A Benders Decomposition Method to Solve an Integrated Logistics Network Designing Problem with Multiple Capacities

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Abstract

In this paper, a new model is proposed for the integrated logistics network designing problem. In many research papers in this area, it is assumed that there is only one option for the capacity of each facility in the network. However, this is not a realistic assumption because generally there may be many possible options for the capacity of the facility that is being established. Usually the cost of establishing a facility depends on its capacity. Moreover, of the majority of the research done in the field of logistics network designing problem only a limited number of options for product recovery is addressed. Specifically, in most of the research papers only one option, i.e. remanufacturing, has been considered. Therefore, a mathematical formulation with multiple options for capacities and product recovery is addressed in this research to obviate this gap. Afterwards a benders decomposition method is developed to efficiently solve the problem. The computational results introduce several random generated problems to be solved with benders algorithm and demonstrate that this algorithm can efficiently solve the proposed model.
Keywords: Logistics network designing problem; Integrated logistics; Multiple capacities; Recycling; Benders decomposition.

1. Introduction

The traditional view of many manufacturers regarding the used products is assuming that they are valueless. They generally do not feel any obligation about what happens to the product discarded by the customer. They design their products to minimize the cost of materials, assembly and distribution but do not consider the costs of repairing, reusing or recycling (Zhou & Wang, 2008). Even though reusing the products discarded by the customers is not a new subject and it has been around in some industries for a long time (Srivastava, 2008), the level of product recovery has significantly risen throughout the last decades (Fleischmann, Beullens, Bloemhof-Ruwaard, & Wassenhove, 2001) and this fact is a reminder of the necessity of reverse logistics.
The reverse logistics can be defined as "the process of planning, implementing, and controlling the efficient and cost effective flow of raw materials, in-process inventory, finished goods and related information from the point of consumption to the point of origin for the purpose of recapturing value or proper disposal" (Hawks, 2006). Such a concept has been around for a long time and many researchers have investigated it from many different viewpoints, most of them developed the traditional models considering real-case applications. For example Roghanian and Pazhuheshfar (2014) solved a stochastic reverse logistics model using genetic algorithm. Also Hatefi and Julai (2014) proposed a robust logistic model under demand uncertainly and facility disruptions. Other related works can be found in Rahmati, Ahmadi, and Karimi (2014) and Mehdizade and Fatehi Kivi (2014).
Nowadays, increasing concerns for environmental issues and passing new laws to protect the environment have highlighted the importance of reverse logistics. In many industries, such as electronic products, considering reverse logistics has become a necessity, especially with continuously decreasing product life cycle in these industries. Therefore designing an efficient reverse logistics network to reuse the products that are at the end of their life cycles is of major importance. Figure 1 illustrates the structures of forward and reverse logistics.
Establishing an efficient reverse logistics requires a well-designed network with a set of activities, such as collecting, inspecting, dismantling, recycling, remanufacturing and repairing (Kannan, Pokharel, & Sasi Kumar, 2009). In order to obtain an optimum network simultaneously considering both forward and reverse is indispensable, because the design of one network affects the optimum design for the other one, therefore optimizing these networks separately will lead to sub-optimality. However, in many research papers in the field

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of logistics network designing problem, the designing of the forward and reverse networks has been separately considered.
One of the major drawbacks in the literature of reverse logistics network design problem is that the majority of the research papers in this field consider a limited number of options for product recovery. According to the work done by Thierry, Salomon, Nunen, and Wassenhove (1995) there are five options for product recovery. These five options are: repairing, refurbishing, remanufacturing, cannibalizing and recycling. The objective of the mentioned options, except for recycling, is to maintain the identity of the product, while in recycling the identity of the product is lost. However, in most of the research papers, only one option is considered. In this paper a new model is proposed to address this drawback. Another important issue that is neglected in the current literature is considering multiple options for the capacity of the facilities in the network. This paper addresses the issue by considering multiple options for the capacity of facilities. For example in drug production industry, some drug utensils should be reused while some others should be recycled. These operations are done using the specific facilities that each of them may have multiple options for capacities. For more detail see Wang, Hung, and O`Neill (2011).

