The manufacture-retailers model is formulated as follows:

$$max\, z_1 = \sum_{i=1}^{I} \sum_{c=1}^{n} D_{ic}(w_{ic} - cm - \Phi) \qquad (2)$$

$$- \sum_{i=1}^{I} \left[\frac{S_i}{C_i}\right] - \sum_{i=1}^{I} H_i[\sum_{c=1}^{n}(\frac{D_{ic}^2 C_i^2}{2rC_i})] - TC_{VMI} ,$$

$$max\, z_2 = \sum_{i=1}^{I} \sum_{c=1}^{n} D_{ic}(p_{ic} - w_{ic} - \xi_{ic}) , \qquad (3)$$

Subject to:
$$TC_{VMI} = \qquad (4)$$
$$\sum_{i=1}^{I} \sum_{c=1}^{n} \left[\frac{SR_c}{C_i}\right] + \sum_{i=1}^{I} \sum_{c=1}^{n} h_{ic} \left(\frac{D_{ic}(1-b_{ic})^2 C_i}{2}\right) +$$
$$\sum_{i=1}^{I} \sum_{c=1}^{n} \pi_{ic}(\frac{D_{ic} b_{ic}^2 C_i}{2}) - \sum_{i=1}^{I} \sum_{c=1}^{n} \xi_{ic} D_{ic} ,$$
$$\sum_{i=1}^{I} \sum_{c=1}^{n} D_{ic} \leq r, \qquad (5)$$
$$p_{ic} > w_{ic} + \xi_{ic} , \qquad \forall i = 1, ... , I, c = 1, ... , n, \qquad (6)$$
$$0 \leq b_{ic} \leq 1 , \qquad \forall i = 1, ... , I, c = 1, ... , n, \qquad (7)$$
$$C_i, w_{ic}, p_{ic} \geq 0, \quad \forall i = 1, ... , I, c = 1, ... , n, \qquad (8)$$

It is clear that the above model is nonlinear with two conflicting objective functions. The first objective function given in Eq. (2) is net profit for manufacturer consisting of the revenue from sale of finished products to retailers at wholesale prices, production and transportation costs, setup costs, holding cost and inventory costs paid by the manufacturer, due to VMI system. Eq. (3) is net profit for all retailers. Eq. (4) is total inventory cost incurred by the manufacturer to manage all retailers' inventory, consist of the difference between all the inventory costs he realizes and the revenue he receives from the retailers for managing their inventory. The inventory costs at each retailer's side are the fixed inventory costs, variable inventory costs, and back-ordering costs. Eq. (5) satisfies that total demand faced by the manufacture does not exceed his production capacity. Eq. (6) shows the least acceptable price in order to assurance at least positive net profit for all retailers. Eq. (7) is to set the limits for the fraction of backlogging rate and Eq. (8) guarantees positive value for each decision variables. Fig. 1 shows how we can obtain average holding costs and backorder costs for retailers. Also, Fig. 2, shows

the total inventory level of manufacture for product i per common replenishment cycle.
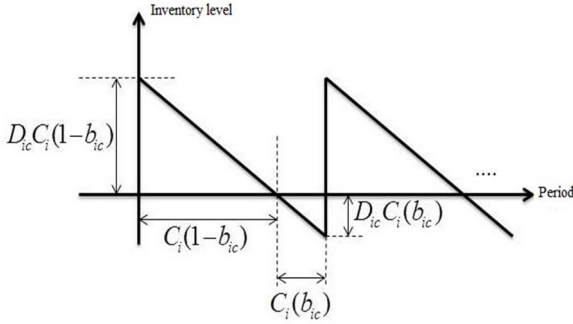

Fig. 1. Inventory level of retailer c for product i per common replenishment cycle
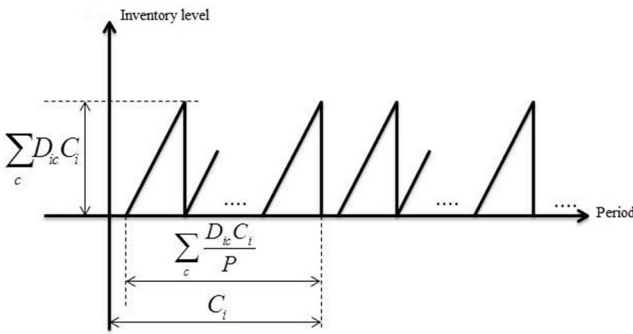

