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Reliability Modelling of the Redundancy Allocation Problem in the Series-parallel Systems and Determining the System Optimal Parameters

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Abstract

Considering the increasingly high attention to quality, promoting the reliability of products during designing process has gained significant importance. In this study, we consider one of the current models of the reliability science and propose a non-linear programming model for redundancy allocation in the series-parallel systems according to the redundancy strategy and considering the assumption that the failure rate depends on the number of the active elements. The purpose of this model is to maximize the reliability of the system. Internal connection costs, which are the most common costs in electronic systems, are used in this model in order to reach the real-world conditions. To get the results from this model, we used meta-heuristic algorithms such as genetic algorithm and simulation annealing after optimizing their operators' rates by using response surface methodology.

Keywords: Reliability, Redundancy allocation problem, Genetic algorithm, simulated annealing, Response surface methodology.

1. Introduction

Industries providing services for human beings use expensive and complicated systems; this makes them vulnerable because a minor failure or problem may have great impacts on customer services and the cost of the industry. Industries like Power generation, aerospace industry, petrochemical industry, military, and automotive industries, etc. are examples of complicated industries (kuo et al. (2001) & Elegbede et al. (2003)).

As humans' ordinary life tends to rely on advanced technology , e.g. GPS, the Internet, and sensor networks, the reliability of either a hardware system or a software service turns into one of the most critical concerns in a system design. Generally, system reliability can be enhanced either by incremental improvements of the component reliability or by the provision of the redundancy components in parallel; both methods result in an increase in system costs. Redundancy allocation problem is one of the important and applicable problems in the reliability science (Ebling et al. (1997) &Arulmozhi et al. (2002) &Prasad et al. (1999).

Redundancy Allocation Problem (RAP) is a mathematical model for evaluating series-parallel system reliability under some given constraints such as cost and weight. In other words, it is a combinatorial optimization problem, which focuses on determining an optimal assignment of the components in a system design. This problem is broadly used in a variety of practical circumstances, especially in the field of electrical engineering and industrial engineering [6-8]. The practical application of RAP is usually involved in circuit design, power plant components replacement, consumer-electronics industry, etc. Engineers put redundancy at some critical parts to ensure the success of launching. Due to the diverse combination of components, RAP known to be NP-hard, proved in (Fyffe et al. (1968) & Chern et al. (1992)).

1.1. Literature review

In recent years, researchers developed reliability models, especially in series-parallel and redundancy problems. The origin and development of the redundancy problem in series-parallel problems is presented in Table 1.

The literature and concepts related to the redundancy allocation problem in series-parallel systems are explained briefly in this section. The proposed model and the coding procedure will be discussed in the next section. The main concept of the response surface methodology and the results of this methodology for optimizing the.

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operators of the genetic algorithm and simulated annealing algorithm for solving the proposed model will be reviewed in section 3. Test results of the genetic algorithm and simulated annealing algorithm are presented in section 4. Section 5 concludes the study and provides suggestions for further researches

- Elements of system and subsystem have intact or failed state.
- For redundant elements, internal connection cost will be used
- Costs and weights of the elements are definite
- The reliability of the elements is deterministic and definite
- None of the elements use preventive maintenance strategy

Table 1
Classification of the Studies on the Issue of redundancy allocation problem

The development of the single-objective model for the Redundancy Allocation Problem (RAP)						
Authors	Decision/Subject	solving methodology			Year	Reference No.
		Exact	Approximate	Heuristic/ Meta-heuristic		
Fyffe et al.	RAP and a Computational Algorithm(Dynamic Programing)	•			1968	[9]
Misra and Sharma	A Geometric Programming Formulation for RAP		•		1973	[11]
Nakagawa and Miyazaki	Surrogate Constraints Algorithm for RAP	•			1981	[8]
Bulfin and Liu	Optimal Allocation for RAP in Large Systems	•		•	1985	[12]
Mohan and Shanker	RAP and Using Random Search Technique for optimizing		•		1987	[13]
Ida Gen and Yokota	Optimization RAP using genetic algorithm(GA)			•	1994	[14]
Ida Gen and Yokota	Optimization RAP with several failure modes using GA			•	1995	[15]
Coit and Smith	Reliability Optimization of Series-Parallel Systems Using GA	•			1996	[16-18]
Hsieh	A Two-Phase Linear Programming Approach for RAP			•	2002	[19]
Kim, Bae and Park	Simulated Annealing Algorithm for Redundancy Optimization			•	2004	[20]
Liang and Smith	An Ant Colony Optimization Algorithm for RAP			•	2004	[21]
Chen and You	Immune Algorithms-Based Approach for RAP			•	2005	[22]
Liang and Wu	A Variable Neighborhood Descent Algorithm(VNA) for RAP			•	2005	[23]
Liang and Chen	A VNA for RAP in series-parallel systems			•	2007	[24]
Studies that Looked At the Problem as Multi Objective						
Yun and Kim	Multi-Level Redundancy Optimization in Series Systems			•	2004	[25]
Coit and Konak	Multiple Weighted Objectives Heuristic for the RAP			•	2006	[26]
Wang et al	Multi-objective Approach to RAP in Parallel- series Systems			•	2009	[27]
Khalili and Amiri	Solving binary-state multi-objective reliability for RAP			•	2012	[28]
Chambari et al	A bi-objective model to optimize reliability and cost of system			•	2012	[29]

2. Problem Formulation

In this section, initially, model assumptions are briefly described. Then indexes, parameters and decision variables are discussed.