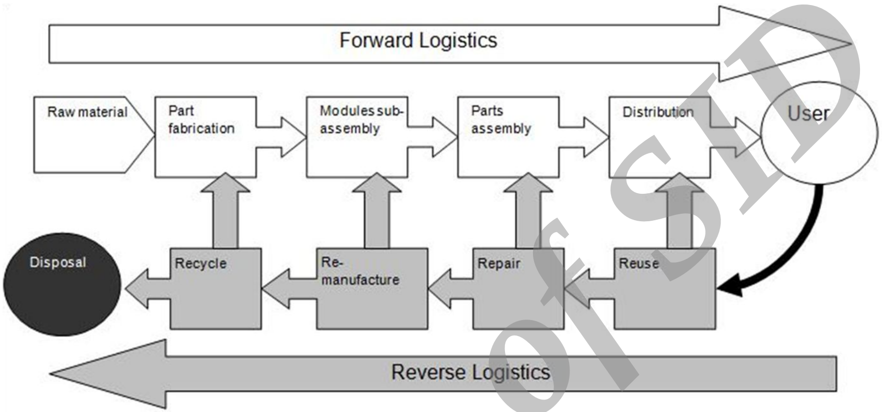

Fig. 1. Forward and reverse logistics

Even though product recovery has been around for a long time, scientific attention to reverse logistics has started since the early 1990s. In recent years various models have been developed in the field of logistics network design problem. These models range from simple schemes without capacity restrictions (Yeh, 2005) to more complex multi-objective models (Du & Evans, 2008). In the literature of supply chain, a great portion of research papers considered designing a forward supply chain from suppliers to customers, but the number of research papers on designing reverse logistics network is limited. Pokharel and Mutha (2009) reviewed the literature of reverse logistics, in this review they indicated that the research in this field has been growing more significantly since 2005. Moreover Melo, Nickel, and Saldanha-Da-Gama (2009) have performed a review in logistics network designing problem area and suggested promising areas for future research.
Jayaraman, Guide Jr, and Srivastava (1999) proposed a mixed integer programming model for a closed loop logistics network designing problem to minimize the costs of the network in objective function. They also discussed the managerial aspects of the models and explained the application of their model in decision making. In their model, a multi-commodity network with capacity constraints is addressed. However, their study is limited because they did not consider an integrated network, moreover they assumed that there is only one option for the capacity of the facilities. Fleischmann et al. (2001) proposed an integer programming model to investigate the effect of product recovery on the design of logistics network. They showed that solving and optimizing an integrated network is more suitable than separately optimizing forward and reverse networks. However they only considered a single commodity network and they only considered a single capacity option for each facility. Realff, Ammons, and Newton (2004) proposed a robust optimization model for carpet recovery and stated that in the United States, carpet recovery can provide 76 million dollars annually. They proposed a reversed multi-commodity logistics network for modelling this problem. Multi-objective approach has been also studied by Altiparmak, Gen, Lin, and Paksoy (2006). It this work, a multi-objective genetic algorithm has been extended to solve a forward supply chain network designing problem considering a forward single commodity network with multiple objectives of minimizing the costs and maximizing the service level and maximizing the usage of facilities.
All aforementioned researches are based on the single-capacity facilities, not considering multiple options. However, it is an obvious fact that real world cases can be found with multiple capacitated facilities. Hence, for reducing this gap between theoretical and practical problems, the extension of this models is a reasonable development. In this regard, Amiri (2006) proposed

