

# Facility Location and Inventory Balancing in a Multi-period Multi-echelon Multi-objective Supply Chain: An MOEA Approach

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## Abstract

A comprehensive and integrated study of any supply chain (SC) environment is a vital requirement that can create various advantages for the SC owners. This consideration causes productive managing of the SC through its whole wide components from upstream suppliers to downstream retailers and customers. On this issue, despite many valuable studies reported in the current literature, considerable gaps still prevail. These gaps include integration and insertion of basic concepts, such as queuing theory, facility location, inventory management, or even fuzzy theory, as well as other new concepts such as strategic planning, data mining, business intelligence, and information technology. This study seeks to address some of these gaps. To do so, it proposes an integrated four-echelon multi-period multi-objective SC model. To make the model closer to the real world problems, it is also composed of inventory and facility location planning, simultaneously. The proposed model has a mixed integer linear programming (MILP) structure. The objectives of the model are reducing cost and minimizing the non-fill rate of customer zones demand. The cost reduction part includes cost values of raw material shipping from suppliers to plants, plant location, inventory holding costs in plants, distribution cost from plants to warehouses or distribution centers (DCs), and shipping costs from DCs to customer zones. Finally, since the literature of SC lacks efficient Pareto-based multi-objective evolutionary algorithms (MOEAs), a new multi-objective version of the biogeography-based optimization algorithm (MOBBO) is introduced to the literature of the SC. The efficiency of the algorithm is proved through its comparison with an existing algorithm called multi-objective harmony search (MOHS).

**Keywords:** Integrated supply chain management, Production-distribution, Facility location problem, inventory balancing, Planning problem, MOBBO, MOHS.

## 1. Introduction

Different definitions have been proposed for supply chain (SC) in the literature. Generally, it is defined as an integrated system of facilities and activities that synchronizes inter-related business functions of material procurement, material transformation to intermediates and final products and distribution of these products to customers (Simchi-Levi, 2000). In other words, the goal of the supply chain management is to integrate suppliers, manufacturers, warehouses, and stores, so that the production of the merchandises and their distribution can be done at the right quantities, to the right locations, and at the right time. The total objective of the system is to minimize system-wide costs while satisfying service level requirements of the customers.

Since SC covers a vast range of concepts and methods, various issues are evolved as the subjects of the industrial and academic research in SC. Some common issues include supplier selection, production-distribution planning, transportation and distribution, facility location

of the distribution centers and logistic warehouse, queuing issues, inventory controlling and balancing. Some researchers have focused on only one of these items while others have considered the combination of two or more of them.

One of the important combinations in the literature is the combination of production area of the SC with the distribution part of it, known as production-distribution problems. In this class, Jayarman and Pirkul (2001) considered factory, distribution center, and demand area within their multi-product deterministic SC. Yilmaz and Cagatay (2006) introduced a three-stage strategic planning for their production-distribution network. In this model, which considers one product, multi-suppliers, multi-manufacturers, and multi-distributors, demand is deterministic and the objective is to minimize cost of production, transportation, and inventory. Some review papers are also presented on this subject (Erenguc et al., 1999, Chen, 2004). Some papers focus on inventory

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subject in their supply chain (Muckstadt and Roundy, 1987; Chan et al., 2002; Levi et al., 2005). Inventory has considerable role in the studies of SCs as the main artery of any supply chain. The basic SC model that considers this item is generally known as single warehouse multi-retailer (SWMR) problem (Muckstadt and Roundy, 1987). Federgruen and Zipkin (1984) studied single period, one warehouse, multi-retailer problem under uncertain demands. Roundy (1985) proposed a policy with 98%

effectiveness in  $O(n \log n)$  time for analyzing a problem which permits no shortage or backlogging. Chan and Kumar (2009 a, b) investigated a manufacturing environment that included warehouse-scheduling problem in a manufacturing environment. Poon et al. (2009) studied order picking operations in warehouses. Another important issue, which is inserted into SC models, is facility location. Javid and Azad (2010) solved an integrated model of facility location, capacity, inventory, and routing. Bidhandi et al. (2009) developed a mixed linear integer programming problem of multi-commodity supply chain. They solved their problem using decomposition methods. Rappold and Van Roo (2009) studied two-echelon supply chain, which combined facility location, inventory allocation, and capacity investment.

Among the presented studies, most of the researches are single objective (Williams 1981, Gen and Syarif, 2005; Tsiakis and Papageorgiou, 2008). Further, some studies focus on multi-objective problems in the SC areas. This class is more realistic, because most of the real world problems, specifically in the complex environment of the SC problems, cope with several goals. In this class, Altıparmak et al. (2006) developed a multi-objective shortage forbidden model that investigated network structure of manufacturers and customer area. Their model tried to minimize costs, deliver time and balance of the capacity of the factories. Jolai et al. (2011) proposed a linear multi-objective production-distribution model. Their model considered a SC with multi-products, levels, and periods. However, they changed their model into a single objective model in their solving approach. Sadeghi et al. (2011) developed a single-vendor single-retailer in a multi-product supply chain. Aliakbari and Seifbarghy (2011) introduced a social responsible supplier selection model. Songsong and Lazaros (2012) also studied a multi-objective production-distribution model. Their model considered a universal SC with three objectives of costs, response, and service level. Taherkhani and Seifbarghy (2012) determined the material flows in a multi-echelon assembly supply chain. Shankar et al. (2013), in their multi-objective production-distribution, proposed single-product four-echelon supply chain architecture. They also considered facility location planning in their problem. However, they did not consider the inventory issue in their integrated model. They solved their model via a multi-objective hybrid particle swarm optimization (MOHPSO) algorithm. Their approach is a Pareto-based

approach in which the multi-objective is not converted into a single objective model. These approaches are more popular these days (Deb et al., 2001). The number of these algorithms is not considerable in the literature of the SC though. Vahdani and Sharif (2013) developed an inexact-fuzzy-stochastic optimization model for a closed loop supply chain network design problem.

According to the literature, this research proposes an integrated model which fills some gaps of the literature. To do so, since, in the literature of SC, specifically for production-distribution planning problem, fewer researches have studied the shortage permitted assumption, this item is considered in our model. Moreover, some other terms of the inventory issue, as the main artery of any SC model, are included in the developed integrated multi-echelon multi-period inventory parts of the model. In addition, to make the model more realistic, it is also encompassed facility location planning. The final structure of this model is as a mixed integer linear programming (MILP) problem. Furthermore, since the literature lacks efficient Pareto-based multi-objective evolutionary algorithms (MOEAs), a new multi-objective version of the biogeography-based optimization algorithm (MOBBO) is introduced to the literature of the SC. Finally, this algorithm is compared with an existing algorithm called MOHS (Rahmati et al. 2013). The results are also evaluated through different statistical and non-statistical tests, tables, and figures.

The paper is organized as follow. The developed integrated model is described in section 2. This section includes problem definition including all parts of the model ranging from assumptions and indices to objective functions and constraints. Section 3 presents the required concepts and operators of the proposed MOEA. Section 4, through different computational experiments, proves efficiency of the proposed algorithms. Section 5 concludes the paper and presents the future works.