2.1. Model assumptions

- The system consists of some series subsystem in which redundant elements are parallel
- Only one kind of element can be assigned to each subsystem
- Each subsystem can only choose on strategy between active or cold standby in redundancy allocation
- For each redundancy strategy, the failure rate of elements depends on active elements
- The failure of the elements is independent
- Inactive elements do not harm the system
- The failure of the switch only happens in response to a failure

2.2. Indexes and parameters and the decision variables

i :	subsystems index $(i = 1, 2, \ldots, s)$
j :	redundant element index $(j = 1, 2, \ldots, m_i)$
k_i :	The kind of the element assigned to i th subsystem index $(k_i \in \{1, 2, \ldots, m_i\})$
k :	$(k_1, k_2, \ldots k_s)$

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n_i :	Number of the elements in i th subsystem $\left(n_i \in \{1,2,\ldots,n_{Max,i}\}\right)$
n :	$(n_1, n_2, \ldots n_s)$
st_i :	Redundancy strategy chosen for i th subsystem $st_i \in \{A, S\}$
st :	$(st_1, st_2, \ldots st_s)$
s :	Number of subsystems
m_i :	number of the kind of the elements for assigning in i th subsystem
$n_{Max,i}$:	Upper limit for n_i
t :	Mission time of the system
$r_{ij}(t)$:	Reliability of j th part assigned to j th subsystem
c_{ij} :	Cost of j th element assigned to i th subsystem
w_{ij} :	Wight of j th element assigned to i th subsystem
C :	Upper limit of the cost of the system
W :	Upper limit of the weight of the system
ρ_i :	switching success probability in failure detection
$F_{i,k_i}^{(j)}(t)$:	Probability function distribution for j th failure in i th subsystem with assigning element k_i to subsystem in time t
A :	Set of subsystems with active strategy
S :	Set of subsystems with cold stand by strategy
λ_j :	Failure rate of the j th kind of the element
λ_{i,n_i} :	Failure rate of the elements in i th subsystem when n_i elements are active
ρ_{ij} :	Internal connection cost of j th element to i th subsystem
$R(t, st, k, n)$:	Reliability of the system in time t with assigning vector k of all kind of elements and vector n of number of the elements

2.3. Formulation the objective of the problem

The purpose of the presented model is to maximize the reliability of a series-parallel system considering different redundancy strategies without component mixing in each subsystem (RAPCM). The failure rate of active elements in a subsystem are dependent on the number of active elements. The redundancy strategies in each subsystem may be active and cold standby.

2.3.1. The effect of the dependence of the failure rate of the elements on active elements in each subsystem

As described in the literature review, Sharifi et al. (2010) considered the effect of the dependence of the failure rates of the elements on active elements in each subsystem. They derived the failure rate of each element in i^{th} subsystem with k_i kind and n_i active element as described in (1) equation. They also proposed that the best value for δ is 0.5.

$$\lambda_{i,n_i} = \frac{n_i - \delta(n_i - 1)}{n_i} \lambda_{k_i} \qquad (1)$$

2.3.2. The total reliability in i^{th} subsystem with cold standby strategy

The reliability of the system in cold standby and considering failure probability in switching time from the/a failed element to the/an intact element exhibited in Eq (2) [33]. The graph of a cold standby system illustrated in figure 1.

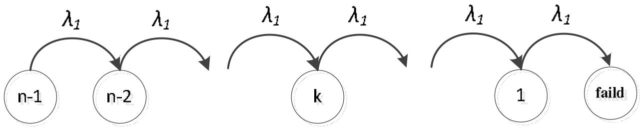

Fig. 1. The graph of a cold standby system

$$R(t,S,k,n) = \prod_{i \in S} \left\{ \begin{aligned} & r_{i,k_i}(t) + \\ & \sum_{j=1}^{(n_i-1)} \int_{u=0}^{t} \rho_i(u) F_{i,k_i}^{(j)}(u) r_{i,k_i}(t-u) du \end{aligned} \right\} \qquad (2)$$

2.3.3. The total reliability on i^{th} subsystem with active strategy

The reliability of a system in active mode is calculated from Eq. (3). In this state, the failure probability in switching time has no effect on calculating reliability. Figure 2 shows the graph of an active system.