Fig. 2. Inventory level of the manufacture for product i per common replenishment cycle

3.5. Weighted sum method

Many scalarzing methods exist for transforming a multi-objective optimization problem to a single objective optimization problem. One of the most popular scalarizing methods is to combine the multiple objectives using a weighted sum method (Hawe & Sykulski, 2008):

$$Maximize\ \hat{f}(x) = \sum_{i=1}^{M} w_i \hat{f}_i(x) \tag{9}$$

where $\hat{f}_i$ is the normalized value of objective f_i and the weights of the objectives $w_i > 0$, are the parameters of the scalarization. Therefore, Eq. (10) is objective function in the manufacture-retailers model as follows:

$$z = w_1 z_1 + w_2 z_2 \tag{10}$$

4. Genetic Algorithm

The proposed manufacture-retailers model is a NLP problem; obviously, solving the NLP problems are hard with exact methods because the NLP is an NP-complete problem (Kuk, 2004). Exact methods are complex and are not very affective for solving the NLP models. In recent years, genetic algorithms (GAs) are developed to solve the NLP problems. GAs are strong tools for solving the NLP models (Yokota et al., 1996). Therefore, a GA is utilized to solve the presented model.

The fundamental principle of GAs was first introduced by Holland, 1992, encode the features of a problem by chromosomes, where each gene represents a feature of the problem. In GA, the crossover and mutation operators are used with the given probabilities. In general, GA consists of the following steps (Ramezanian et al., 2012):

Step 1: Initialize a population of chromosomes.
Step 2: Evaluate the fitness of each chromosome.
Step 3: Create new chromosomes by applying genetic operators such as reproduction, crossover and mutation to current chromosomes.
Step 4: Evaluate the fitness of the new population of chromosomes.
Step 5: If the termination condition is satisfied, stop and return the best chromosome; otherwise, go to Step 3.

4.1 Chromosome Representation

In a GA, a chromosome is a string or trail of genes, which is considered as the coded figure of a solution (appropriate or none appropriate). Designing a suitable chromosome is the most important stage in applying the GA in the solution process of the problem (Ramezanian et al., 2012). In this research, the chromosomes are provided by a $I \times (3n + 1)$ matrix. Fig. 3 illustrates the general form of a chromosome.

p_{11}	p_{12}	p_{1n}	b_{11}	b_{12}	b_{1n}	w_{11}	w_{12}	w_{1n}	C_1
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
p_{I1}	p_{I2}	p_{In}	b_{I1}	b_{I2}	b_{In}	w_{I1}	w_{I2}	w_{In}	C_I

Fig. 3. The chromosome presentation

4.2 Evaluation and Initial Population

When GA is employed for an optimization problem, a fitness value, which is the value of the objective function (which is defined in Section 3), is needed to be assigned for a chromosome, as soon as it is generated. An initial population (or a batch of chromosomes) is generated randomly. Some of the generated chromosomes may not be feasible, so the generation of the chromosomes is controlled via penalty method to generate feasible chromosomes.

4.3 Selection

The selection provides the opportunity to deliver the gene of a good solution to next generation. Various selection operators can be used to select the parents. In this study, the roulette wheel selection is employed where chromosome selection in mating pool is based on their probability selection. The probability selection of each chromosome is evaluated based on its fitness value.

4.4 Crossover

Crossover is a process in which chromosomes exchange genes through the breakage and reunion of two chromosomes to generate a number of children. In this

study, to explore solution space an arithmetic crossover is chosen. Arithmetic crossover generates an offspring by linear combining two selective parents. Let $X_1 = (x_{11}, x_{12}, \dots x_{1n})$ and $X_2 = (x_{21}, x_{22}, \dots x_{2n})$ be two selected parents. Therefore, two offspring are obtained based on the following equations:

$$
\begin{aligned}
y_{1i} &= \alpha_i x_{1i} + (1 - \alpha_i)x_{2i}, \quad i = 1, \dots, n, \\
y_{2i} &= \alpha_i x_{2i} + (1 - \alpha_i)x_{1i}, \quad i = 1, \dots, n,
\end{aligned} \tag{11}
$$

where $\alpha_i \in (0,1)$.