## 2. Problem Definition

In this section, the integrated SC model is described. The proposed SC model of this research is a multi-echelon multi-period model which encompasses inventory and facility location planning simultaneously. The inventory part of the model includes four-echelon multi-period inventory cost as the objective function and inventory balancing among different echelon within different periods. The structure of the proposed model is as MINLP. Figure 1 illustrates a simple structure of this model, schematically. The rest of this section defines the required definitions and notations and then formulates the main model in different subsections.

### 2.1. Notations

l: Number of suppliers ( $h = 1, 2, \dots, l$ )

n: Number of potential plant locations ( $i = 1, 2, \dots, n$ )

t: Number of warehouse (DC) locations ( $e = 1, 2, \dots, t$ )  
 m: Number of customer zones (markets) or demand points ( $j = 1, 2, \dots, m$ )  
 p: Number of components ( $c = 1, 2, \dots, p$ )  
 s: Number of time periods ( $k = 1, 2, \dots, s$ )

## 2.2. Parameters

$D_{jk}$ : Average demand from markets  $j$  at time period  $k$   
 $K_{ik}$ : Potential capacity of plant  $i$  at time period  $k$   
 $K_{ek}$ : Potential capacity of warehouse  $e$  at time period  $k$   
 $S_{chk}$ : Supply capacity by supplier  $h$  from component  $c$  at time period  $k$   
 $F_i$ : Annual fixed cost of keeping open of plant  $i$   
 $F_e$ : Annual fixed cost of keeping open of warehouse  $e$   
 $C_{hcik}$ : Cost of making and shipping a components  $c$  from supply source  $h$  to plant  $i$  at time period  $k$   
 $C_{iek}$ : Cost of producing and shipping one unit from plant  $i$  to warehouse  $e$  at time period  $k$   
 $C_{ejk}$ : Cost of throughput and shipping one unit from warehouse  $e$  to customer  $j$  at time period  $k$   
 $IC_{ik}$ : Inventory holding cost of one unit in plant  $i$  at time period  $k$   
 $IC_{ek}$ : Inventory holding cost of one unit in warehouse  $e$  at time period  $k$

$IC_{cik}$ : Inventory holding cost of component  $c$  in plant  $i$  at time period  $k$

$\alpha_c$ : Consume coefficient of component  $c$

## 2.3. Decision variables

$Y_i$ : 1, if plant  $i$  is open, 0 otherwise

$Y'_e$ : 1, if warehouse  $e$  is open, 0 otherwise

$X_{hcik}$ : Quantity of component  $c$  shipped from supplier  $h$  to plant  $i$  at time period  $k$

$X'_{ejk}$ : Quantity shipped from warehouse  $e$  to customer zone  $j$  at time period  $k$

$X''_{iek}$ : Quantity shipped from plant  $i$  to warehouse  $e$  at time period  $k$

$I_{cik}$ : Inventory quantity of component  $c$  in plant  $i$  at time period  $k$

$I'_{ek}$ : Inventory quantity in warehouse  $e$  at time period  $k$

$I''_{ik}$ : Inventory quantity in plant  $i$  at time period  $k$

## 2.4. The main model

This subsection presents the main proposed model as follows.

$$\begin{aligned} \text{Min } Z_1 = & \sum_{i=1}^n f_i \times Y_i + \sum_{e=1}^t f_e \times Y'_e + \sum_{i=1}^n \sum_{h=1}^l \sum_{c=1}^p \sum_{k=1}^s C_{hcik} \times X_{hcik} + \sum_{i=1}^n \sum_{e=1}^t \sum_{k=1}^s C_{iek} \times X''_{iek} \\ & + \sum_{e=1}^t \sum_{j=1}^m \sum_{k=1}^s C_{ejk} \times X'_{ejk} + \sum_{c=1}^p \sum_{i=1}^n \sum_{k=1}^s IC_{cik} \times I_{cik} + \sum_{i=1}^n \sum_{k=1}^s IC_{ik} \times I''_{ik} + \sum_{e=1}^t \sum_{k=1}^s IC_{ek} \times I'_{ek} \end{aligned} \quad (1)$$

$$\text{Min } Z_2 = 1 - \frac{\sum_{e=1}^t \sum_{j=1}^m \sum_{k=1}^s X'_{ejk}}{\sum_{j=1}^m \sum_{k=1}^s D_{jk}} \quad (2)$$

s.t.

$$\sum_{i=1}^n X_{hcik} \leq S_{chk} \quad \forall h=1, 2, \dots, l, c=1, 2, \dots, p, k=1, 2, \dots, s \quad (3)$$

$$\sum_{e=1}^t X'_{ejk} \leq D_{jk} \quad \forall j=1, 2, \dots, m, k=1, 2, \dots, s \quad (4)$$

$$\sum_{e=1}^t X''_{iek} \leq K_{ik} Y_i \quad \forall i=1, 2, \dots, n, k=1, 2, \dots, s \quad (5)$$

$$\sum_{j=1}^m X'_{ejk} \leq K_{ek} Y'_e \quad \forall e=1, 2, \dots, t, k=1, 2, \dots, s \quad (6)$$

$$\sum_{h=1}^l X_{hcik} - \sum_{e=1}^t X''_{iek} \geq 0 \quad \forall i=1, 2, \dots, n, c=1, 2, \dots, p, k=1, 2, \dots, s \quad (7)$$

$$\sum_{i=1}^n X''_{iek} - \sum_{j=1}^m X'_{ejk} \geq 0 \quad \forall e=1, 2, \dots, t, k=1, 2, \dots, s \quad (8)$$

$$I_{cik} = I_{cik-1} + \sum_{h=1}^l X_{hcik} - \sum_{e=1}^t \alpha_c \times X'_{iek} \quad \forall c=1, 2, \dots, p, i=1, 2, \dots, n, k=1, 2, \dots, s \quad (9)$$

$$I''_{ik} = I''_{ik-1} + K_{ik} - \sum_{e=1}^t X''_{iek} \quad \forall i=1, 2, \dots, n, k=1, 2, \dots, s \quad (10)$$

$$I'_{ek} = I'_{ek-1} + \sum_{i=1}^n X''_{iek} - \sum_{j=1}^m X'_{ejk} \quad \forall e=1, 2, \dots, t, k=1, 2, \dots, s \quad (11)$$

$$X_{hcik}, X''_{iek}, X'_{ejk}, I_{ick}, I''_{ik}, I'_{ek} \geq 0$$

$$Y_i, Y'_e \in \{0,1\} \quad (12)$$

$$I_{ci0}, I''_{i0}, I'_{e0} = 0$$

In this model, Eq.1 models the first objective function, which minimizes the total cost in supply chain. Total cost includes raw material shipping from suppliers to plants, plant location, inventory holding costs in plants, distribution cost from plants to warehouses or distribution centers (DCs), throughput and shipping costs from DCs to customer zones.

The objective function (2) is minimizing the non-filled rate of customer zones demand.

Equation (3) ensures that the total quantity shipped from a supplier at each period cannot exceed the supply capacity.

Equation (4) indicates that the demand at customer zone should be satisfied to the maximum extend.

Equation (5) shows that no plant can supply more than its capacity if the plant is opened.