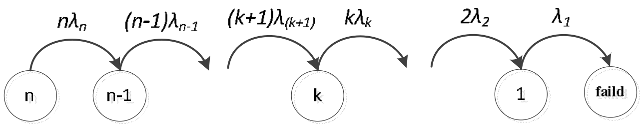

Fig. 2. The graph of an active system

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$$R(t,A,k,n)= \prod_{i\in A}\left\{\begin{array}{l}\left\{\prod_{j=1}^{m}\lambda_{i,n_i}\right\}\times \\ \sum_{i=k}^{m}\left\{\dfrac{n_i!\,e^{-i\lambda_{i,n_i}t}}{i(k-1)!\,\lambda_{i,n_i}\left\{\prod_{\substack{\theta=k\\ \theta\neq i}}^{m}\left(\theta\lambda_\theta - i\lambda_{i,n_i}\right)\right\}}\right\}\end{array}\right\} \quad (3)$$

2.3.4. Total reliability of the system

According to the system instruction, the total reliability of a system is calculated by multiplication of the reliability of the subsystems in cold standby and active strategy, as shown in Eq. (4).

$$R(t,st,k,n)=R(t,A,k,n)\times R(t,S,k,n) \quad (4)$$

2.4. Model's constraints formulation

2.4.1. Cost constraint

Problems in the real word always face this constraint. We consider a more effective kind of this constraint. Total cost of a system consists of assignment of the redundant elements and internal connection of the elements. Internal connection cost has an exponential nature because of limited space in electronic systems. Surcharging each redundant element to each subsystem has progressive costs. This constraint is shown in equation (5).

$$\sum_{i=1}^{s}\sum_{j=1}^{m_i}\left\{c_{ij}\left(n_i+e^{\rho_{i,k_i}n_i}\right)\right\}\leq C \quad (5)$$

2.4.2. Weight constraint

This constraint is the same as redundancy allocation base model and is shown in equation (6).

$$\sum_{i=1}^{s}\sum_{j=1}^{m_i}\left(w_{ij}n_i\right)\leq W \quad (6)$$

The proposed model of the problem is as follows:

$$Max\ \ Z = R(t,st,k,n)$$

$$S.t: \quad \sum_{i=1}^{s}\sum_{j=1}^{m_i}\left\{c_{ij}\left(n_i+e^{\rho_{i,k_i}n_i}\right)\right\}\leq C \qquad \sum_{i=1}^{s}\sum_{j=1}^{m_i}\left(w_{ij}n_i\right)\leq W \quad (7)$$

Finally, the objective of the model is to define the best strategy, number and kind of redundant parts assigned to each subsystem considering the constraints.

2.5. Problem coding

Recently, there have appeared different coding for the redundancy allocation problem, but we used the most efficient method, proposed by Tavakkoli-Moghaddam et al. (2008). In this study, each solution consists of a $(3\times s)$ matrix in which the 1st row shows redundancy strategy, the 2nd row shows kind, and the 3rd row shows the number of the redundant elements for each subsystem. In other words, for each subsystem there is a column, which shows redundancy strategy kind and the number of the elements. Figure 3 shows the described coding with 14 subsystems in which possibility of choosing three or four different kinds of the elements for each subsystem exists. We used this type of coding in the genetic algorithm and simulated annealing for solving the problem.

Subsystem No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14
The selected redundancy strategy for each subsystem	A	S	S	A	A	S	A	S	A	S	S	A	A	S
Allocated component type to each subsystem	1	1	2	3	4	2	1	3	2	1	3	1	2	4
Number of components allocated to each subsystem	2	2	1	5	3	2	1	1	1	4	2	3	1	2

Fig. 3. Coding provided for the proposed model

3. Tuning of the Parameters

A genetic algorithm is an attempt to solve a problem by using a randomly created initial population and selecting the best of these current programs to create "child" programs that attempt to combine successful features of their parents. By continuing to use this process of natural selection, an efficient program is created over several generations and finally a high-quality solution is achieved (Goldberg et al. (1989) & Holland, J (1975)).

Simulated Annealing (SA) is motivated by an analogy to annealing in solids. The idea of SA originated from a paper published by Metropolis et al. in 1953.

Parameter tuning algorithms means that we are always trying to find the optimal operators' rate. Cross over, mutation and number of the solution in each population

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are some operators of the genetic algorithm, which we want to optimize their values and whose symbols are $npop$, p_c, p_m respectively. The important operators in SA are initial temperature (T_0), the number of neighbors for a solution $(nmove)$ and the intensity of neighborhood (mu_0) all of which are the operators of SA.

Response surface methodology (RSM) is a combination of mathematical and statistical techniques for analyzing problems that face with several variables. The objective is to optimize the response. If the result of a process (Y) is affected by variables vector (X), the objective function is $y = f(x_1, x_2, \dots, x_n) + \varepsilon$, where ε defines the observed error in the response Y. If the expected value of a response is $E(y) = f(x_1, x_2, \dots, x_n) = \eta$, then the surface $\eta = f(x_1, x_2, \dots, x_n)$ is the response surface. The process of this methodology for each algorithm is as follows:

Step 1: Algorithm is tested by using different values of the independent variables (rate of the operators) to get the response variable (the function fitting algorithm for system reliability);

Step 2: Estimating the regression coefficients;

Step 3: Creation of the response surface model by using the estimated coefficients; and Step 4: Optimizing the model to determine the optimal values of the operators' rate.

