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A Benders' Decomposition Approach for Dynamic Cellular Manufacturing System in the Presence of Unreliable Machines

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Abstract

In order to implement the cellular manufacturing system in practice, some essential factors should be taken into account. In this paper, a new mathematical model for cellular manufacturing system considering different production factors including alternative process routings and machine reliability with stochastic arrival and service times in a dynamic environment is proposed. Also because of the complexity of the given problem, a Benders' decomposition approach is applied to solve the problem efficiently. In order to verify the performance of proposed approach, some numerical examples are generated randomly in hypothetical limits and solved by the proposed solution approach. The comparison of the implemented solution algorithm with the conventional mixed integer linear and mixed integer non linear models verifies the efficiency of Benders' decomposition approach especially in terms of computational time.
Keywords: Cellular manufacturing system, Benders' decomposition approach, Machine reliability, Machine utilization factor.

1. Introduction

Cellular Manufacturing System (CMS) is a practical manufacturing strategy which is derived from the Group Technology (GT) concept. CMS can be used in many industrial plants in order to increase both flexibility and productivity of manufacturing systems. Major benefits of CMS implementation include quality and efficiency improvement, material handling cost reduction, work in process inventory reduction, setup cost reduction and so on. Four main decisions should be considered in order to design a CMS.

a) Cell Formation (CF): grouping machines (parts) into manufacturing cells in order to decrease inter-intra cell part trips.
b) Group Layout (GL): an optimal layout of machines and cells within cells and manufacturing space respectively should be determined during this optimization process.
c) Group Scheduling (GS): scheduling process of different parts on corresponding machines based on process routings.
d) Operator Assignment (OA): assigning operators to machines and cells in such a way that a minimum value of operator related costs be obtained.

As pointed out by Dimopoulos et al. (2000), a CMS problem is known as a NP-hard problem; hence, optimization of these decisions has been an important concern in recent years, especially in complicated models in which two or more of these decisions are considered concurrently. Accordingly, many researchers have investigated the CMS design problem from both designing and optimization aspects.

Onwubolu and Mutingi (2001) proposed a mathematical model for CF problem with the aim of cell load variation minimization. Moreover; Jabal-Ameli and Arkat (2008) introduced a pure integer mathematical model to solve the CF problem. In their study, machine reliability and Alternative Process Routings (APR) are considered. The integration of CF problem with Production Planning (PP) and system reconfiguration was investigated by Kioon et al. (2009). Satoglu and Suresh (2009) introduced a goal programming approach to solve a hybrid CMS. Three steps are proposed. First, parts with erratic demands should be determined and selected as special parts. These parts are processed in a functional layout of the manufacturing environment because of their demand's erratic nature. In the second step, a mathematical model is proposed to solve the CF problem. At last, considering the obtained machine-cell solution, the OA problem is solved using a goal programming technique. Mahdavi et al. (2010) integrated a dynamic CF problem with PP and OA. The overall goals were minimization of inventory holding and backorder costs, inter cell part trip cost, machine

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reconfiguration and operator related costs including hiring, firing and salary. Aryanezhad et al. (2009) developed a new mathematical model which solves the CF and OA problems simultaneously. Part routing flexibility, machine flexibility and also promotion of workers from one skill level to another have been considered. Bagheri and Bashiri (2014) developed a new mathematical model which integrates the CF problem with inter-cell layout and RA problems. In their research, it was shown that these problems are inter-related and should be solved concurrently. However, incorporating many production factors in the mathematical programming approaches increases the problem complexity strictly. Finding an optimal solution using conventional approaches like Branch and Bound (B&B) algorithm, especially for large size problems, is almost intractable due to the combinatorial nature of the CMS problem. Hence, in recent years many heuristic and meta-heuristic approaches have been introduced and implemented to solve the problem efficiently.

Krishnan et al. (2012) investigated the inter-intra cell layout problem in a CM environment. Their research includes three basic steps; at first a mathematical model is proposed for grouping the machines into cells in order to inter-cell part total movements be minimized. The second step addresses two heuristic procedures to grouping the parts into cells based on machine grouping solution. At last, a genetic algorithm is implemented to determine the best inter-intra cell layout. Wu et al. (2010) proposed a water flow- like meta-heuristic to solve the CF problem. The proposed approach was verified in terms of both solution effectiveness and efficiency aspects in comparison with other solution methods. Also, a multi objective Particle Swarm Optimization (PSO) algorithm was implemented by Tavakkoli-Moghaddam et al. (2007) to solve a CMS design problem. The model objectives were optimal labor allocation and cell utilization maximization. Kia et al. (2012) proposed a Simulated Annealing (SA) approach to solve the CF and inter-intra GL problems concurrently. Furthermore, Jolai et al. (2011) developed an electromagnetism-like approach to solve the CF problem integrated with inter-intra GL problem. Tavakkoli-Moghaddam et al. (2005) investigated the efficiency of three basic meta-heuristics including Genetic Algorithm (GA), SA and Tabu search (TS) in solving a dynamic CMS problem. Saidi-Mehrabad and Mirnezami-ziabari (2011) developed a new mathematical model for dynamic CMS considering some essential objectives such as cell load variation minimization and also utilization rate of human resource maximization by considering the constant and variable costs of machine, inter - cell material handling costs by assuming the operation sequence and machine relocation costs. Moreover, they applied a multi objective PSO to solve the problem efficiently. Bagheri and Bashiri (2014) developed a hybrid GA and Imperialist Competitive Algorithm (ICA) to solve a CF problem. Saidi-Mehrabad and Mirnezami-ziabari (2011) developed a new mathematical model for dynamic CMS considering some essential objectives such as cell load variation minimization and also utilization rate of human resource maximization by considering the constant and variable costs of machine, inter - cell material handling costs by assuming the operation sequence and machine relocation costs. Moreover, they applied a multi objective PSO to solve the problem efficiently.

An essential drawback of these algorithms is their incapability in finding an optimal solution especially for large scale problems which is a necessary concern in many industrial plants. So introducing and implementing new exact algorithms in which an optimal solution can be obtained in a reasonably computational time, is an interesting subject to be investigated. Also according to the literature, many real world production elements such as machine reliability, process uncertainty and APRs are not incorporated in many studies. Ghotboddini et al (2011) investigated the possibility of applying Benders' decomposition approach on a CMS design problem. However, essential production factors like machine reliability and APRs are not considered in their research. In this paper, first a mathematical model based on these essential factors is proposed. The objectives of presented mathematical model are: inter-intra cell part trips reduction, minimization of machine breakdown and operator related costs including hiring, firing and training cost and maximization of machines utilization factor. Then, because of complexity of the given mathematical model, Benders' decomposition approach, which is an exact solution method, is implemented to solve the problem efficiently.