designing a supply chain network that included finding the optimal locations of facilities and distribution warehouses so that the cost of the network is minimized. This study is the first research considering multiple options for the capacity of the facilities in the network. However, this study considers a single-commodity forward supply chain network. Ko and Evans (2007) highlighted the importance of concurrently considering the forward and reverse networks and proposed an integrated multi-commodity logistics network. They constructed a non-linear integer programming model to solve this problem. Since this problem is NP-hard, they proposed a genetic algorithm to solve this problem. However, they did not consider multiple options for the capacities. They considered only one option for product recovery, Zhou and Wang (2008) also studied designing of a generic integrated logistics network considering two product recovery options in their model. They developed a mathematical model and a branch and bound method to solve this problem. However, they didn't consider any limit for the capacity of the facilities. Pishvaee, Jolai, and Razmi (2009) proposed a stochastic optimization model for an integrated logistics network under uncertainty aiming to minimize the expected value of costs. However, their study considers only one option for product recovery and the capacity of the facilities. Mutha and Pokharel (2009) proposed a mathematical model, the reverse logistics network designing problem, with the recovery options of remanufacturing, recycling and disposal. Alumur, Nickel, Saldanha-Da-Gama, and Verter (2012) proposed a multi-period reverse logistics network designing problem. They developed an integer programming model to solve this problem and suggested that their model can be used for real-world problems. They considered a multi-commodity network and aimed to maximize the profit. They also showed the advantages of their model in comparison to static models through many different scenarios. But the network considered in their model is not integrated and only one product recovery option is considered.

Some other related works can be found in the literature, where all of their contributions are to use multiple objective functions, locating facilities, considering forward and reverse models simultaneously. However, one can hardly find any research regarding multiple capacity options for facilities. Also, very few studies considered more than one option for product recovery. Therefore, because of the existing of the discussed gap, this paper introduces an integrated logistics network designing problem with multiple options for the capacities of the facilities and several options for product recovery. Because of the complexity of the proposed problem, Benders decomposition algorithm is used to solve this model. To cognize why this model is NP-hard, one can refer to Ko and Evans (2007). Therefore for providing a perceivable description of our work, the rest of this paper is organized as follows:

In section 2, the proposed model is illustrated and a mathematical formulation is proposed to solve this problem. After developing a benders decomposition method in Section 3, the model is experimented and computational results are presented in Section 4. Finally, Section 5 is assigned to conclusion remarks and also future activities.

2. Problem Description and Mathematical Formulation

In this section the discussed problem is described and a mathematical model is proposed to solve it. This part of the article is divided into two subsections where subsection 2.1 describes the preliminaries of the problem and subsection 2.2 formulates the mathematical model.

1.1 Problem description

The problem considered in this study involves managing the reverse flow in the forms of repairing, remanufacturing, recycling and disposal. There are four types of entities in the network: customers, distribution centers, central recovery centers (CRCs) and production plants. In order to reduce the costs of network Pishvaee et al. (2009) and Lee and Dang (2008) suggested that the distribution and collection facilities use the same resources for transporting materials, production, human resources and infrastructures. In the proposed problem in this study it is assumed that the customers return the used products to the hybrid distribution-collection centers and then the returned products are sent to the CRCs. The reverse flows are managed in CRCs (Srivastava, 2008). In the CRCs the returned products are inspected and assigned to perform one of the following actions: repairing, remanufacturing, disposing, and recycling. According to Thierry et al (1995) these actions are defined as:

Repairing: the objective is to restore the products to a working condition. The quality of the repaired products is generally lower than the brand new products.

Remanufacturing: The objective is to enhance the quality of the used product to reach the standard of a brand new product. In this process the used products are completely dismantled and all of its components are inspected.

Recycling: The objective is to use the raw material of the used product. In recycling the identity of the product is lost.

In the proposed problem, the repairable products are repaired in the CRCs and sent back to the hybrid distribution-collection centers. The products that are assigned to be remanufactured, are sent to the production plants and after remanufacturing are sent back to the hybrid distribution-collection centers. The products that are assigned to be recycled are sent to the recycling centers and the disposable products are sent to the disposal centers. The logistics network proposed in this paper have 6 layers including manufacturing/remanufacturing plants, hybrid

distribution-collection centers, CRCs, disposal centers and recycling centers. In the forward flow, the products are transported from production plants to the hybrid distribution-collection centers and then to the customers.
In most of the previous studies, neglecting the capacity of the facilities in the network (production plants, hybrid distribution-collection centers, CRCs and disposal centers) is one of the major drawbacks. In this paper, multiple options are considered for the facilities in the network. The demand and returns of the customers are assumed to be deterministic. With these conditions, the proposed model is defined as a 6 layered multi-commodity network designing problem. In the next section a mixed integer linear programming model is proposed to solve this problem.