The results of performing a stepwise approach for each algorithm are calculated using Minitab-16 and are presented in tables 2 and 3, respectively. The response variable for each algorithm is equal to the reliability of the system

Table 2
The results for the different rates of GA operators

Test No.	npop	pc	pm	Response
1	50	0.4	0.050	0.95369
2	100	0.4	0.050	0.96210
3	50	0.8	0.050	0.95406
4	100	0.8	0.050	0.95401
5	50	0.4	0.200	0.95805
6	100	0.4	0.200	0.95876
7	50	0.8	0.200	0.96009
8	100	0.8	0.200	0.96166
9	50	0.6	0.125	0.95697
10	100	0.6	0.125	0.95239
11	75	0.4	0.125	0.96152
12	75	0.8	0.125	0.95872
13	75	0.6	0.050	0.95463
14	75	0.6	0.200	0.96140
15	75	0.6	0.125	0.95859
16	75	0.8	0.050	0.95746
17	75	0.8	0.200	0.95820
18	75	0.4	0.050	0.95770
19	75	0.4	0.125	0.96091

Table 3
The results for the different rates of SA operators

Test No.	nmove	T0	mu0	Response
1	5	10000	0.150	0.95346
2	10	10000	0.150	0.96366
3	5	20000	0.150	0.95563
4	10	20000	0.150	0.96247
5	5	10000	0.300	0.95142
6	10	10000	0.300	0.96218
7	5	20000	0.300	0.95282
8	10	20000	0.300	0.96074
9	5	15000	0.225	0.95181
10	10	15000	0.225	0.96418
11	7	10000	0.225	0.95723
12	7	20000	0.225	0.9582
13	7	15000	0.150	0.95859
14	7	15000	0.300	0.95805
15	7	15000	0.225	0.96212
16	7	20000	0.150	0.96188
17	7	10000	0.150	0.95813
18	7	10000	0.300	0.95934
19	7	15000	0.150	0.95418

The ANOVA results obtained through Minitab16 for GA and SA are provided in tables 4 and 5, respectively.

Table 4
ANOVA test for GA in 19 times repeat

Source	FD	Seq SS	Adj SS	Adj MS	F	P
Regression	9	0.000116	0.000116	0.000013	2.47	0.097
Linear	3	0.000053	0.000053	0.000018	3.38	0.068
npop	1	0.000004	0.000004	0.000004	0.70	0.423
pc	1	0.000003	0.000003	0.000003	0.60	0.459
pm	1	0.000046	0.000046	0.000046	8.84	0.016
Square	3	0.000031	0.000031	0.000010	1.99	0.187
npop*npop	1	0.000012	0.000026	0.000026	5.04	0.051
pc*pc	1	0.000019	0.000015	0.000015	2.87	0.125
pm*pm	1	0.000000	0.000000	0.000000	0.03	0.869
Interaction	3	0.000032	0.000032	0.000011	2.04	0.179
npop*pc	1	0.000007	0.000007	0.000007	1.39	0.269
npop*pm	1	0.000005	0.000005	0.000005	0.89	0.371
pc*pm	1	0.000020	0.000020	0.000020	3.84	0.082
Residual	9	0.000047	0.000047	0.000005		
Lack-of-Fit	5	0.000039	0.000039	0.000008	4.14	0.097
Pure	4	0.000008	0.000008	0.000002		
Total	18	0.000163				

Table 5
ANOVA test for SA in 19 times repeat

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Regression	9	0.000249	0.000249	0.000028	5.02	0.012
Linear	3	0.000239	0.000239	0.000080	14.46	0.001
Nmove	1	0.000231	0.000231	0.000231	41.97	0.000
T0	1	0.000000	0.000000	0.000000	0.07	0.803
mu0	1	0.000007	0.000007	0.000007	1.34	0.276
Square	3	0.000005	0.000005	0.000002	0.27	0.844
nmove*nmove	1	0.000003	0.000000	0.000000	0.07	0.794
T0*T0	1	0.000001	0.000001	0.000001	0.22	0.652
mu0*mu0	1	0.000000	0.000000	0.000000	0.00	0.969
Interaction	3	0.000005	0.000005	0.000002	0.32	0.812
nmove*T0	1	0.000005	0.000005	0.000005	0.87	0.375
nmove*mu0	1	0.000000	0.000000	0.000000	0.06	0.810
T0*mu0	1	0.000000	0.000000	0.000000	0.02	0.881
Residual	9	0.000050	0.000050	0.000006		
Lack-of-Fit	5	0.000008	0.000008	0.000002	0.14	0.972
Pure	4	0.000042	0.000042	0.000011		
Total	18	0.000298				