The rest of the paper is organized as follows: In the next section, a non-linear mathematical model (MINLP), based on the aforementioned objectives, is proposed and in order to obtain a linear (MIP) model, a linearization technique is applied. Benders' decomposition approach which is an effective optimization tool is discussed in section 3. In section 4, the efficiency of proposed model is verified and analyzed by some numerical examples followed by conclusion in section 5.

2. The Mathematical Model

2.1 problem description and crucial assumptions

In many studies existing in the literature, it is assumed that a machine type is always reliable and can process in its production horizon without any interruption and breakdown. However, this assumption is not valid in many real industrial plants. In this paper, it is assumed that machines reliability follows an exponential distribution with a known failure (breakdown) rate. Also breakdown cost for each machine type is known and is based on its repairing cost, install/uninstall cost, etc. The reliability of a machine type during its operating time can be calculated as:

$$R = \exp(-\lambda t)$$

Where λ is its failure rate which is known and fixed during the production horizon. Based on exponential distribution behavior, the Mean Time Between Failures (MTBF) can be calculated as follows:

$$MTBF = \frac{1}{\lambda}$$

Considering these definitions, a machine's total breakdowns over its production periods can be determined by multiplying MTBF by its total processing time. Ultimately, by multiplying this value by the machine breakdown cost, total machine failure cost can be determined.

In many industrial plants, there are Alternative Process Routings (APR) for some part types. It seems that considering this factor is necessary to have a real production system. In this paper, the routings are selected in such a way that machine breakdown cost will be reduced.

In this paper, the part uncertainty concept appears in arrival time of the parts and also machine service time is supposed to follow an exponential distribution with a known rate. Accordingly utilization factor or probability that each machine type is busy can be considered as essential parameters to evaluate the performance of CMS.

Also operator related assumptions are as follows:

1. An operator can be assigned to only one cell. The operator transmission between cells is not allowed.
2. An operator can be assigned to more than one machine based on his/her capability.
3. An operator can be trained to operate with specific machine in a production period by spending a training cost. Training process is performed between periods and it takes zero time.

2.2 Notation

Indices and their relative upper bounds:

I Number of machines
J Number of parts
C Number of machine cells
H Number of production periods
K number of available operators
L_j Number of available routings for part type j
i Index for machines ($i = 1,..,I$)
j Index for parts ($j = 1,..,J$)
c, c' Index for machine cells ($c = 1,..,C$)
h Index for production periods ($h = 1,..,H$)
k Index for operators ($k = 1,..,K$)
l Index for operations required by part j in period h ($d = 1,..,L_j$)

Input parameters:

D_j^h Demand value for part j in period h
t_{ji}^h Processing time of part j on machine i in period h
K_{lj} Number of machines in routing l of part type j

$\{U_{lj}^{(1)}, U_{lj}^{(2)}, U_{lj}^{(3)}, ..., U_{lj}^{(K_{lj})}\}$ Machine index in routing l of part type j

A_1 Inter-cell part trip unit cost
A_2 Intra-cell part trip unit cost
λ'_{jh} Mean arrival rate for part j in period h
μ_i^h Number of parts served by machine i per unit time in production period h
u_c, l_c The upper and lower machine capacity for cell c
u_i, l_i The maximum and minimum number of operators required by machine i
u_k, l_k The maximum and minimum number of machines which can be assigned to operator k
$MTBF_i = \frac{1}{\lambda_i}$ Mean time between failures for machine type i based on its exponential distribution parameter λ_i
BR_i Breakdown cost for machine type i
a_{ki} Training cost for operator k to operate with machine i
HR_k Hiring cost of operator k
FR_k Firing cost of operator k
$LowEm$ Minimum number of operators should be hired in each production period

Decision variables:

$$X_{ic}^h = \begin{cases} 1; & \text{if machine type i is located in cell c in period h;} \\ 0; & \textit{Otherwise} \end{cases}$$

$$Z_{lj}^h = \begin{cases} 1; & \text{If routing l of part type j is selected} \\ & \text{as process plan in period } h \\ 0; & \textit{Otherwise} \end{cases}$$

ρ_i Utilization factor for machine i (or the probability that the machine i is busy)

$$Em_k^t = \begin{cases} 1; & \text{If operator } k \text{ \textit{is to be} employed in period } t \\ 0; & \textit{Otherwise} \end{cases}$$

$$r_{ki}^t = \begin{cases} 1; & \text{If operator } k \text{ is assigned to machine } i \text{ in period } t \\ 0; & \textit{Otherwise} \end{cases}$$

$$S_{kc}^t = \begin{cases} 1; & \text{If operator } k \text{ is assigned to cell } c \text{ in period } t \\ 0; & \textit{Otherwise} \end{cases}$$

$$Y_{ki}^t = \begin{cases} 1; & \text{If operator } k \text{ is unable to operate with machine} \\ & i \text{ in period } t\ (t \geq 2). \\ 0; & \textit{Otherwise} \end{cases}$$

2.3 Objective function and constraints

The MINLP model for the CMS design is presented as follows:

$$\min \quad OF1 =$$

$$\sum_{h=1}^{H}\sum_{j=1}^{J}\sum_{l=1}^{L_j}\sum_{i=1}^{K_{lj}-1}\sum_{c=1}^{C}\sum_{\substack{c'=1\\c'\neq c}}^{C} A_1 Z_{lj}^{h} D_j^{h} X_{(U_{lj}^{i})c}^{h} X_{(U_{lj}^{i+1})c'}^{h} \quad (1\text{-}1)$$

$$+\sum_{h=1}^{H}\sum_{j=1}^{J}\sum_{l=1}^{L_j}\sum_{i=1}^{K_{lj}-1}\sum_{c=1}^{C} A_2 Z_{lj}^{h} D_j^{h} X_{(U_{lj}^{i})c}^{h} X_{(U_{lj}^{i+1})c}^{h} \quad (1\text{-}2)$$