2.1 Mathematical model

In this section a mathematical model is proposed to solve the proposed problem. The following notations are used to formulate the problem:

Sets:

$I=\{1,\ldots,N_p\}$	Candidate locations for manufacturing/remanufacturing plants
$J=\{1,\ldots,N_d\}$	Candidate locations for the hybrid distribution-collection centers
$K=\{1,\ldots,N_r\}$	Candidate locations for CRCs
$O=\{1,\ldots,N_x\}$	Candidate locations for disposal centers
$R=\{1,\ldots,N_e\}$	Candidate locations for recycling centers
$L=\{1,\ldots,N_c\}$	Set of customers
$M=\{1,\ldots,N_g\}$	Set of products
$H=\{1,\ldots,N_h\}$	Set of possible capacities

Parameters:

f_{ih}^{p}	Fixed cost of establishing a manufacturing/remanufacturing plant i with capacity level h
$f_{\mathrm{j}h}^{\mathrm{d}}$	Fixed cost of establishing a hybrid distribution-collection center j with capacity level h
$f_{\mathrm{kh}}^{\mathrm{r}}$	Fixed cost of establishing a CRC k with capacity level h
$f_{\mathrm{oh}}^{\mathrm{x}}$	Fixed cost of establishing a disposal center o with capacity level h
$f_{\mathrm{rh}}^{\mathrm{e}}$	Fixed cost of establishing a recycling center r with capacity level h
$C_{\mathrm{mijl}}^{\mathrm{f}}$	Forward flow- Variable cost of supplying one unit of the demand of customer l for the product m with manufacturing plant i and hybrid distribution-collection center j
$C_{\mathrm{mkjl}}^{\mathrm{f}}$	Forward flow- Variable cost of supplying one unit of the demand of customer l for the product m with CRC k and hybrid distribution-collection center j
$C_{\mathrm{mljk}}^{\mathrm{r}}$	Reverse flow- Variable cost of retrieving one unit of the product m returned by customer l at hybrid distribution-collection center j which is repaired at CRC k
$C_{\mathrm{mljki}}^{\mathrm{r}}$	Reverse flow- Variable cost of retrieving one unit of the (re-manufacturable) product m returned by customer l at hybrid distribution-collection center j to CRC k which is sent to remanufacturing plant i.
$C_{\mathrm{mljo}}^{\mathrm{r}}$	Reverse flow- Variable cost of retrieving one unit of the (disposable) product m returned by customer l at hybrid distribution-collection center j to CRC k which is sent to disposal center o.
$C_{\mathrm{mlj}kr}^{\mathrm{r}}$	Reverse flow- Variable cost of retrieving one unit of the (recyclable) product m returned by customer l at hybrid distribution-collection center j to CRC k which is sent to recycling center r.
$C_{\mathrm{ml}}^{\mathrm{u}}$	Penalty cost of not supplying one unit of the demand of customer l for product m
$C_{\mathrm{ml}}^{\mathrm{w}}$	Penalty cost of not retrieving one unit of the returns of customer l for product m
C_o	Cost of disposal for one unit
C_r	Cost of recycling
C_m	Cost of manufacturing/remanufacturing one unit of product m in plant i
C_{km}	Cost of inspection of one unit product m at CRC k
C_{rpm}	Cost of repairing one unit of product m at CRC k
d_{ml}	Demand of customer l for product m
r_{ml}	Return of customer l for product m
β_m	Maximum percentage of repairable products.
γ_m	Minimum percentage of disposable products.
η_m	Maximum percentage of recyclable products.
cap_{ih}^{p}	The capacity of manufacturing plant i at capacity level h
cap_{rih}^{p}	The capacity of remanufacturing plant i at capacity level h
cap_{jh}^{d}	The distribution capacity of hybrid distribution-collection center j at capacity level h
cap_{rjh}^{d}	The collection capacity of hybrid distribution-collection center j at capacity level h
cap_{rh}^{e}	The recycling capacity of recycling center r at capacity level h
cap_{kh}^{r}	The capacity of CRC k at capacity level h
cap_{roh}^{x}	The disposal capacity of disposal center o at capacity level h