The Equation in (6) represents that no warehouse can supply more than its capacity if the warehouse is opened.

Equation (7) ensures that the quantity shipped out of a plant cannot exceed the component quantity received.

Consequently, equation (8) ensures that the quantity shipped out of a warehouse cannot exceed the quantity received.

Equations (9) and (10) represent the inventory balance constraint in plant.

Equation (11) is the inventory balance equation for DC. For example, in this equation, inventory of one unit in warehouse  $e$  and at time period  $k$  ( $I'_{ek}$ ) is equal to the inventory at one time unit of previous period ( $I'_{ek-1}$ ) plus amount of one unit shipped from plants to DC  $e$  at time

period  $k$  ( $\sum_{i=1}^n X''_{iek}$ ) minus amount of one unit shipped from DC  $e$  to customer zones (CZs), at time period  $k$  ( $\sum_{j=1}^m X'_{ejk}$ ). Finally, constraint (12) shows positive and binary variables.

Figure 1 illustrates the diagram of this supply chain schematically.

### 3. Solving Methodology

As mentioned earlier, the developed model of this research has MINLP structure. It is proved that simpler model than this model are NP-Hard (Shankar et al., 2013). Therefore, a meta-heuristic approach is proposed to solved the problem. This approach introduces MOBBO algorithm to the SC area. This algorithm is a MOEA based on biogeography optimization (BBO) algorithm as the single objective version. BBO is a population-based optimization algorithm (Simon, 2008). Therefore, it has different similarities with other existing population-based algorithms like genetic algorithm (GA) or particle swarm optimization (PSO). Generally, in this type of algorithm we have a set of individuals that is called population. The individual in this algorithm is called habitat or island. Any feature of the individual (like gene in GA) here is known as a SIV. The fitness value of the individuals here is measured by high suitability index (HSI). However, it has some distinctive differences with the existing population-based algorithms. For instance, in this algorithm, instead

of fitness value migration rates are used to guide the algorithm.

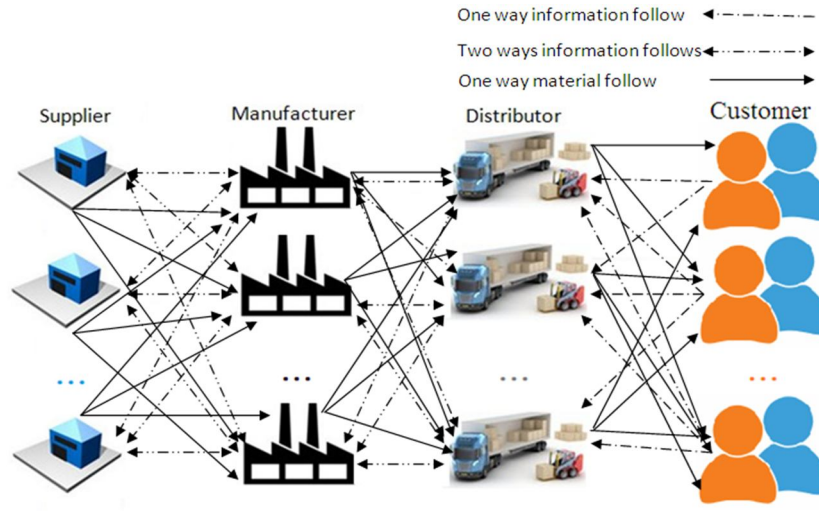


Fig. 1. A four layers diagram of the proposed supply chain

Actually, in biogeography science migration is divided into two different performances of the species, including emigration and immigration. For each of these performances, a specific rate is also considered called emigration rate ( $\mu_j$ ) and immigration rate ( $\lambda_i$ ).

Emigration rate determines how likely a species (emigrating species) shares its features with other species (immigrating species). Likewise, immigration rate determines how likely a species (immigrating species) accepts features from other species (emigrating species). In a relation with HSI, it can be expected that features migrate from high-HSI habitats (emigrating habitat) to low-HSI habitats (immigrating habitat). Therefore, by using migration rates, the aim of the BBO is to guide the optimization process in a way that the HSI is maximized (Rahmati and Zandieh, 2012).

Now, before explaining the operators of this algorithm, since this paper is going to introduce multi-objective version of the BBO, the fundamental principles and definitions of MOAs are introduced.

### 3.1. Multi-objective principles

In a multi-objective problem like  $f(\vec{x}) = [f_1(\vec{x}), \dots, f_m(\vec{x})]$  subject to  $g_i(\vec{x}) \leq 0, i = 1, 2, \dots, c, \vec{x} \in X$ , solution  $\vec{a}$  can dominate solution  $\vec{b}$  ( $\vec{a}, \vec{b} \in X$ ) if following two conditions are held simultaneously:

$$1) f_i(\vec{a}) \leq f_i(\vec{b}), \quad \forall i = 1, 2, \dots, m$$

$$2) \exists i \in \{1, 2, \dots, m\} : f_i(\vec{a}) < f_i(\vec{b})$$

Now, in different iterations of the MOEA, a set of solutions that cannot dominate each other is known as Pareto solutions set or Pareto front. Improving this Pareto front, during different iterations to achieve Pareto optimal

front, is the goal of MOEA. Improvement in multi-objective environment has two signs, including (1) improving the convergence to the optimal front, or (2) improving the diversity of the existing solutions of a Pareto front. Therefore, it is expected that the final obtained Pareto front of an MOEA has an appropriate convergence and diversity. To evaluate these two types of the improvement in a MOEA, different types of measures can be used, some of which will be introduced and used in the next section.

### 3.2. Representation, Initialization and decoding scheme of the habitats

In MOBO, like any other population-based algorithm, the optimization process starts with initializing the initial population. The proposed habitat is composed of three rows by which some constraints are satisfied based on the values of decision variables. The first row represents that facilities such as plant and warehouse are open or close in binary representation. The second row includes the quantity shipped through suppliers, plants, DCs and CZs. The third row indicates the amount of inventory level in plants and warehouses. The general form of a habitat structure is represented in Figure 2.

In the first row of this habitat, potential location of plants and warehouses are coded as binary variables. The amount of shipped components and one unit of product are calculated according to supply capacity of suppliers, plants, DCs and demand of markets at each time period in second row of solution. For example, quantity shipped from a plant to DCs at time period  $k$  is less than plant capacity. In addition, this amount should be less equal than the component amount shipped from suppliers to plant.