The response surface model for GA is presented in Eq. (8) and the optimal solution with the contour plats of response is illustrated in Figure 4. In addition, the response surface model for SA is presented in Eq. (9) and

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the optimal solution with the contour plats of response is shown in Figure 4.

$$Max\ Response = 0.00093 \times npop - 0.0718997 \times p_c - 0.0146765 \times p_m - 0.000049 \times npop^2 + 0.0584768 \times p_c^2 + 0.0416128 \times p_m^2 - 0.00019 \times npop \times p_c - 0.000405 \times npop \times p_m + 0.105500 \times p_c \times p_m + 0.943854 \quad (8)$$

$$S.t: \quad 50 \le npop \le 100$$
$$0.4 \le p_c \le 0.8$$
$$0.05 \le p_m \le 0.2$$

Global Solution:

$npop = 70.7071$

$p_c = 0.8$

$p_m = 0.2$

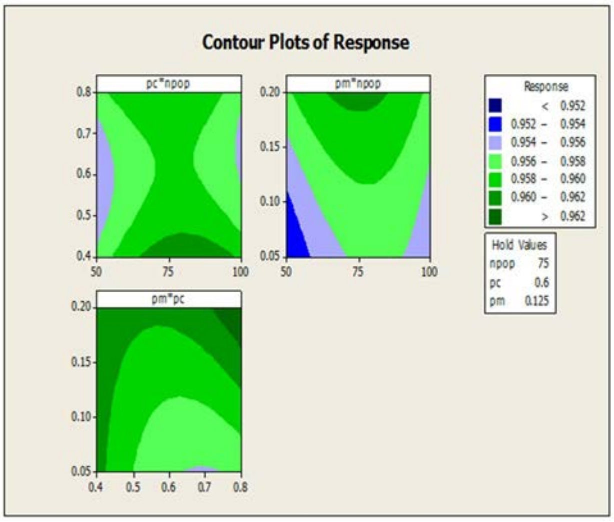

Fig. 4. Optimal solution with the contour plats of response for GA

$$Max\ Response = 0.00352566 \times nmove + 0.000013 \times T_0 - 0.009964 \times mu_0 - 0.000061 \times nmove^2 - 2.65010E(-11) \times T_0^2 - 0.00102268 \times mu_0^2 - 6.20000E(-8) \times nmove \times T_0 + 0.00109333 \times nmove \times mu_0 - 3.4E(-7) \times T_0 \times mu_0 + 0.93017 \quad (9)$$

$$S.t: \quad 5 \le nmove \le 10$$
$$10000 \le T_0 \le 20000$$
$$0.15 \le mu_0 \le 0.3$$

Global Solution:

$nmove = 10$

$T_0 = 13232.3$

$mu_0 = 0.15$

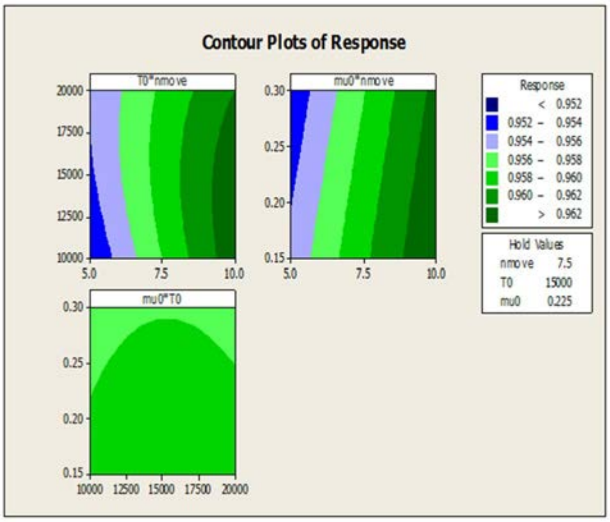

Fig. 5. Optimal solution with the contour plats of response for SA

4. Numerical results

In order to test the power of the presented algorithms, we consider a simple example. The optimal solution of this example is calculated by searching all the example solutions (feasible and infeasible). This example consists of six sub-systems. The parameters of this example are the parameters of sub-systems 1 to 6 in Coit example presented in Table 8 (Coit ET AL. (1996)). The other parameters of this example are presented in Table 6.

Table 6
The specification of the designed problem

Example specification	s	k	$n_{Max,i}$	C	W
	6	2	4	100	100

According to the example parameters, number of example solutions is equal to $\left(2^6 \times 2^6 \times 5^6 = 64{,}000{,}000\right)$. The results of two SA and GA algorithms and the optimal solution are presented in Table 7 and Figure 7.