4.5 Mutation

Mutation generates an offspring solution by randomly modifying the parent's features. It helps to keep a reasonable level of diversity in the population, and serves the search by jumping out of local optimal solutions. In this research, an exchange mutation is chosen. This mutation swaps the value of the two random selected genes of current solution together.

4.6 Stopping criterion

The search process stops if the number of generations is greater than a maximum number of generations or the some specified number of generations without improvement of best-known solution is reached.

5. Parameter Tuning

In this section, the Response Surface Methodology (RSM) is utilized to optimize GA parameters. RSM is a collection of mathematical and statistical techniques that is useful in the modeling and analyzing of problems. A response of interest is influenced by several variables, and the objective is to optimize this response (Najafi et al., 2009). Usually, the first step is to fit a first-order model and conduct a test of lack of fit. However, because the first-order model was inadequate, a second-order model is used. The most popular second-order model is the central composite design (CCD) (Najafi et al., 2009). In this research, there are 2^{k-1} factorial points (fractional factorial), n_c central points, and $2k$ axial points. The second-order model that is used in the CCD is:

$$
E(Y) = \beta_0 + \sum_{i=1}^{k} \beta_i X_i + \sum_{i=1}^{k} \beta_{ii} X_i^2 \\
+ \sum_{i<j}^{k} \sum_{i<j}^{k} \beta_{ij} X_i X_j \tag{12}
$$

where $E(Y)$ is the expected value of the response variable, $\beta_0, \beta_i, \beta_j$ are the model parameters, X_i and X_j are the input variables that affect the response Y, and k is the number of factors being studied. In this research, k factors that affect the response are population size (PopS), the maximum number of generations (MaxG), the crossover probability (P_c), the mutation probability (P_m) and the problem size (ProS). Three levels values of these parameters are listed in Table 1.

Table 1
The GA parameter levels

Factors	Range	Min	Medium	Max
ProS	3-7	3	5	7
PopS	20-100	20	60	100
MaxG	200-700	200	450	700
P_c	0.5-0.8	0.5	0.65	0.8
P_m	0.1-0.2	0.1	0.15	0.2

In order to evaluate the GA parameters, three different problem sizes 3, 5, 7 products with two retailers are considered. We chose $w_1 = 0.5$ and $w_2 = 0.5$ in which $w_1 + w_2 = 1$. The values of model parameters are generated from second column in Table 2.

Table 2
Data generation

Parameters	Values in RSM	Values in examples
k_c	3500	$U(2000, 5000)$
e_c	1.9	$U(1.4, 2.4)$
ξ_{ic}	2.45	$U(1.5, 3.4)$
cm	3	3
Φ	2	2
P	2000	2000
S_i	30	$U(20, 40)$
SR_c	27.5	$U(10, 45)$
π_{ic}	150	$U(100, 200)$
H_i	3.75	$U(2.5, 5)$
h_{ic}	1.75	$U(0.5, 3)$

In Table 2, term "U" is related to the uniform distribution. These examples are coded with MATLAB 7.8 (R2009a) software. The design matrix of the selected central composite design along with the experimental results is shown in Table 3.

The PTYPE column of Table 3 represents the type of the design points (''-1'' for the axial points, ''0'' for the central points and ''1'' for the factorial points). The last column of Table 3 represents the best fitness value for each problem. The experimental results are analyzed with Minitab 16.2.2 software. Second-order coefficients ($P < 0.05$) are listed in Table 4.

The ANOVA is presented in Table 5.