Decision Variables:

X_{mijl}^{f}	Percentage of the demand of customer l for product m that is supplied by manufacturing plant i through CRC j
X_{mkjl}^{f}	Percentage of the demand of customer l for product m that is supplied by CRC k through distribution-collection center j
U_{ml}	Percentage of the demand of customer l for product m that is unanswered
X_{mljk}^{r}	Percentage of returns of customer l for product

m that is repaired at CRC k through distribution-collection center j

X^{r}_{mljki} Percentage of returns of customer l for product m that is remanufactured at plant i through distribution-collection center j and CRC k

X^{r}_{mljko} Percentage of returns of customer l for product m that is disposed at disposal center o through distribution-collection center j and CRC k

X^{r}_{mljkr} Percentage of returns of customer l for product m that is recycled at recycling center r through distribution-collection center j and CRC k

W_{ml} Percentage of the returns of customer l for product m that is unanswered

Y^{e}_{rh} Equals to 1 if recycling center r with capacity level h is established; Zero otherwise.

$Y^{\mathrm{p}}_{\mathrm{ih}}$ Equals to 1 if manufacturing/remanufacturing plant i with capacity level h is established; Zero otherwise.

$Y^{\mathrm{d}}_{\mathrm{jh}}$ Equals to 1 if hybrid distribution-collection center j with capacity level h is established; Zero otherwise.

Y^{r}_{kh} Equals to 1 if CRC k with capacity level h is established; Zero otherwise.

Y^{x}_{oh} Equals to 1 if disposal center o with capacity level h is established; Zero otherwise.

Using these notations, the problem can be formulated as follows:

$$\min \sum_{i\in I}\sum_{h\in H} f^{p}_{ih}y^{p}_{ih} + \sum_{j\in J}\sum_{h\in H} f^{d}_{jh}y^{d}_{jh} + \sum_{k\in K}\sum_{h\in H} f^{r}_{kh}y^{r}_{kh} + \sum_{o\in O}\sum_{h\in H} f^{x}_{oh}y^{x}_{oh} + \sum_{r\in R}\sum_{h\in H} f^{e}_{rh}y^{e}_{rh} + \sum_{m\in M}\sum_{i\in I}\sum_{j\in J}\sum_{l\in L} C^{f}_{mijl}d_{ml}X^{f}_{mijl}$$
$$+ \sum_{m\in M}\sum_{k\in K}\sum_{j\in J}\sum_{l\in L} C^{f}_{mjkl}d_{ml}X^{f}_{mkjl} + \sum_{m\in M}\sum_{l\in L}\sum_{j\in J}\sum_{k\in K} C^{r}_{mljk}r_{ml}X^{r}_{mljk}$$
$$+ \sum_{m\in M}\sum_{l\in L}\sum_{j\in J}\sum_{k\in K}\sum_{i\in I} C^{r}_{mljki}r_{ml}X^{r}_{mljki} + \sum_{m\in M}\sum_{l\in L}\sum_{j\in J}\sum_{k\in K}\sum_{o\in O} C^{r}_{mljko}r_{ml}X^{r}_{mljko}$$
$$+ \textstyle\sum_{m\in M}\sum_{l\in L}\sum_{j\in J}\sum_{k\in K}\sum_{r\in R} C^{r}_{mljkr}r_{ml}X^{r}_{mljkr} + \sum_{m\in M}\sum_{l\in L} C^{u}_{ml}d_{ml}U_{ml} + \sum_{m\in M}\sum_{l\in L} C^{w}_{ml}r_{ml}W_{ml} \tag{1}$$

S.T.