Table 7
The results of solving the designed problem using different method

Solution method	Fitness function	The number of surveyed solutions	Computational time (Second)
Counting rule	0.954033	64,000,000	30,840
Genetic algorithm	0.954033	1780	9.4
Simulated annealing algorithm	0.954033	2790	14.2

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$$\underbrace{\begin{bmatrix} A & S & S & S & S & S \\ 2 & 2 & 2 & 1 & 2 & 2 \\ 3 & 2 & 4 & 4 & 4 & 4 \end{bmatrix}}_{\text{Best Solution}}$$

Fig. 6. Best solution of example

In order to evaluate the genetic and simulated annealing algorithm, we used the example, cited in Coit et al. (2003). Consider a series-parallel system with 14 series subsystems. Each subsystem can consist of up to six elements. In addition, for each sub-system, three or four different component types are available to allocate and all components are CFR[1]. Cost, weight and failure rates of the elements are presented in Table 8.

Each subsystem can choose one of the redundancy strategies: active or cold standby. In subsystems with cold standby strategy, the switch reliability is 0.99. The objective is to maximize the reliability of the system in time 100 under cost (C=130, Max) and weight (W=170, Max) constraint.

In order to find the best solution for the algorithms, the algorithms were implemented 10 times and the best feasible solution in these steps was concluded as the best solution. The results are shown in Table 9. In this table, the results of the algorithms and the best results are compared, and the convergence of the algorithms are shown in figures 8 and 9 for GA and SA, respectively.

Table 8
Values of the parameters

i	Choice 1 (j=1)			Choice 2 (j=2)			Choice 3 (j=3)			Choice 4 (j=4)		
	λ_{ij}	C_{ij}	W_{ij}	λ_{ij}	C_{ij}	W_{ij}	λ_{ij}	C_{ij}	W_{ij}	λ_{ij}	C_{ij}	W_{ij}
1	0.00532	1	3	0.000726	1	4	0.004990	2	2	0.00818	2	5
2	0.00818	2	8	0.000619	1	10	0.004310	1	9	*		
3	0.01330	2	7	0.011000	3	5	0.012400	1	6	0.00466	4	4
4	0.00741	3	5	0.012400	4	6	0.006830	5	4	*		
5	0.00619	2	4	0.004310	2	3	0.008180	3	5	*		
6	0.00436	3	5	0.005670	3	4	0.002680	2	5	0.000408	2	4
7	0.01050	4	7	0.004660	4	8	0.003940	5	9	*		
8	0.01500	3	4	0.001050	5	7	0.010500	6	6	*		
9	0.00268	2	8	0.000101	3	9	0.000408	4	7	0.000943	3	8
10	0.01410	4	6	0.006830	4	5	0.001050	5	6	*		
11	0.00394	3	5	0.003550	4	6	0.003140	5	6	*		
12	0.00236	2	4	0.007690	3	5	0.013300	4	6	0.011	5	7
13	0.00215	2	5	0.004360	3	5	0.006650	2	6	*		
14	0.01100	4	6	0.008340	4	7	0.003550	5	6	0.00436	6	9

Table 9
The results of GA and SA on 14 problems that was used by Coit[35]

Subsystem No.	Results obtained using GA			Optimal solution			Results obtained using SA		
	k_i	n_i	Redundancy	k_i	n_i	Redundancy	k_i	n_i	Redundancy
1	2	3	Cold standby	3	4	Cold standby	2	2	Cold standby
2	2	3	Cold standby	1	2	Cold standby	2	2	Cold standby
3	4	4	Cold standby	4	3	Cold standby	4	4	Cold standby
4	3	5	Cold standby	3	3	Cold standby	1	5	Cold standby
5	2	6	Active	2	3	Active	2	6	Active
6	4	2	Cold standby	2	2	Cold standby	4	2	Cold standby
7	2	4	Cold standby	1	2	Cold standby	2	4	Cold standby
8	2	6	Cold standby	3	2	Cold standby	2	6	Cold standby
9	2	2	Cold standby	1	2	Cold standby	2	2	Active
10	3	3	Cold standby	2	3	Cold standby	3	3	Active
11	1	4	Cold standby	3	2	Cold standby	1	5	Active
12	1	4	Cold standby	4	2	Cold standby	1	5	Active
13	1	5	Active	2	2	Active	1	4	Active
14	3	4	Cold standby	3	2	Cold standby	3	5	Active
Reliability	**0.9726**			**0.9863**			**0.9698**		

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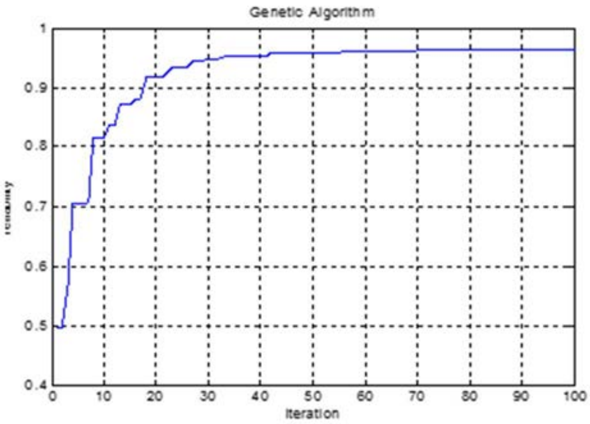