Table 3
Design matrix of the central composite design

Run	PTYPE	ProS	PopS	MaxG	P_c	P_m	Fitness
1	-1	7	60	450	0.65	0.15	6247.26
2	1	7	100	200	0.80	0.10	6345.54
3	1	7	100	200	0.50	0.20	6112.45
4	0	5	60	450	0.65	0.15	1238.43
5	1	7	100	700	0.80	0.20	6331.24
6	0	5	60	450	0.65	0.15	1009.22
7	0	5	60	450	0.65	0.15	1235.65
8	-1	5	20	450	0.65	0.15	1126.28
9	-1	5	60	450	0.50	0.15	1135.25
10	-1	5	60	450	0.80	0.15	1106.65
11	-1	3	60	450	0.65	0.15	1198.55
12	1	3	100	700	0.80	0.10	2001.65
13	1	7	100	700	0.50	0.10	6159.78
14	-1	5	100	450	0.65	0.15	1113.67
15	0	5	60	450	0.65	0.15	1235.35
16	1	3	100	200	0.50	0.10	987.64
17	1	3	20	700	0.50	0.10	978.00
18	0	5	60	450	0.65	0.15	998.34
19	1	3	20	200	0.80	0.10	1234.80
20	1	3	20	200	0.50	0.20	1005.65
21	1	7	20	700	0.80	0.10	6223.15
22	0	5	60	450	0.65	0.15	1235.64
23	1	7	20	200	0.50	0.10	5948.65
24	-1	5	60	700	0.65	0.15	1135.38
25	-1	5	60	450	0.65	0.10	1256.98
26	1	3	100	700	0.50	0.20	931.32
27	1	7	20	200	0.80	0.20	6897.14
28	-1	5	60	450	0.65	0.20	1298.88
29	0	5	60	450	0.65	0.15	1254.54
30	1	3	100	200	0.80	0.20	956.56
31	-1	5	60	200	0.65	0.15	1465.44
32	1	7	20	700	0.50	0.20	6100.50
33	1	3	20	700	0.80	0.20	904.66

Table 4
Multiple regression analysis for fitness

Term	Coef	SE Coef	T	P
Constant	9756.100	1325.200	7.362	0.000
ProS	-5165.400	202.200	-25.547	0.000
PopS	19.600	7.100	2.768	0.017
MaxG	-1.100	1.300	-0.867	0.003
ProS*ProS	616.600	18.400	33.547	0.000
ProS*Pc	318.200	96.100	3.310	0.006
ProS*Pm	812.300	288.400	2.817	0.016
PopS*Pc	-10.700	4.800	-2.223	0.046
PopS*Pm	-38.000	14.400	-2.635	0.022

S = 115.365   R-Sq = 99.91%   R-Sq (adj) = 99.75%

Table 5
Analysis of variance for fitness

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Regression	20	172115701	172115701	8605785	646.61	0
Linear	5	122013254	10307990	2061598	154.9	0
Square	5	49599610	49599610	9919922	745.35	0
Interaction	10	502836	502836	50284	3.78	0.016
Residual error	12	159709	159709	13309		
Lack-of-fit	6	79714	79714	13286	1	0.502
Pure error	6	79995	79995	13333		
Total	32	172275410				

The level of significance is 5%. The (R^2) value of 99.91% and the F-value for the regression was significant at a level of 5% ($P < 0.05$), while the lack of fit was not significant at the 5% ($P > 0.05$), indicating the good predictability of the model. The estimated regression of the model fitness is given in Eq. (13)

$$fitness = 9756.1 - 5165.4 * (ProS) + 19.6 * (PopS) \tag{13}$$
$$- 1.1 * (MaxG) + 616.6 * (ProS)^2$$
$$+ 318.2 * (ProS) * (Pc)$$
$$+812.3 * (ProS) * (Pm) - 10.7 * (PopS) * (Pc)$$
$$-38 * (PopS) * (Pm)$$

Furthermore, the optimal values of GA parameters are presented in Table 6 where the problem size (ProS) is five products.

Table 6
Optimal values of GA parameters

Parameters	Value
PopS	100
MaxG	200
P_c	0.78
P_m	0.2

6. Numerical Examples

In order to assess the applicability of proposed model, 10 numerical examples with different size of retailers, different finished products and randomly generated data are considered. These random data are generated according to the information provided in Table 2 (the third column). Both objectives are equally important. In order to evaluate the performance of the proposed GA, the numerical examples are solved using GAMS 23.5 software on an Intel(R), core (TM) i7, 3.23 GHz lap top with 512 Mb RAM. The GA is implemented for 10 independent runs for each instance. The best objective function values of these examples and related the CPU times are considered. Then, the obtained solutions of GA and GAMS are compared together, and results are reported in Table 7.