$$-\sum_{h=1}^{H}\sum_{i=1}^{I}\rho_i^{h} \quad (1\text{-}3)$$

$$+\sum_{h=1}^{H}\sum_{j=1}^{J}\sum_{l=1}^{L_j}\sum_{i=1}^{K_{lj}} \frac{Z_{lj}^{h} D_j^{h} t_{j(U_{lj}^{i})}}{MTBF_{U_{lj}^{i}}} BR_i \quad (1\text{-}4)$$

$$+\sum_{h=1}^{H}\sum_{c=1}^{C}\sum_{i=1}^{I}\sum_{k=1}^{K} Em_k^{h} r_{ki}^{h} X_{ic}^{h} S_{kc}^{h} Y_{ki}^{h} a_{ki} \quad (1\text{-}5)$$

$$+\sum_{h=1}^{H}\sum_{k=1}^{K}(Em_k^{h} HR_k + (1-Em_k^{h}) FR_k) \quad (1\text{-}6)$$

Subjected To :

$$\sum_{i=1}^{I} X_{ic}^{h} \geq Low_c \quad \forall c,h \quad (2)$$

$$\sum_{i=1}^{I} X_{ic}^{h} \leq UP_c \quad \forall c,h \quad (3)$$

$$\sum_{c=1}^{C} X_{ic}^{h} = 1 \quad \forall i,h \quad (4)$$

$$\sum_{l=1}^{L_j} Z_{lj}^{h} = 1 \quad \forall j,h \quad (5)$$

$$\rho_i^{h} = \frac{\sum_{j=1}^{J}\sum_{l=1}^{L_j} Z_{lj}^{h}\lambda'_{jh}}{\mu_{(U_{lj}^{i})}^{h}} \quad \forall i,h \quad (6)$$

$$\rho_i^{h} \leq 1 \quad \forall i,h \quad (7)$$

$$\sum_{k=1}^{K} Em_k^{h} \geq lowEm \quad \forall h; \quad (8)$$

$$r_{ki}^{h} \leq Em_k^{h} \quad \forall k,i,h; \quad (9)$$

$$S_{kc}^{h} \leq Em_k^{h} \quad \forall k,c,h; \quad (10)$$

$$\sum_{k=1}^{K} r_{ki}^{h} \leq u_i \quad \forall i,h; \quad (11)$$

$$\sum_{k=1}^{K} r_{ki}^{h} \geq l_i \quad \forall i,h; \quad (12)$$

$$\sum_{i=1}^{I} r_{ki}^{h} \leq Em_k^{h} u_k \quad \forall k,h; \quad (13)$$

$$\sum_{i=1}^{I} r_{ki}^{h} \geq Em_k^{h} l_k \quad \forall k,h; \quad (14)$$

$$\sum_{c=1}^{C} S_{kc}^{h} = Em_k^{h} \quad \forall k,h; \quad (15)$$

$$r_{ki}^{h} \leq \sum_{c=1}^{C} X_{ic}^{h} S_{kc}^{h} \quad \forall k,i,h \quad (16)$$

$$Y_{ki}^{h+1} = (1-r_{ki}^{h}) \times Y_{ki}^{h} \quad \forall h = 1,..H-1, \forall k,i; \quad (17)$$

$$\rho_j^{h} \geq 0 \quad \forall j,h \quad (18)$$

$$r, S, Em, Z, X, Y \in \{0,1\} \quad (19)$$

The first and second terms of objective function minimize inter-intra cell part trips, respectively. Term (1-3) maximizes utilization factor (the probability that each machine is busy). Actually, in order to increase the total production efficiency, a large value of utilization factor is desirable. Term (1-4) minimizes the machine breakdown cost. As stated previously, some part types have different process routings from which one routing should be selected for each part type according to its production characteristics. One such important characteristic stated in this paper is machine reliability which can be appeared as machine breakdown cost. Term (1-5) in the objective function minimizes operators' training cost which is based on the operators' capabilities in working with different machines. At last operators' hiring and firing costs should be minimized by Term (1-6).

Cells lower and upper capacity bounds are restricted by terms (2) and (3), respectively. Constraint (4) is to ensure that each machine is assigned to only one cell. Constraint (5) implies that just a single process routing should be selected for each part type. Constraints (6) computes utilization factor for each machine type. Constraint (7) guarantees that utilization factor for each machine cannot exceed one. As stated by Ghezavati and Saeidi-Mehrabad (2011), maximum utilization factor for a machine type is always one. Constraint (8) ensures that minimum numbers of operators are hired. Constraints (9) and (10) state that an operator can be assigned to a machine and a cell, respectively, only if is hired in that period. Minimum and maximum number of operators required by each machine type is restricted by constraints (11) and (12), respectively. The maximum and minimum numbers of machines dedicated to each operator are restricted by constraints (13) and (14), respectively. Constraint (15) ensures that an operator can be assigned to a machine in the same cell. Actually, this constraint restricts the operator transmission between cells. Constraint (16) guarantees that each hired operator should be assigned to only one cell. Training effect is taken into account by constraint (17) and it states that the trained operator in a period will not need to learn again to work with the same machine in the next periods. Constraints (18) and (19) define variables type.

2.4 Model linearization

The proposed mathematical model is a non-linear model because of the product of binary decision variables in terms (1-1), (1-2), (1-5) and constraints (16) and (17). Non-linear models are basically much harder to be solved than linear models. So the model has been reformulated as a mixed-integer linear programming model by introducing and implementing new variable sets. Also some additional constraints should be added to the model.