Fig. 7. Convergence of GA

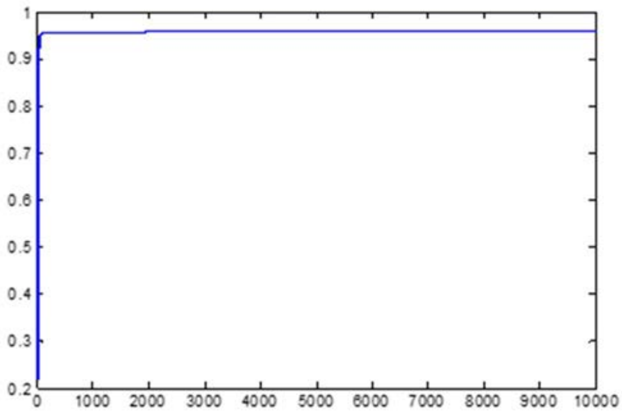

Fig. 8. Convergence of the SA

For comparing the results of two algorithm, we used ANOVA technique in Minitab-16. The results of one-way ANOVA technique presented in figures 9 and10.

```
Source     DF        SS         MS     F      P
algorithm   1  0.0005348  0.0005348  6.82  0.011
Error      64  0.0050178  0.0000784
Total      65  0.0055527
S = 0.008855   R-Sq = 9.63%   R-Sq(adj) = 8.22%
                              Individual 95% CIs For Mean Based on
                              Pooled StDev
Level   N     Mean    StDev   -----+---------+---------+---------+----
GA     33  0.96114  0.00870                       (---------*---------)
SA     33  0.95545  0.00901   (---------*---------)
                              -----+---------+---------+---------+----
                                 0.9540    0.9570    0.9600    0.9630
```

Fig. 9. One-way ANOVA: reliability versus algorithm

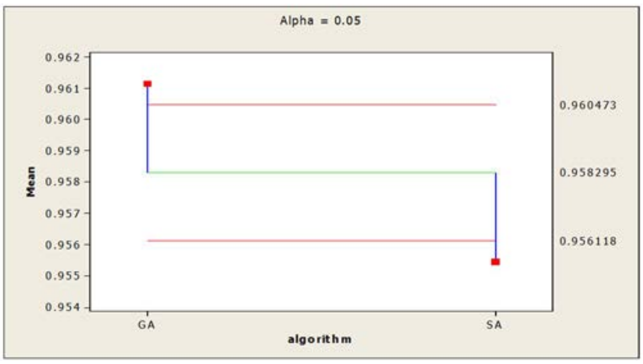

Fig. 10. One-Way Normal ANOVA for reliability

As figures 9 and 10 indicate, the performance of GA is better than SA.

For further analysis of the performance of the two algorithms, we solved 33 problems which were presented by Nakagawa and Miyazaki [8]. The parameters of these 33 problems are similar to the parameters of the solved problem but the upper limit of system weight was changed from 159 to 191. The results of problems solutions are presented in Table 10. Each problem was solved with both algorithms 5 times and the standard deviation of all solutions for each problem is near zero. The schematic standard deviation of both algorithms is presented in Figure 11.

As we expected, the cost of system in the presented model is more than the cost of the model solved by Nakagawa and Miyazaki because of the cost of internal connection in the presented model.

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Table 10
The results of GA and SA on 33 problems used by Nakagawa & Miyazaki [8]