To compare objective values obtained by GA with the results of the GAMS, a quality measure, the percent deviation of solution, is defined according to the following equations set:

$$\%Deviation_{Obj.} = \frac{z_{GA} - z_{GAMS}}{z_{GAMS}} \times 100 \tag{14}$$

The performance of the GA is illustrated in Fig. 4, with respect to the solution deviance with the objective values obtained by GAMS.

In the Table 7 and Fig. 4, the comparisons focused on the objective values, and CPU times reveal that increasing the size of retailers, increases z and CPU times. Nevertheless, it can be seen that the quality of solutions obtained by GA is near to the GAMS results. Therefore, these comparisons prove the effectiveness of our approach. Table 8 shows the computational results by GA for an example with two products and three retailers.

Table 7
Comparison of GAMS and GA results

| products	Retailers	GAMS		GA		
		z ($)	CPU time(s)	Best ($)	CPU time(s)	% Dev. obj.
3	2	2481.864	0.098	2434.130	46.13	1.923
2	3	1872.378	0.086	1831.001	44.11	2.210
3	3	3350.679	0.120	3154.794	58.39	5.846
5	3	2789.081	0.190	2660.450	38.59	4.612
3	5	2017.491	1.228	2003.650	39.82	0.686
5	5	7284.784	1.929	7198.001	40.66	1.191
3	7	6052.431	2.301	5994.210	48.77	0.962
5	7	12339.969	2.877	11385.200	63.20	7.737
3	9	6885.329	5.457	6611.547	41.45	3.976
5	9	13341.413	8.721	12984.780	51.19	2.673
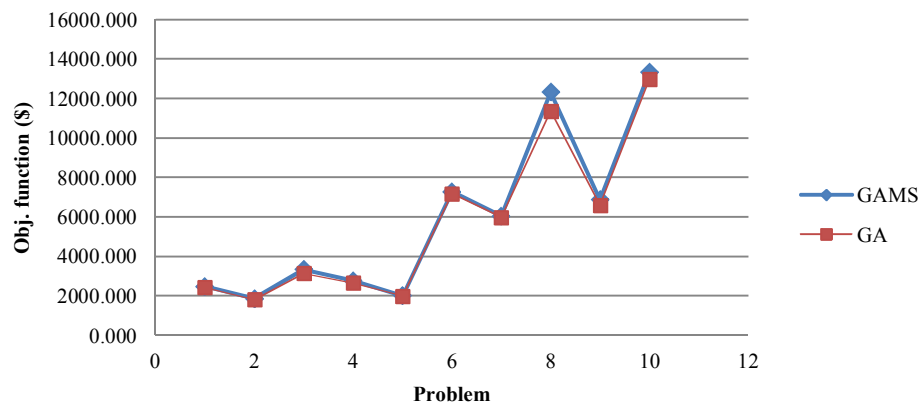
Fig. 4. Percent deviance of objective values obtained by GA and GAMS

Table 8
Computational results by GA

Retailer	Product 1				Product 2			
	w	p	b	D	w	p	b	D
1	7.654	10.738	0.007	39.254	5.548	11.257	0.008	35.649
2	11.986	17.395	0.014	49.849	11.262	16.087	0.006	56.315
3	4.588	14.922	0.017	27.660	12.122	14.277	0.007	29.754
Replenishment cycle	0.914				1.067			

7. Conclusion

Previous research on this topic mainly included a single objective optimization model where the objective was to minimize the total supply chain cost or maximize the total supply chain benefits. This paper developed a bi-objective mathematical model to solve a VMI problem in a two-echelon supply chain that was formulated with a single manufacturer and multiple retailers. This model is a non-linear mathematical model that maximizes manufacture profit and retailer's profit. The manufacturer and its retailers determine wholesale prices, retail prices, replenishment cycles and backorder quantity to maximize their profits. It is assumed that the demand is a function of retail price. Then, bi-objective problem was converted to single-objective using weighted sum method. Since the model of the problem was NP-hard, a GA was developed to solve it and RSM is applied to tune the GA parameters. Finally, several numerical examples were presented to describe the sufficiency of the proposed strategy. It can be seen that the implemented algorithm obtains good solutions within a reasonable computational time. In future research, this work can be expanded by addressing the problem in variation of the common replenishment cycle.

8. References

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