Let define new binary variables, $Q^h_{lj(U^i_{lj})(U^{i+1}_{lj})cc'}$, $B_{lj(U^i_{lj})(U^{i+1}_{lj})c}$, XS^h_{kic}, RY^h_{ki} and N^h_{kic} which are computed by following equations:

$$Q^h_{lj(U^i_{lj})(U^{i+1}_{lj})cc'} = Z^h_{lj} X^h_{(U^i_{lj})c} X^h_{(U^{i+1}_{lj})c'}$$

$$B_{lj(U^i_{lj})(U^{i+1}_{lj})c} = Z^h_{lj} X^h_{(U^i_{lj})c} X^h_{(U^{i+1}_{lj})c}$$

$$XS^h_{kic} = X^h_{ic} S^h_{kc}$$

$$RY^h_{ki} = r^h_{ki} Y^h_{ki}$$

$$N^h_{kic} = Em^h_k XS^h_{kic} RY^h_{ki}$$

By considering these equations, the following auxiliary constraints should be added to the proposed model:

$$Q^h_{lj(U^i_{lj})(U^{i+1}_{lj})cc'} \le Z^h_{lj} \tag{20}$$

$$Q^h_{lj(U^i_{lj})(U^{i+1}_{lj})cc'} \le X^h_{(U^i_{lj})c} \tag{21}$$

$$Q^h_{lj(U^i_{lj})(U^{i+1}_{lj})cc'} \le X^h_{(U^{i+1}_{lj})c'} \tag{22}$$

$$Q^h_{lj(U^i_{lj})(U^{i+1}_{lj})cc'} \ge Z^h_{lj} + X^h_{(U^{i+1}_{lj})c'} + X^h_{(U^i_{lj})c} - 2 \tag{23}$$

$$B_{lj(U^i_{lj})(U^{i+1}_{lj})c} \le Z^h_{lj} \tag{24}$$

$$B_{lj(U^i_{lj})(U^{i+1}_{lj})c} \le X^h_{(U^i_{lj})c} \tag{25}$$

$$B_{lj(U^i_{lj})(U^{i+1}_{lj})c} \le X^h_{(U^{i+1}_{lj})c} \tag{26}$$

$$B_{lj(U^i_{lj})(U^{i+1}_{lj})c} \ge Z^h_{lj} + X^h_{(U^i_{lj})c} + X^h_{(U^{i+1}_{lj})c} - 2 \tag{27}$$

$$XS^h_{kic} \le X^h_{ic} \tag{28}$$

$$XS^h_{kic} \le S^h_{kc} \tag{29}$$

$$XS^h_{kic} \ge X^h_{ic} + S^h_{kc} - 1 \tag{30}$$

$$RY^h_{ki} \le r^h_{ki} \tag{31}$$

$$RY^h_{ki} \le Y^h_{ki} \tag{32}$$

$$RY^h_{ki} \ge r^h_{ki} + Y^h_{ki} - 1 \tag{33}$$

$$N^h_{kic} \le Em^h_k \tag{34}$$

$$N^h_{kic} \le XS^h_{kic} \tag{35}$$

$$N^h_{kic} \le RY^h_{ki} \tag{36}$$

$$N^h_{kic} \ge Em^h_k + XS^h_{kic} + RY^h_{ki} - 2 \tag{37}$$

The ultimate linear model can be presented as follows:

$$\min \quad OF2 =$$

$$\sum_{h=1}^{H}\sum_{j=1}^{J}\sum_{l=1}^{L_j}\sum_{i=1}^{K_{lj}-1}\sum_{c=1}^{C}\sum_{\substack{c'=1 \\ c'\ne c}}^{C} A_1 D^h_j Q^h_{lj(U^i_{lj})(U^{i+1}_{lj})cc'} \tag{1-7}$$

$$+\sum_{h=1}^{H}\sum_{j=1}^{J}\sum_{l=1}^{L_j}\sum_{i=1}^{K_{lj}-1}\sum_{c=1}^{C} A_2 D^h_j B_{lj(U^i_{lj})(U^{i+1}_{lj})c} \tag{1-8}$$

$$-\sum_{h=1}^{H}\sum_{i=1}^{I} \rho^h_i \tag{1-3}$$

$$+\sum_{h=1}^{H}\sum_{j=1}^{J}\sum_{l=1}^{L_j}\sum_{i=1}^{K_{lj}} \frac{Z^h_{lj} D^h_j t_{j(U^i_{lj})}}{MTBF_{U^i_{lj}}} BR_i \tag{1-4}$$

$$+\sum_{h=1}^{H}\sum_{c=1}^{C}\sum_{i=1}^{I}\sum_{k=1}^{K} a_{ki} N^h_{kic} \tag{1-9}$$

$$+\sum_{h=1}^{H}\sum_{k=1}^{K} (Em^h_k HR_k + (1 - Em^h_k) FR_k) \tag{1-6}$$

Subjected to:

Unaltered set constraints (2) – (15), (18), new auxiliary constraints (20) – (37) and also:

Set constraint (16) is replaced by:

$$r^h_{ki} \le \sum_{c=1}^{C} XS^h_{ikc} \quad \forall k, i, h \tag{38}$$

Set constraint (17) is replaced by:

$$Y^{h+1}_{ki} = Y^h_{ki} - RZ^h_{ki} \quad \forall h = 1,..H-1, \forall k, i; \tag{39}$$

Set constraint (19) is replaced by:

$$r, S, Em, Z, X, Y, Q, B, N, XS, RY \in \{0,1\} \tag{40}$$

3. Solution approach

3.1 Benders' decomposition solution approach

The combinatorial nature of CMS design problem causes a difficulty in finding an optimum even a feasible solution in a reasonably computational time. As stated earl, many heuristic and meta-heuristic approaches have been introduced in recent years to solve this NP-hard problem. However, finding the optimal solution in many industrial plants is a necessary concern. Since non-linear models are basically much harder to be solved efficiently, we first transformed the proposed model into a Mixed Integer Linear Programming (MILP) model. There are several exact algorithms such as Bender's decomposition and Lagrangian relaxation to solve MILP models efficiently.

Benders' decomposition solution approach, which was originally proposed by Benders (1962), partitions the problem into two smaller problems including master and sub problems, instead of solving the original problem directly. Actually, the computational time difficulty of optimization process depends on a number of variables and also constraints strictly. Hence the fundamental approach is iteration of solving process between master and sub problems. A master problem is a MILP model which includes all integer variables of the original problem and a continuous variable. Also a sub problem or a dual sub problem is an LP in which all integer variables are fixed to their values obtained by the master problem. Accordingly, integer solution is obtained by solving the master problem and then these variables are fixed to the corresponding values in the dual sub problem. Dual sub problem generates an optimal even a feasible cut for the corresponding integer solution which should be added to the master problem. This iterative process continues on tile the upper and lower bounds are as close as it desirable.