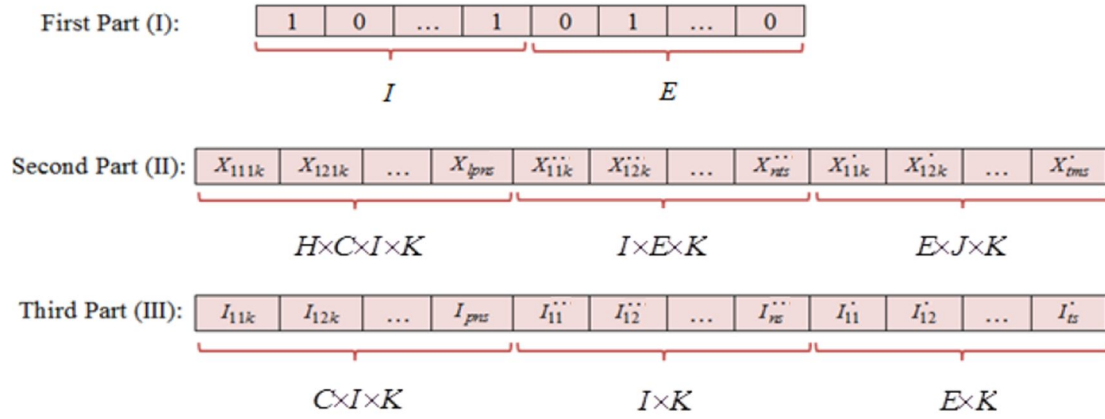


Fig. 2. The habitat structure of the MOBBO

In the third row of habitat, inventory levels in plants and warehouses are decoded considering the second row of the habitat. For example, the amount of one unit that is delivered to DC must be equal to the amount of one unit that leaves from and is stored in this DC. In order to prevent the negative and infeasible solutions in this part, until the inventory quantity is negative, a random number between zero and one is generated. If the value of this number is less than a predetermined value, a plant is selected randomly, and the amount of shipped items is equal to the minimum total of this value with inventory deficit and plant capacity to ship. Otherwise, a customer zone (CZ) is selected randomly and the amount of shipped from DC is equal to maximum difference of this amount with inventory deficit and zero value to generate feasible solutions of both states.

### 3.3. Sorting strategy

This operator is the first main factor that distinguishes MOBBO from its single version. In this part, after decoding the habitats and calculating their HSI's, instead of sorting the population according to the HSI's values, a multi-objective strategy is used. This strategy is proposed by Deb et al. (2000). In this strategy, an operator called *FNDS* is used for assigning ranks to individuals of the population due to domination concept. Then, another operator called *CD* is used to estimate density of solutions which are laid surrounding a particular solution in the same rank. Now, according to these two operators the population is sorted. To do so, in the case of the different ranks (or the individuals from different fronts), the one with lower rank is better. However, if the ranks are the same, the one with higher *CD* is preferred.

### 3.4. Selection strategy and migration operator

This operator is the second main factor that distinguishes MOBBO from its single objective version. In this selection strategy, a binary tournament selection is used

for selecting the emigrating habitat. To do so, after calculating of the *CDs* and *FNDSs*, if a specific habitat needs to be immigrated, two habitats are selected randomly. Then, if they are from different ranks (or different front), the one with lower rank is selected; otherwise, the one with higher *CD* is selected as the emigrating habitat. A scheme of this selection strategy can be seen in Figure 7.

Now, to implement migration operator, it is necessary to calculate  $\lambda_i$  and  $\mu_j$ . The method of this calculation is the third (and the last) main factor that distinguishes MOBBO from its single version. After sorting the population, immigration rate  $\lambda_i$  and emigration rate  $\mu_j$  can be evaluated as Eq.13 and Eq. 14, respectively. In these equations,  $k_i$  represents rank of  $i$ th habitat after sorting all habitats according to multi-objective strategy and  $n$  represents size of the population. Of course, it should be mentioned that,  $k_i$  ranging from 1 to  $n$  and the higher values are of more interest.

$$\lambda_i = I(1 - \frac{k_i}{n}) \quad (13)$$

$$\mu_i = E(\frac{k_i}{n}) \quad (14)$$

The inverse relation between these two rates is shown in Figure 3. In this figure,  $E$  and  $I$ , represents maximum number of the migration rates and are usually set at zero, and  $S_i$  denotes the state of the species amount. According to what was mentioned and this figure, by increasing of the number of species (or going for a more suitable habitat), the immigration rate is decreasing and the emigration rate is increasing. It means that the features in a more suitable habitat with miscellaneous species are more likely to be emigrated rather than to be immigrated. This is the main concept of the migration.

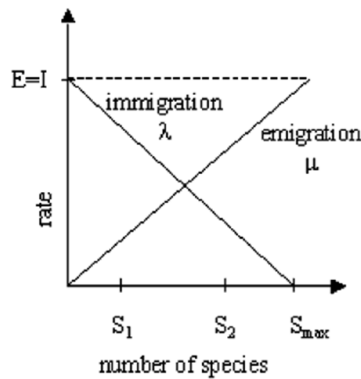


Fig. 3. The variation of migration rates toward number of the species (Simon, 2008)

Figure 3 summarizes the concepts that were proposed in this sub section to do a quite migration process. According to this figure, it is clear that a good habitat (with low  $\lambda_i$ ) is less likely to be immigrated, while as, a poor habitat (with high  $\lambda_i$ ) is so likely to be. Finally, to

perform migration operator, the structure introduced for the single objective BBO is used. The Uniform neighborhood structure is used for conducting the migration as Figure 4. In this figure,  $H$  represents habitat and  $n$  denotes number of the SIVs of each habitat.

To show how Uniform migration works, a scheme is plotted in Figure 5. In this structure, the immigrating habitat accept features from sharing or emigrating habitat for those cells that their random vector numbers (Rand is 0 or 1) is 1.

### 3.5. Mutation operators

In this research, for mutation structure Mask operator is implemented. The scheme of this operator is illustrated in Figure 5.

```

Generate a random number (Rand  $\in [0,1]$ )
If Rand  $\leq \lambda_j$ ,  $H_j$  should be immigrated
  For j=1:n
    Select  $H_i$  through binary tournament selection (as emigrating habitat)
    Perform Uniform operator {SIV( $H_i$ )  $\leftarrow$  ..... SIV( $H_j$ )}
  End
End

```

Fig. 4. The migration operator and the selection strategy

Immigrating Habitat	220	140	75	150	...	0
Emigrating Habitat	280	0	310	0	...	138
Random Vector	1	0	0	1	...	0
Immigrated Habitat	280	140	75	0	...	0

Fig. 5. Uniform migration operator scheme

Habitat	220	140	75	150	...	0
Mask Vector	0.45	0.52	0.29	0.74	...	0.58
Mutated Habitat	195	140	103	150	...	0

Fig. 6. Mask mutation operator scheme

In this figure, a Mask vector is generated randomly with the number from the interval  $[0,1]$ . Now, for those cells of the Mask vector, which have the values less than 0.5, the habitat is mutated. For conducting this mutation, the

mentioned cell is regenerated and its value is assigned randomly.



### 3.6. The MOBBO's optimization process

The evolution process of the MOBBO is illustrated schematically in Figure 7. This process is started by initializing the initial population of the habitat  $R_t$ . Then, BBO's operators, including migration and mutation, are implemented on  $R_t$  to create the new population  $Q_t$ . The blending of  $P_t$  and  $Q_t$  creates  $R_t$ . In this step, habitats of  $R_t$  are sorted in several fronts by means of the explained strategy in sub section 3.3. Now, to create population of the next iteration  $P_{t+1}$ , while the capacity of  $P_{t+1}$  is not exceeded, the fronts are added to  $P_{t+1}$ , according to increasing order of their ranks. But, when without a front,  $P_{t+1}$  has fewer members than population size and with it,  $P_{t+1}$  has more members than population size, the habitats must be selected partially to reach the predetermined population size. In this situation, the habitats of the front are sorted in decreasing order of their CDs, and the habitats of next iteration are chosen from top of the front.