Prob No.	Upper bound of W	Results obtained using GA				Results obtained using SA			
		W	Cost	R	Standard deviation	W	Cost	R	Standard deviation
1	159	178	195	0.96700	0.02395	186	219	0.95081	0.02901
2	160	169	195	0.96946	0.01599	160	191	0.94572	0.01314
3	161	184	193	0.94843	0.01909	173	212	0.96411	0.01210
4	162	183	217	0.96966	0.01784	169	215	0.95006	0.01765
5	163	166	212	0.96207	0.01707	163	205	0.95638	0.01160
6	164	175	222	0.94763	0.02017	186	200	0.94582	0.02198
7	165	163	206	0.95252	0.02461	170	212	0.95852	0.03124
8	166	163	212	0.95977	0.02421	171	220	0.94865	0.01619
9	167	180	200	0.97085	0.01757	188	193	0.96003	0.01176
10	168	158	214	0.97105	0.01297	171	206	0.96106	0.01321
11	169	165	226	0.94926	0.01863	174	209	0.96277	0.01359
12	170	186	212	0.97121	0.01391	187	224	0.95411	0.02185
13	171	178	229	0.97084	0.01652	181	213	0.94344	0.02032
14	172	180	190	0.95811	0.01101	180	215	0.94766	0.01584
15	173	189	208	0.96661	0.02760	183	192	0.96758	0.02484
16	174	184	210	0.94883	0.02751	184	209	0.94543	0.01274
17	175	183	218	0.95639	0.01871	169	195	0.96503	0.01000
18	176	184	217	0.96972	0.01704	159	216	0.95667	0.02296
19	177	167	229	0.96639	0.01182	188	220	0.96999	0.01772
20	178	172	200	0.97091	0.02212	162	194	0.94327	0.02150
21	179	167	212	0.96270	0.01032	186	218	0.95388	0.01737
22	180	165	192	0.94596	0.01890	189	217	0.94410	0.01052
23	181	168	200	0.96793	0.01306	186	191	0.96899	0.01884
24	182	180	211	0.97022	0.02895	180	227	0.94113	0.02110
25	183	172	209	0.96333	0.01710	176	211	0.96355	0.01285
26	184	176	202	0.96546	0.01231	173	198	0.96478	0.02349
27	185	165	195	0.96506	0.01867	169	212	0.96628	0.03120
28	186	178	203	0.95559	0.03147	161	217	0.94346	0.01197
29	187	184	223	0.96270	0.01123	169	200	0.95263	0.01679
30	188	160	215	0.94962	0.01866	186	207	0.94856	0.02164
31	189	191	199	0.96406	0.01222	161	221	0.96428	0.01984
32	190	178	209	0.94586	0.03146	163	226	0.95355	0.03068
33	191	190	229	0.95248	0.01985	168	205	0.96750	0.02586

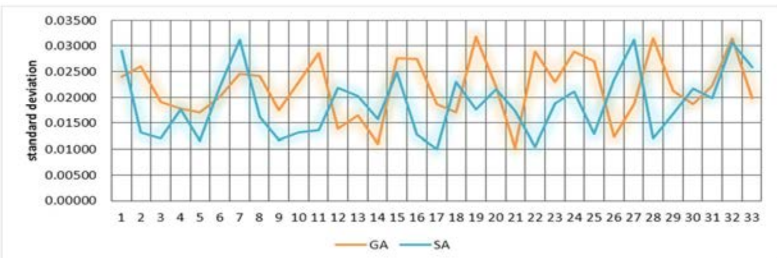

Fig. 11. Schematic presentation of the standard deviation of each problem obtained using GA and SA

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A hypothesis test for checking the equality of algorithms performance was done using MINITAB 16 and the results are presented in Figures 12 and 13. Obviously, the performance of GA is better than SA for solving the presented model.

One-way ANOVA: reliability versus algorithm

```
Source     DF        SS         MS     F      P
algorithm   1  0.0005348  0.0005348  6.82  0.011
Error      64  0.0050178  0.0000784
Total      65  0.0055527
S = 0.008855   R-Sq = 9.63%   R-Sq(adj) = 8.22%
                              Individual 95% CIs For Mean Based on
                              Pooled StDev

Level   N     Mean    StDev   -----+---------+---------+---------+----
GA     33  0.96114  0.00870                      (---------*---------)
SA     33  0.95545  0.00901   (---------*---------)
                              -----+---------+---------+---------+----
                                0.9540    0.9570    0.9600    0.9630
```

Fig. 12. One-way ANOVA: Reliability versus algorithm

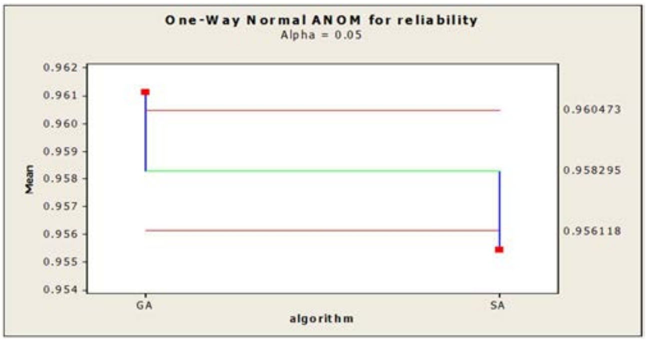

Fig. 13. One-way normal ANOM for reliability

5. Conclusion

In this research, an integer nonlinear programming model for the redundancy allocation, without component mixing was presented considering the dependence of the component failure rates work and the interconnection cost of the system. In other words, the purpose of this paper is to allocate components and the redundancy strategy to any subsystem without allocating the component mixing to any subsystems in order to maximize system reliability under certain physical restrictions. Since, this issue is considered as an NP hard problem, Meta heuristic genetic algorithms and Annealing Simulation, after optimizing their function rates, using "response surface" methodology was engaged in solving it. Therefore, instead of randomly determining the rate of the operators, scientific methods were used which led to the gradation in the quality of the results obtained from the two algorithms. Finally, in order to evaluate the performance of these algorithms, several numerical examples were solved using these algorithms.

Presenting multi-objective models, considering new restrictions such as volume and factors such as weight, possible cost, and more than two active and cold standby strategies for each subsystem are among research that will contribute to the development of the future models.

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