A general MILP model has the following form:

$$\min \quad Z = c^T x + d^T y$$

$$S.t.$$

$$Ay \geq b$$

$$Ex + Fy \geq h$$

$$x \geq 0, y \in S$$

By representing the number of continuous variables, integer variables, constraints which contain only integer variables and mixed integer-continuous constraints as *n, p, m* and *q,* respectively, we considered A as a $(m\times p)$, E as a $(q\times n)$, F as a $(q\times p)$, c^T as a $(1\times n)$, d^T as a $(1\times p)$ and h as a $(q\times 1)$ matrices. So the master, sub and dual sub problems can be derived as follows:

Table 1
Sub and dual sub problems parameters

Master Problem (MP):	Sub Problem (SP):	Dual Sub Problem (DSP):
$\min \quad Z_{lower} = c^T x + d^T y$	$\min \quad c^T x$	$\max \quad (h - F\hat{y})^T u$
$S.t.$	$S.t.$	$S.t.$
$Z_{lower} \geq d^T y$	$Ex \geq h - F\hat{y}$	$E^T u \leq c$
$Ay \geq b$	$x \geq 0$	$u \geq 0$
$y \in S$	$\hat{y}$: Solution of the master problem	

During the optimization process, an upper bound solution for the original problem can be obtained as equation (41) in terms of DSP. In this equation $\hat{u}^p$ is an optimal dual solution.

$$\hat{Z}_{upper} = d^T \hat{y} + (h - F\hat{y})^T \hat{u}^p \tag{41}$$

If $\left|\hat{Z}_{upper} - \hat{Z}_{lower}\right| \leq \varepsilon$ then the process should be stopped. The corresponding solution is optimal. Otherwise an optimality cut as $Z_{lower} \geq d^T y + (h - Fy)^T \hat{u}^p$ should be added to the MP. Moreover; during the optimization process if an unbounded solution is obtained for DSP (an infeasible solution for SP), an infeasibility cut as $(h - Fy)^T \hat{u}^r \leq 0$ will be added to MP in next iteration.

The flowchart for the Benders' decomposition is depicted in Figure 1.

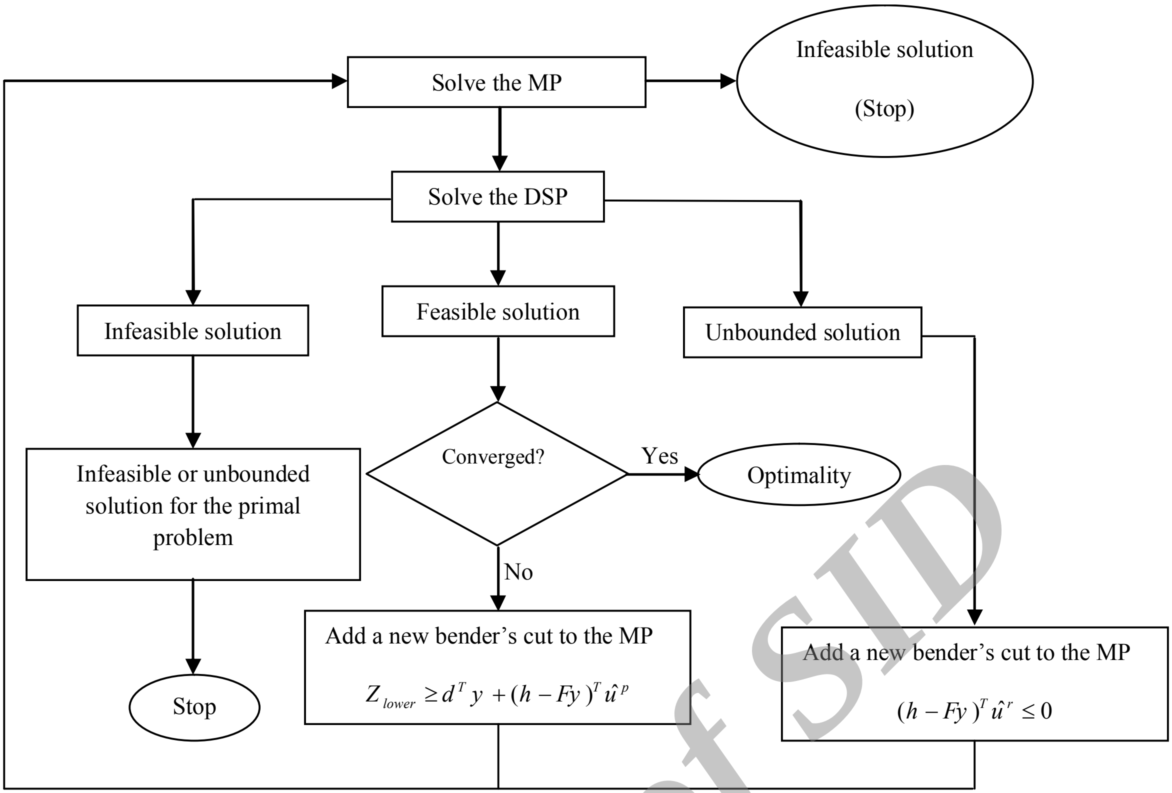

Fig. 1. Flow chart of benders' decomposition algorithm

3.2 Driving MP and DSP for the proposed MILP

According to the previous section, we apply the benders' decomposition approach to the proposed MILP model. So the master and dual sub problems are derived as follows.

3.2.1 DSP

In the following formulation, variables which are fixed in their values are shown by hats (^) symbols. We define a real variable set θ^1 and θ^2 for the DSP driving. Actually, DSP is obtained after driving the SP. Also the following formulation computes Equation (1) directly.