In fact, the most differentiation of the MOBBO with the NSGAII (Deb et al., 2000) is their evolution operators. In other words, the searching heart of the NSGAII is GA, but the searching heart of MOBBO is BBO. In other terms, instead of some simple differences, they guide the multi-objective process similarity. The pseudo code of the MOBBO is also presented in Figure 8. In this figure, the searching heart of the MOBBO, which is BBO, is separated in the middle of the pseudo code.

### 3.7 The MOHS

As mentioned above, MOBBO is compared with the MOHS from the literature (Rahmati, 2013). MOHS is a Pareto-based multi-objective version of the single objective harmony search (HS) algorithm, which is reinforced by the same operators as the ones implemented in this study to get a multi-objective. This algorithm mimics the improvising process of musicians. In HS, three different operators are used, including harmony memory operator, pitching operator, and random operator. The more elaborate description of this algorithm is found in Rahmati et al. (2013). However, it is required to mention that for the pitching operator of the MOHS in this study, a structure just like what explained for the mutation of the MOBBO is used.

## 4. Computational Results

In this section, to assess the developed model and the proposed algorithm, first some test problems are generated. Then, the algorithms are compared based on the whole generated test problems. This comparison is made through utilizing different types of the statistical and non-statistical tests and various explanatory illustrations.

### 4.1. Test problem generating

In the model study, the parameters are generated as Table 1. In this table,  $U(1000,1500)$  represents a random number generated in the interval (1000,1500) from uniform distribution.

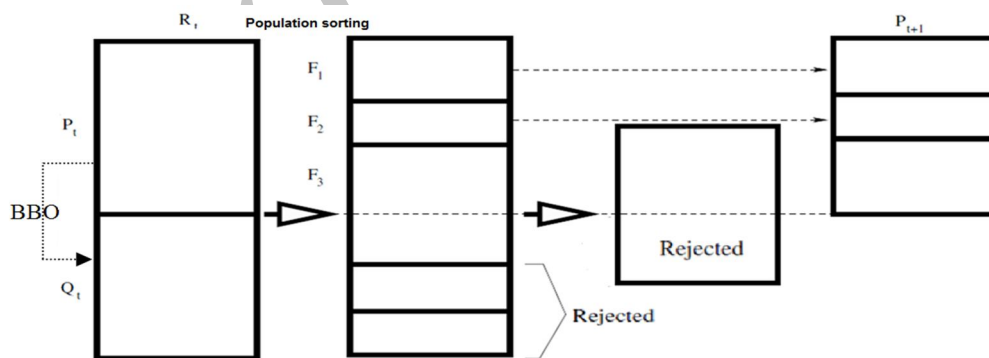


Fig. 7. Evolution process of the MOBBO



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Parameter setting:  $E = 1$ ,  $I = 1$ ,  $m_{max} = 1$ , Pop. size, Num. iteration

Initialization: Generating habitats as size as population size

Population evaluation: Calculate the habitats HIS

Sort the population according multi-objective sorting strategy

Calculate  $\lambda_j$ ,  $\mu_j$ ,  $P_j$ , &  $m_j$  according to habitat's rank after sorting

For  $i=1$ : Num. iteration

$P_t$  = population

For  $j=1$ : Pop. Size

Generate  $Rand \in [0,1]$

If  $Rand \leq \lambda_j$

$H_i(SIV)$  = Choose a habitat via binary tournament selection

Perform migration operator  $H_j(SIV) \leftarrow H_i(SIV)$

Else

The habitat keeps unchanged

End if

Generate  $Rand \in [0,1]$

If  $Rand \leq m_j$

Perform mutation operator

End if

End for

$Q_t$  = new population

$R_t = P_t \cup Q_t$

Perform Non-dominate sorting on  $R_t$

Calculate crowding distance of  $R_t$  and perform sorting on  $R_t$

Create  $P_{t+1}$  as size as population size (population =  $P_{t+1}$ )

Calculate  $\lambda_j$ ,  $\mu_j$ ,  $P_j$ , &  $m_j$  according to new habitat's rank

End for

---

Fig. 8. The pseudo code of MOBBO

Table1  
Input data of the model

Parameter	Distribution function	Parameter	Distribution function
$D_{jk}$	U(1000,1500)	$C_{ejk}$	U(30,50)
$F_i$	U(2000,8000)	$IC_{cik}$	U(5,10)
$F_e$	U(2000,8000)	$IC_{ik}$	U(8,12)
$C_{hcik}$	U(30,50)	$IC_{ek}$	U(10,15)
$C_{iek}$	U(40,70)	$\alpha_c$	U(0,1)

Then, to test the generated model, 14 test problems were created via the information presented in Table 2. In this table, the factors that make a distinctive problem are number of suppliers (l), plant location (n), DCs (T), and

CZs (m). Moreover, 5 raw material types and at 3 time periods are considered.

#### 4.2. Outputs of the algorithms on the generated test problems

In this subsection, after defining some required definitions, the outputs of the algorithms are evaluated. The definitions include the implemented metrics and the tests used.

##### 4.2.1. Multi-objective metric description

Generally, two main features are considered to evaluate the performance of a MOEA.

- 1) The first feature assesses whether the final Pareto front of the algorithm is converged to the Pareto optimal front or not.
- 2) The second feature evaluates the diversification of the set of solutions of the Pareto front.

Table 2  
Generated test problems

Test Problem	<i>L</i>	<i>N</i>	<i>T</i>	<i>M</i>
1	2	2	3	5
2	3	3	4	6
3	3	7	6	7
4	5	7	7	10
5	5	7	9	12
6	7	9	12	15
7	8	10	12	17
8	8	12	15	20
9	10	15	18	25
10	12	18	20	30
11	10	20	22	35
12	10	20	25	40
13	10	20	25	48
14	10	20	25	70

Table 3  
Utilized performance measures

Metric	Metric calculation	Metric brief description
Diversity (Zitzler, 1999) (↑)	$D = \sqrt{\sum_{j=1}^m (\max_{i=1:n} f_j^i - \min_{i=1:n} f_j^i)^2}$	It is used to evaluate the spread of the front.
Spacing (Schott, 1995) (↓)	$S = \sqrt{\frac{1}{n+1} * \sum_{i=1}^n (d_i - \bar{d})^2}$ $d_i = \min_{k \in n \wedge k \neq i}  f_j^i - f_j^k $ $\bar{d} = \sum_{i=1}^n \frac{d_i}{n}$	It is used to measure the uniformity of the solutions within a front.
Mean ideal distance (MID) (Rahmati et al., 2012) (↓)	$MID = \frac{c_i}{NOS} \text{ where } c_i = \sqrt{\sum_{j=1}^m (f_j^i)^2}$	It is used to measure the closeness of solutions in a Pareto front with an ideal point which is usually considered as (0, 0)
Simultaneous metric (SM) (Rahmati et al., 2013) (↓)	$SM = \frac{MID}{D}$	This metric considers the two features of the MOEAs simultaneously.
Number of the non-dominated solutions in final Pareto ( <i>NOS</i> ) (Rahmati et al., 2012) (↑)	-	Measures number of the Pareto solutions.