$$\max \quad OF3 = \hat{Z}_{upper} \tag{1-10}$$

$$\sum_{h=1}^{H}\sum_{i=1}^{I}\theta_{ih}^{1}\frac{\sum_{j=1}^{J}\sum_{l=1}^{L_j}\hat{Z}_{lj}^{h}\lambda'_{jh}}{\mu_{(U_{lj}^{i})}^{h}} \tag{1-11}$$

$$-\sum_{h=1}^{H}\sum_{i}^{I}\theta_{ih}^{2} \tag{1-12}$$

$$\sum_{h=1}^{H}\sum_{j=1}^{J}\sum_{l=1}^{L_j}\sum_{i=1}^{K_{lj-1}}\sum_{c=1}^{C}\sum_{\substack{c'=1\\c'\neq c}}^{C}A_1 D_j^h \hat{Q}_{lj(U_{lj}^{i})(U_{lj}^{i+1})cc'}^{h} \tag{1-13}$$

$$+\sum_{h=1}^{H}\sum_{j=1}^{J}\sum_{l=1}^{L_j}\sum_{i=1}^{K_{lj-1}}\sum_{c=1}^{C}A_2 D_j^h \hat{B}_{lj(U_{lj}^{i})(U_{lj}^{i+1})c} \tag{1-14}$$

$$+\sum_{h=1}^{H}\sum_{j=1}^{J}\sum_{l=1}^{L_j}\sum_{i=1}^{K_{lj}}\frac{\hat{Z}_{lj}^{h}D_j^h t_{j(U_{lj}^{i})}}{MTBF_{U_{lj}^{i}}}BR_i \tag{1-15}$$

$$+\sum_{h=1}^{H}\sum_{c=1}^{C}\sum_{i=1}^{I}\sum_{k=1}^{K}a_{ki}\hat{N}_{kic}^{h} \tag{1-16}$$

$$+\sum_{h=1}^{H}\sum_{k=1}^{K}(E\hat{m}_k^h HR_k + (1-E\hat{m}_k^h)FR_k \tag{42}$$

S.t.

$\theta_{ih}^{1}+\theta_{ih}^{2}\geq 1 \quad \forall i,h$

3.2.2 MP

The master problem for the proposed mathematical model includes optimization of binary variables and finding a lower bound for the original problem.

$\min \quad OF4 = \hat{Z}_{lower}$

Subjected to:

Constraints (2) – (5), (8)-(15) and (20) – (37)

The optimality cut as :

$$\hat{Z}_{lower} \geq \sum_{h=1}^{H}\sum_{i=1}^{I}\hat{\theta}_{ih}^{1}\frac{\sum_{j=1}^{J}\sum_{l=1}^{L_j} Z_{lj}^{h}\lambda'_{jh}}{\mu_{(U_{lj}^{i})}^{h}} - \sum_{h=1}^{H}\sum_{i}^{I}\hat{\theta}_{ih}^{2}$$

$$+\sum_{h=1}^{H}\sum_{j=1}^{J}\sum_{l=1}^{L_j}\sum_{i=1}^{K_{lj}-1}\sum_{c=1}^{C}\sum_{\substack{c'=1\\c'\neq c}}^{C} A_1 D_j^h Q_{lj(U_{lj}^{i})(U_{lj}^{i+1})cc'}^{h}$$

$$+\sum_{h=1}^{H}\sum_{j=1}^{J}\sum_{l=1}^{L_j}\sum_{i=1}^{K_{lj}-1}\sum_{c=1}^{C} A_2 D_j^h B_{lj(U_{lj}^{i})(U_{lj}^{i+1})c} \tag{43}$$

$$+\sum_{h=1}^{H}\sum_{j=1}^{J}\sum_{l=1}^{L_j}\sum_{i=1}^{K_{lj}}\frac{Z_{lj}^{h} D_j^h t_{j(U_{lj}^{i})}}{MTBF_{U_{lj}^{i}}} BR_i$$

$$+\sum_{h=1}^{H}\sum_{c=1}^{C}\sum_{i=1}^{I}\sum_{k=1}^{K} a_{ki} N_{kic}^{h}$$

$$+\sum_{h=1}^{H}\sum_{k=1}^{K}(Em_k^h HR_k + (1-Em_k^h)FR_k$$

And a feasibility cut as:

$$\sum_{h=1}^{H}\sum_{i=1}^{I}\hat{\theta}_{ih}^{1}\left(\frac{\sum_{j=1}^{J}\sum_{l=1}^{L_j} Z_{lj}^{h}\lambda'_{jh}}{\mu_{(U_{lj}^{i})}^{h}}\right) - \sum_{h=1}^{H}\sum_{i}^{I}\hat{\theta}_{ih}^{2} \leq 0 \tag{44}$$

4. Numerical Examples

In order to evaluate the performance of the proposed solution approach and also applicability of the proposed mathematical model, some numerical examples are generated randomly in hypothetical limits and is solved using Gams 23.5-Cplex on a Core i5 PC with 1 GB RAM. Examples information and also comparison between the proposed Benders' decomposition method and conventional solution approaches are reported in table 2. Considering the last example which is a large size one, the MILP model could not find an optimal solution after 1000 seconds. Another essential point is that implemented Benders' decomposition method has high efficiency in comparison to the conventional MIP model. Computational time of Benders' decomposition approach has lower slope by increasing the problem size in comparison to the MIP and MINLP models.

To analyze the proposed mathematical model in more details, input information of P5 are reported in tables 3-5. Minimum and maximum numbers of operators required by each machine type are 1 and 4, respectively ($L_i = 1, U_i = 4$). Also the minimum and maximum machine capacities for each cell are 1 and 3, respectively ($L_c = 1, U_c = 3$).The maximum and minimum numbers of machines which can be assigned to an operator are 2 and 4, respectively ($L_k = 2, U_k = 4$).

Table 2
The input information of different examples

Example number	Number of parts	Number of machines	Number of cells	Number of operators	Total available routes	Objective function- Computational time: Benders' decomposition approach (OF3-OF4)	Objective function- Computational time(s): Conventional MIP model (OF2)	Objective function- Computational time(s): MINLP model (OF1)
P1	4	4	2	5	8	85500-0.1	85500-0.325	85500-10
P2	5	5	3	8	15	6857-0.108	6857-0.99	6857-1208.3
P3	5	6	3	8	15	9536-1.3	9536-1.6	9536-54.4
P4	6	6	3	10	17	9377-1.7	9377-1.8	9377-363.3
P5	8	7	3	10	20	14576-4	14576-12.88	16771-16*60

Table 3
The input information of part-machine matrix (period1)-P5

Parts	**Routes**	**Process sequence**	**Processing time of each operation(*t*)**	**Demand** (D_j^1)	λ'
	Rout1	1-3-4-2-7-6	0.6, 0.3, 0.5, 0.7, 0.5, 0.4		
1	Rout2	1-3-5	0.6, 0.5, 0.2	90	2
	Rout 3	1-3-4-2-5-6	0.6,0.3,0.5,0.7, 0.2,0.4		
	Rout 1	5-6	0.1, 0.3		
2	Rout 2	5-7-1	0.1, 0.5, 0.1	100	6
	Rout 3	4-2-6	0.05,0.1,0.3		
	Rout 1	6-5-2-7-4-3	0.6, 0.1, 0.2, 0.2, 0.6, 0.6		
3	Rout 2	6-5-2-7-4-2	0.6, 0.1, 0.2, 0.2, 0.6,0.2	20	5
	Rout 3	6-5-4	0.6, 0.1, 0.2, 0.6		
4	Rout 1	2-4-5	0.7, 0.3, 0.6	100	4
	Rout 2	1-3-6	0.2, 0.6, 0.2		
	Rout 1	1-2-7-5-3	0.5, 0.7, 0.5, 0.5, 0.5		
5	Rout 2	1-3-5	0.5,0.5, 0.5	70	5
	Rout 3	1-2-4	0.5,0.7, 0.3		
	Rout 4	1-3-2-1	0.5, 0.5, 0.7, 0.5		
6	Rout 1	1-2-5	0.1, 0.4, 0.3	10	5
	Rout 2	1-6-7	0.1,0.5,0.1		
7	Rout 1	7-6-1-4-7-5-3	0.1, 0.4, 0.2, 0.5, 0.1 0.7, 0.6	30	3
8	Rout 1	5-2-7-6-4-3-7	0.1, 0.1, 0.4, 0.5, 0.1, 0.7,0.4	60	4
	Rout 2	1-2-1-4-5	0.3, 0.1, 0.2, 0.1,0.1		

Table 4
The input information of part-machine matrix (period2)-P5

Parts	Routes	Process sequence	Processing time of each operation(t)	Demand (D_j^1)	λ'
1	Rout1	2-5	0.7, .03	20	2
	Rout2	2-4-1	0.7, 0.1, 0.2		
	Rout 3	2-6-3	0.7, 0.2, 0.3		
2	Rout 1	7-6	0.5, 0.4	50	2
	Rout 2	4-3-2	0.4, 0.2, 0.3		
	Rout 3	4-2-6	0.4,0.3,0.4		
3	Rout 1	6-7	0.1, 0.2	100	2
	Rout 2	7-1-7	0.2,0.05,0.1		
4	Rout 1	2-4-5	0.7, 0.3, 0.6	80	2
	Rout 2	1-3-6	0.2, 0.6, 0.2		
5	Rout 1	5-7-3-7	0.1, 0.5, 0.3, 0.5	100	3
	Rout 2	1-2	0.8,0.1		
	Rout 3	4-1-4	0.1,0.8,0.1		
	Rout 4	5-1-3	0.1, 0.8, 0.3		
6	Rout 1	5-3-7-1	0.6, 0.2, 0.6, 0.2	70	3
	Rout 2	5-3-7-5	0.6, 0.2, 0.6, 0.6		
7	Rout 1	6-3-5-2-7-1	0.3, 0.6, 0.7, 0.6, 0.4, 0.4	10	3
8	Rout 1	5-1-2-7-3-7	0.1, 0.5, 0.2, 0.1, 0.4, 0.1	90	3
	Rout 2	5-1-2-6	0.1,0.5,0.2,0.5		

Table 5
Operator related information-P5

		Hiring(firing) cost	**Machine** Capabilities (1-Z) - training cost (a) of operators						
			1	**2**	**3**	**4**	**5**	**6**	**7**
Operators	**1**	100(80)	1-0	0-7	0-9	0-5	0-6	1-0	0-6
	2	100(80)	0-8	1-0	0-9	1-0	1-0	0-4	0-5
	3	80(60)	1-0	1-0	0-9	0-4	0-6	0-4	0-6
	4	40(20)	0-7	0-6	0-8	0-3	0-6	0-3	0-4
	5	30(10)	1-0	0-7	0-8	0-5	0-6	0-4	0-6
	6	40(20)	0-6	1-0	0-9	0-4	0-6	0-3	1-0
	7	50(20)	1-0	1-0	0-7	0-5	0-6	0-4	0-5
	8	50(20)	0-6	0-5	0-9	0-5	0-6	1-0	0-4
	9	50(20)	1-0	1-0	0-9	0-4	0-6	0-4	0-6
	10	50(25)	0-10	1-0	0-7	0-3	0-6	0-3	0-4
$\mu_i^1 - \mu_i^2$	-	-	20-20	15-15	20-20	10-10	16-16	16-16	15-15
$MTBF_i$	-	-	1.5	1.03	0.83	0.81	0.99	0.98	1.04
BR_i	-	-	9	12	20	25	14	17	20

Table 6
Optimal routings obtained for different part types-P5

Part number	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Optimal Routing in period 1	2	1	2	2	1	1	1	2
Optimal Routing in period 1	1	1	1	2	2	1	1	2

Table 7
Optimal cell formation and machine busy time solution-P5

Machine type	Machine busy time factor Period 1	Cell in which machine is located Period 1	Machine busy time factor Period 2	Cell in which machine is located Period 2
1	0.7	2	0.5	3
2	0.667	2	0.533	2
3	0.4	1	0.3	3
4	0.6	3	0	1
5	0.875	2	0.5	1
6	0.5	1	0.625	1
7	0.533	3	0.533	1