In the literature of MOEAs, different metrics are suggested to conduct the evaluation of these two features. In this paper, four metrics are implemented that are summarized in Table 3. Following notations are used in this table. Further information about the metrics can be found in the mentioned references in the Table3.

$d_i$  : denotes the space between two neighbor solutions  $n$  : denotes the number of the existing solutions in the Pareto fronts  
 $m$ : denotes the number of the objective functions  
 $f_j^i$  : denotes the  $j^{th}$  objective function of the  $i^{th}$  solution  
The notation ↑ in this table and the rest of the section indicates that higher values are superior, whereas ↓ indicates superiority of smaller values.

## 4.2.2. Multi-objective metric outputs

In this subsection, by calculating the mentioned metrics of the previous subsection, for each metric, the algorithms are compared. To do so, different types of tests and evaluations are implemented.

Initially, the outputs of the metrics are calculated and summarized in Tables 4 and 5. Table 4 presents the obtained outputs for the three metrics MID, Diversity (D),

and Simultaneous metric (SM). This classification of the metrics into two different tables has two main reasons. First, the capacity of the page and the required explicitness of the table restrict us to bring all of the outputs in a single table. Second, since SM is calculated according to the MID and D, these three metrics are summarized in the same table.

Table 4  
Outputs of the algorithms for D, MID, and SM

#	MOBBO			MOHS		
	D ↑	MID ↓	SM ↓	D ↑	MID ↓	SM ↓
1	37089919	116817607	3.15	4863111	140704362	28.9
2	85258423.8	232383888.08	2.72	3931181	285549361	72.64
3	100528185.8	244919108.29	2.44	1732035	270661007.4	156.27
4	237305065.8	506813072.98	2.14	12412308	525206050.67	42.31
5	218198676	418153581.68	1.92	10011440	487599249.5	48.70
6	363509452.8	694231255.66	1.91	8793775	703080279.14	79.95
7	240635984	765000430.47	3.18	17087936	1020700520.12	59.73
8	396141571	833286334.48	2.10	17411574	1023289907.67	58.77
9	401220433	841811371.28	2.1	12496975	1336703362.8	106.96
10	608459886.8	1279128933.90	2.1	27712129	1862619510.43	67.21
11	439634592.8	1086028563.19	2.47	16160360	343138292.67	21.23
12	810747438.2	1130304421.07	1.39	4899421	320653169.4	65.45
13	803906450.4	976073079.17	1.21	90112170	1702157889.67	18.89
14	708410855.2	963197776.08	1.36	27649181	1471504243.33	53.22
Av.	<b>389360495.3</b>	<b>720582102</b>	<b>2.15</b>	18233828	820969086.1	62.87

Table 5  
Outputs of the algorithms for the S and NOS

#	MOBBO		MOHS	
	S ↓	NOS ↑	S ↓	NOS ↑
1	4483419.29	9	562989.33	10
2	2080295.65	12	198540.41	5
3	3011930.48	21	302815.05	5
4	10792034.16	20	2719804.73	6
5	4535812.75	19	600856.05	6
6	12228262.90	21	1109200.02	7
7	9948226.54	15	1875813	8
8	22566905.14	21	3312008.03	6
9	10286308.40	21	882741.21	5
10	17300678.64	21	1662489.19	7
11	19858926.3	19	6428230.16	3
12	48343852.17	22	383265.53	5
13	78422484.49	21	17173029.98	6
14	39535365.57	21	8660361.48	3
Av.	20242464	<b>18.78</b>	<b>3276582</b>	5.86

In these two tables, the last row, named average (Av.), calculates the average of each metric on all test problems for a specific algorithm. In this row, the superior algorithm is bolded. For example, for NOS in Table 5, MOBBO is superior and is bolded.

According to these two tables, following results can be obtained:

1. For NOS, MOBBO is superior. Figure 9 supports this superiority.

This type of figure is also plotted for other metrics in Figures 9-13 to present the manner of the outputs for different algorithms on the test problems more explicitly. In fact, these figures present the same concepts and

evaluations, as presented in Tables 5 and 6, graphically. These figures are called stacked column charts. Each column of these figures compares the obtained value of each metric by the algorithms. The columns are divided into two parts; the upper part is the value obtained from MOBBO whereas the beneath part represents the value obtained from the MOHS. Clearly, in each figure 15 columns that are related to the outputs of the algorithms on the 14 test problems, on that specific metric, exist.

2. For S, MOHS is superior. Figure 9 supports this superiority. It is clear that for most of the columns MOHS part has less value than MOBBO part.

3. For MID, MOBBO is superior. Figure 11 supports this superiority. It is clear that for most of the columns, MOBBO part has less value than MOHS part.
4. For D, MOBBO is superior. Figure 12 supports this superiority. In this metric, for most of the columns MOHS part has less value than MOBBO part
5. For SM, MOBBO is superior. Figure 13 supports this superiority. Again, in this metric, MOBBO has considerable superiority.

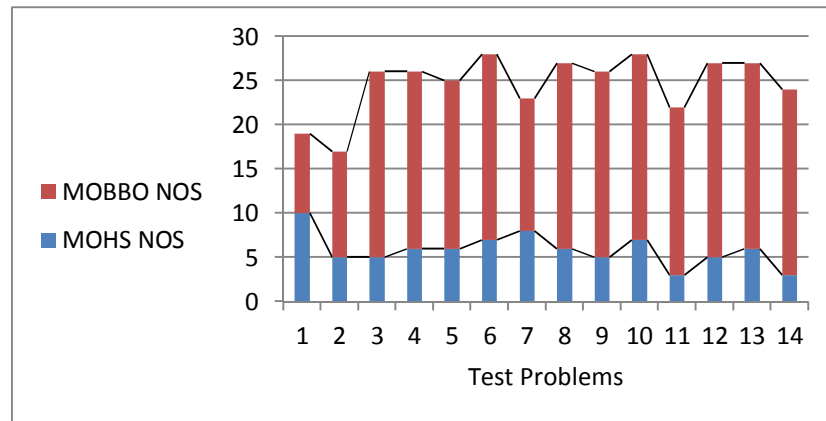


Fig. 9. The outputs of the algorithms for NOS (↑)

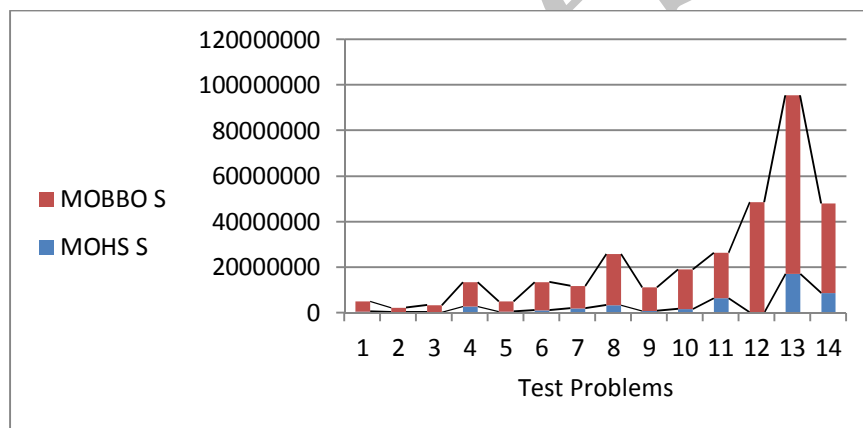


Fig. 10. The outputs of the algorithms for S (↓)