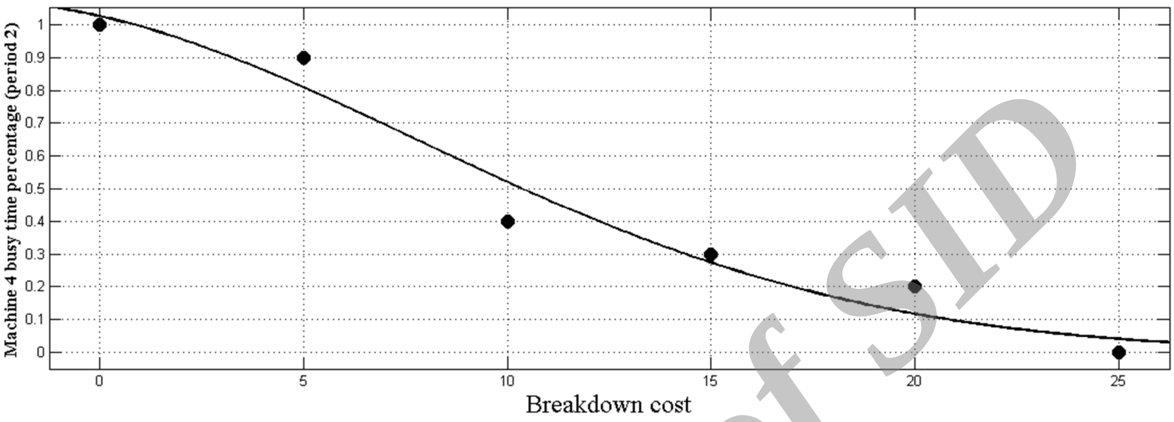

Fig. 2. Effect of Breakdown cost on machine 4 utilization factor in period 2

Optimal cell formation and routing selection solutions obtained for this example are reported in tables 6-7. According to these tables, it is obvious that machine 4 is not applied in period 2. So its busy time is zero. Considering its operating costs, it was a predictable result. Actually machine 4 has minimum service rate irrespective of other production elements such as processing time and high operator training costs. Figure 2 illustrates different values of utilization factor for machine 4 in period 2 versus different breakdown cost values. It can be inferred from this figure that breakdown cost has a significant impact on routing selection process which in turn affects the machine utilization factor.

5. Conclusion

A new mathematical model for considering both cell formation and operator assignment problems was proposed in this paper. Some essential manufacturing elements such as machine reliability, alternative process routings and machine utilization factor were taken into account. Since the proposed model was a MINLP model, in order to find an optimal solution in a reasonably computational time, a linearization method was applied to decrease its complexity. As the proposed model is known as a NP-hard optimization problem, a Benders' decomposition method was applied to solve the model efficiently. In order to examine the efficiency of the applied optimization method, some numerical examples were generated randomly and solved using the Gams optimization package. Based on the computational time view point, Benders' decomposition approach has lower slope by increasing the problem size in comparison to the MIP and MINLP models. Incorporating more manufacturing concepts such as production planning and group layout in the provided framework is suggested as future studies. Moreover; applying the modified Benders' decomposition method on nonlinear models and comparing its performance with the original Benders' decomposition approach may be investigated in the future.

References

Ameli, M. S. and Arkat, J. (2008). Cell formation with alternative process routings and machine reliability consideration. International Journal of Advanced Manufactirong Technology, 35, 761–768.

Aryanezhad, M. B., Deljoo, V. and Mirzapour Al-e-hashem, S. M. (2009). Dynamic cell formation and the worker assignment problem: a new model. International Journal of Advanced Manufacturing Technology, 41, 329–342.

Bagheri, M., Bashiri, M. (2014). A new mathematical model towards the integration of cell formation with operator assignment and inter-cell layout problems in a dynamic environment. Applied Mathematical Modeling, 38, 1237-1254.

Bagheri, M., Bashiri, M. (2014). A hybrid Genetic and Imperialist Competitive Algorithms (GICA) approach to dynamic Cellular Manufacturing System . Proceedings of the

Institution of Mechanical Engineers, Part B: Journal of Engineering Manufacture, 228(3), 2014, 458-470.

Benders, J. F. (1962). Partitioning procedures for solving mixed-variables programming problems. Numerische Mathematik, 4, 238–252.

Dimopoulos, C., Zalzala, A. (2000). Recent developments in evolutionary computations for manufacturing optimization: problems, solutions, and comparisons. IEEE Transactions on Evolutionary Computation, 4, 93–113.

Ghezavati, V. R., and Saidi-Mehrabad, M. (2011). An efficient hybrid self-learning method for stochastic cellular manufacturing problem: A queuing-based analysis. Expert Systems with Applications, 38, 1326-1335.

Ghotboddini, M., Rabbani, M., and Rahimian, H. (2011). A comprehensive dynamic cell formation design: Benders' decomposition approach. Expert Systems with Applications, 38, 2478–2488.

Jolai, F., Tavakkoli-mogaddam, R., Golmohammadi, A. and Javadi, B. (2011). An Electromagnetism-like algorithm for cell formation and layout problem. Expert System with Application, 39, 2172-2182.

Kia, R., Baboli, A., Javadian, N., Tavakkoli-Moghaddam, R., Kazemi, M. and Khorrami, J. (2012). Solving a group layout design model of a dynamic cellula rmanufacturing system with alternative process routings, lot splitting and flexible reconfiguration by simulated annealing. Computers & Operations Research, 39, 2642-2658.

kioon, S. A., Bulgak, A. A. and Bektas, T. (2009). Integrated cellular manufacturing systems design with production planning and dynamic system reconfiguration. European Journal of Operational Research, 192, 414–428.

Krishnan, K. k., Mirzaei, S., Venkatasamy, V., and Pillai, V. M. (2012). A comprehensive approach to facility layout design and cell formation. International Journal of Advanced Manufacturing Technology , 59, 737-753.

Mahdavi, I., Aalaei, A., Paydar, M. M. and Solimanpur, M. (2010). Designing a mathematical model for dynamic cellular manufacturing systems considering production planning and worker assignment. Computers and Mathematics with Applications, 60, 1014-1025.

Onwubolu, G.C., and Mutingi, M. (2001). A genetic algorithm approach to cellular manufacturing Systems. Computers & industrial engineering, 39, 125-144.

Purcheck, G.F.K. (1974). A mathematical classification as a basis for the design of group technology production cells. Production Engineer, 54, 35–48.

Saidi-Mehrabad, M., and Mirnezami-ziabari, S. M. (2011). Developing a Multi-objective Mathematical Model for Dynamic Cellular Manufacturing Systems. Journal of Optimization in Industrial Engineering , 7, 1-9.

Satuglu, S. I. and Suresh, N. C. (2009). A goal-programming approach for design of hybrid cellular manufacturing systems in dual resource constrainted environment. Computers & Industrial Engineering, 56, 560-575.

Tavakkoli-Moghaddam, R., Aryanezhad, M. B., Safaei, N. and Azaron, A. (2005). Solving a dynamic cell formation problem using meta-heuristics. Applied Mathematics and Computation, 170, 761–780.

Tavakkoli-mogaddam, R., Javadian, N., Javadi, B. and Safaei, N. (2007). Design of a facility layout problem in cellular manufacturing systems with stochastic demands. Applied Mathematical Computions, 184, 721-728.

Wu, H., Chung, S-H. Chang, C-C. (2010). A water flow-like algorithm for manufacturing cell formation problems. European Journal of Operations research, 205, 346-360.