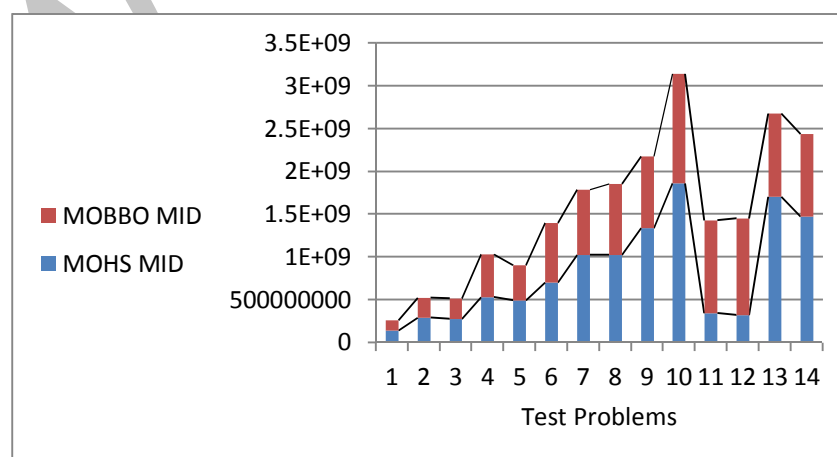
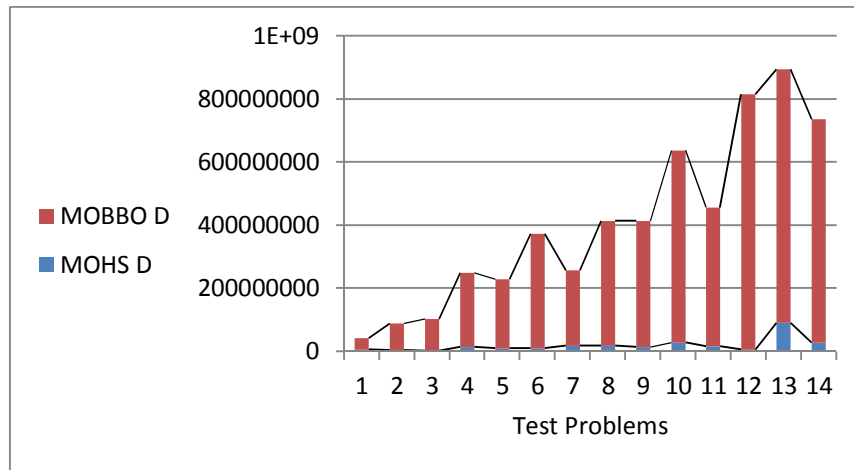
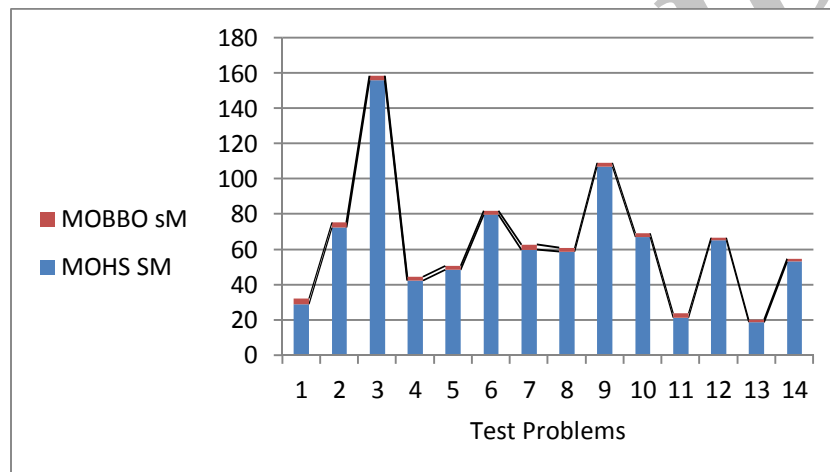


Fig. 11. The outputs of the algorithms for MID (↓)

Fig. 12. The outputs of the algorithms for D ( $\uparrow$ )Fig. 13. The outputs of the algorithms for SM ( $\downarrow$ )

Up to this part, the superiority of the proposed algorithms is recognized on different metrics. However, the superiority recognition requires statistical approval. Hence, two types of statistical tests are implemented which are called 2-sample  $t$  test and Mann–Whitney test. These two tests are alternative parametric and nonparametric tests that are used for comparing two populations of data statistically (Chambari et al., 2012). The outputs of the statistical tests are summarized in Table 6. This table presents the outputs of the two considered types of tests. In this table,  $P$ -values of the tests are reported. Theoretically, if the  $P$ -value is less than our considered significant level, which is 0.05, the null hypothesis ( $H_0$ ) is rejected.

The outputs of this table are consistent with the previous obtained results and confirm them. It means both of the statistical and non-statistical tests approve that for NOS, D, and SM, MOBBO is superior whereas for the S, MOHS wins. For MID, non-statistical tests and

evaluations indicate the superiority of MOBBO. However, statistical tests show that this difference is not significant. Moreover, the accuracy of results is also more reinforced by checking the same outputs of the parametric and non-parametric tests.

For illustrating the results of the statistical tests more explicitly, the box-plots are also plotted for each metric in Figure 14. According to this figure, it is clear that for the cases that null hypothesis is rejected, which algorithm is superior and shows why in MID there is no significant difference graphically.

Finally, to have a better sense of the pattern of Pareto solutions in the final Pareto optimal front of each algorithm and comparing the front pattern of the algorithms, Figure 15 is plotted. In this figure, a sample final Pareto front is plotted for all test problems. Each section of the figure plots the front of the two algorithms for that specific test problem.

Table 6  
The summarized statistical test results

	Mann-Whitney test		t-Test	
	P-value	Result	P-value	Result
NOS	0.00	MOBBO outperforms MOHS	0.00	MOBBO outperforms MOHS
S	0.012	MOHS outperforms MOBBO	0.0003	MOHS outperforms MOBBO
MID	0.59	$H_0$ is not rejected	0.7304	$H_0$ is not rejected
D	0.00	MOBBO outperforms MOHS	0.00	MOBBO outperforms MOHS
SM	0.00	MOBBO outperforms MOHS	0.00	MOBBO outperforms MOHS

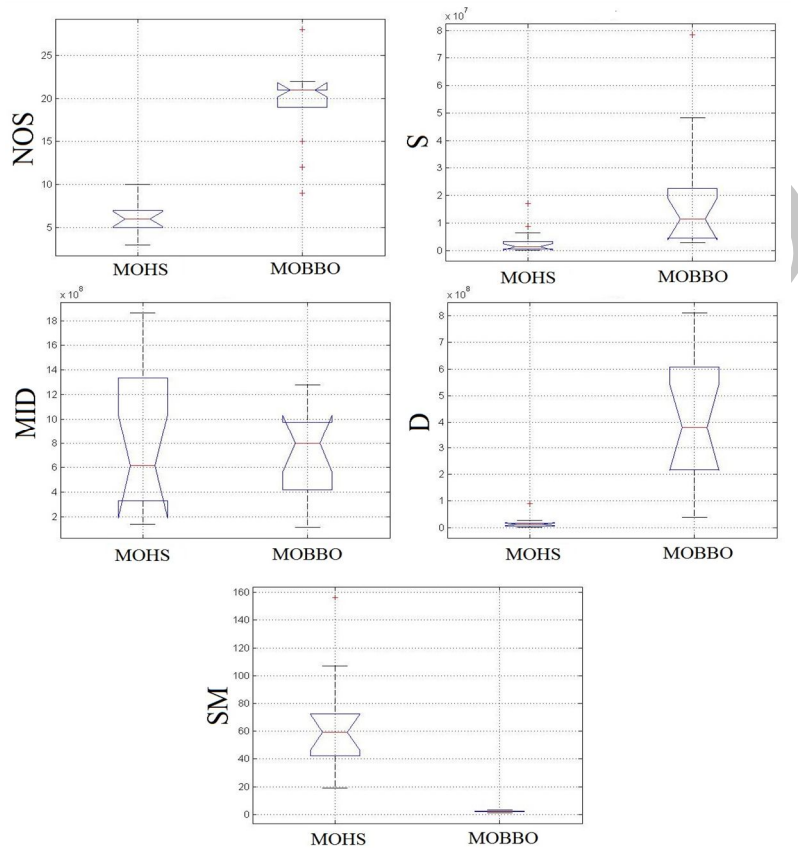


Fig. 14. The box plots of the statistical tests

In this way, it is easy to assess and understand why the mentioned results are obtained in each part of the evaluations. Besides, it shows clearly that how much

MOBBO is superior to reach to the mentioned two main features of the MOEAs in comparison with MOHS.

## 5. Conclusion

This study investigated an integrated four-echelon multi-period multi-objective SC model, which includes inventory and facility location planning simultaneously. The model has two objective functions: the minimization of the total cost with miscellaneous cost terms and minimization of non-fill rate of customer zones demand. Then, since the proposed MINLP model belongs to NP-hard class of the optimization problem, a new MOEA, called MOBBO was developed for solving the problem. MOBBO was validated through a comparison with an algorithm of the literature. This comparison was conducted via both statistical and non-statistical tests on various generated test problems by different multi-objective metrics. Furthermore, different types of statistical and non-statistical figures were implemented. The results of these evaluations prove the high superiority of the MOBBO for solving the model.

Future work can include other practical integrations in terms of queuing theory, facility location, inventory management, strategic planning, data mining, business intelligence, and information technology. Future research can also consider other terms such as queuing considerations, value chain optimization, green designing of the SC for making the model more realistic.

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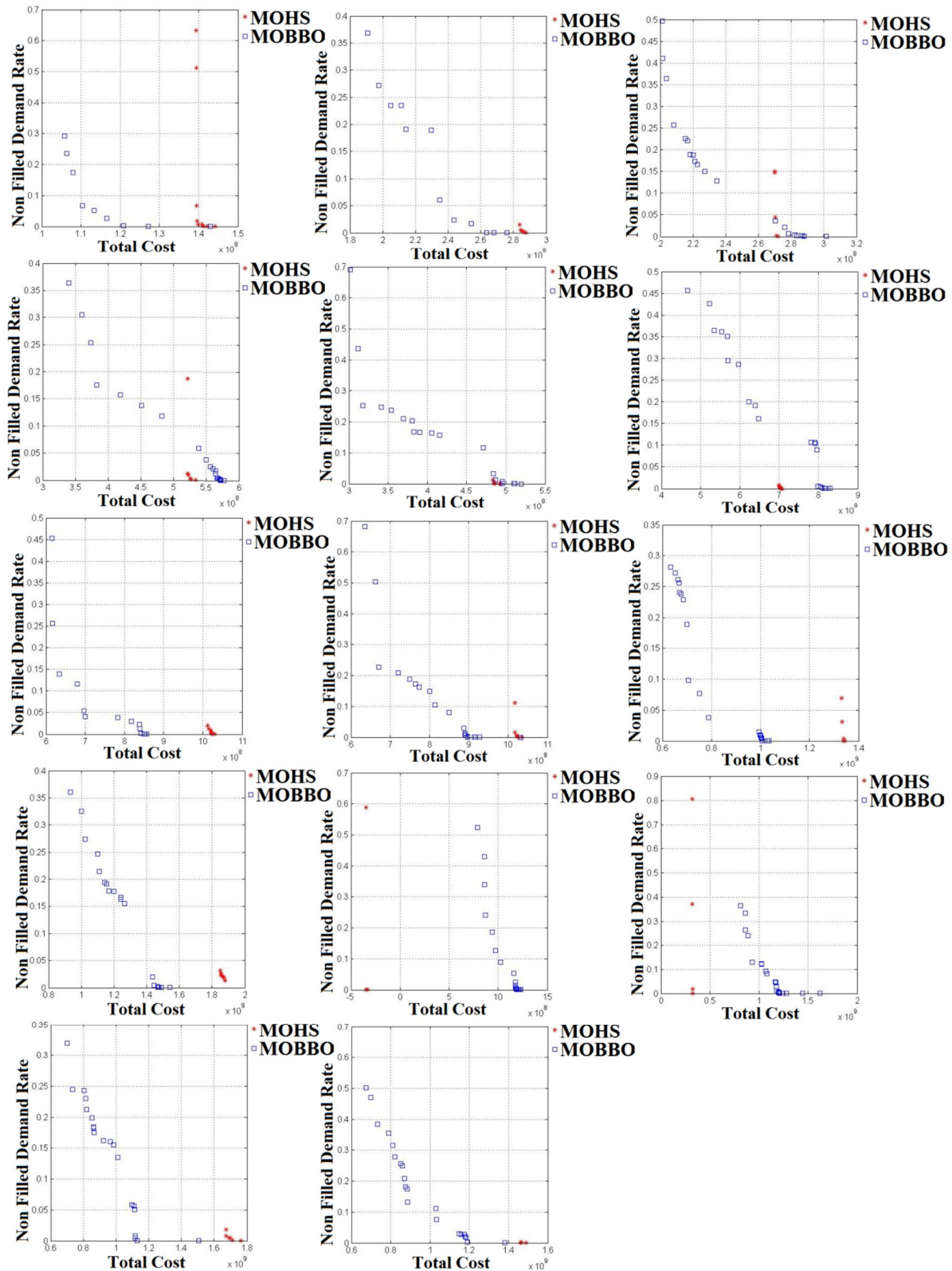


Fig. 15. A sample of the final Pareto front of the algorithms

- with Multi-product and Multi-constraint, *Journal of Optimization in Industrial Engineering* 9, 57-67.
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