$$\textstyle\sum_{l\in L}\sum_{j\in J} X^{r}_{mljk}r_{ml} = \sum_{l\in L}\sum_{j\in J} X^{f}_{mkjl}d_{ml} \qquad \forall m\in M, k\in K \tag{2}$$
$$\textstyle\sum_{i\in I}\sum_{j\in J} X^{f}_{mijl} + \sum_{k\in K}\sum_{j\in J} X^{f}_{mkjl} + U_{ml} = 1 \qquad \forall m\in M, l\in L \tag{3}$$
$$\textstyle\sum_{j\in J}\sum_{k\in K}\left(\sum_{i\in I} X^{r}_{mljki} + \sum_{o\in O} X^{r}_{mljko} + \sum_{r\in R} X^{r}_{mljkr} + X^{r}_{mljk}\right) + W_{ml} = 1 \quad \forall m\in M, l\in L \tag{4}$$
$$\textstyle\sum_{j\in J}\sum_{k\in K}\sum_{l\in L} X^{r}_{mljki}r_{ml} \le \sum_{j\in J}\sum_{l\in L} X^{f}_{mijl}d_{ml} \qquad \forall m\in M, i\in I \tag{5}$$
$$\gamma_m\left(X^{r}_{mljk} + \textstyle\sum_{i\in I} X^{r}_{mljki} + \sum_{o\in O} X^{r}_{mljko} + \sum_{r\in R} X^{r}_{mljkr}\right) \le \sum_{o\in O} X^{r}_{mljko} \quad \forall m\in M, l\in L, j\in J, k\in K \tag{6}$$
$$\beta_m\left(X^{r}_{mljk} + \textstyle\sum_{i\in I} X^{r}_{mljki} + \sum_{o\in O} X^{r}_{mljko} + \sum_{r\in R} X^{r}_{mljkr}\right) \ge X^{r}_{mljk} \quad \forall m\in M, l\in L, j\in J, k\in K \tag{7}$$
$$\eta_m\left(X^{r}_{mljk} + \textstyle\sum_{i\in I} X^{r}_{mljki} + \sum_{o\in O} X^{r}_{mljko} + \sum_{r\in R} X^{r}_{mljkr}\right) \ge \sum_{r\in R} X^{r}_{mljk} \quad \forall m\in M, l\in L, j\in J, k\in K \tag{8}$$
$$\textstyle\sum_{j\in J} X^{f}_{mijl}d_{ml} \le \sum_{h\in H} cap^{p}_{ih}Y^{p}_{ih} \qquad \forall m\in M, l\in L, i\in I, l\in L \tag{9}$$
$$\textstyle\sum_{j\in J} X^{f}_{mijl}d_{ml} + \sum_{k\in k} X^{f}_{mkjl}d_{ml} \le \sum_{h\in H} cap^{d}_{jh}Y^{d}_{jh} \qquad \forall m\in M, l\in L, j\in J \tag{10}$$
$$\sum_{j\in J} X^{r}_{mljk}r_{ml} + \sum_{j\in J}\sum_{i\in I} X^{r}_{mljki}r_{ml} + \sum_{j\in J}\sum_{o\in O} X^{r}_{mljko}r_{ml} + \sum_{j\in J}\sum_{r\in R} X^{r}_{mljkr}r_{ml} \le \sum_{h\in H} cap^{r}_{kh}Y^{r}_{kh}$$
$$\forall m\in M, k\in K, l\in L \tag{11}$$
$$\sum_{k\in K} X^{r}_{mljk}r_{ml} + \sum_{k\in K}\sum_{i\in I} X^{r}_{mljki}r_{ml} + \sum_{k\in K}\sum_{o\in O} X^{r}_{mljko}r_{ml} + \sum_{k\in K}\sum_{r\in R} X^{r}_{mljkr}r_{ml} \le cap^{d}_{rj}Y^{d}_{k}$$
$$\forall m\in M, j\in J, l\in L \tag{12}$$
$$\textstyle\sum_{j\in J}\sum_{k\in K} X^{r}_{mljki}r_{ml} \le \sum_{h\in H} cap^{p}_{rih}Y^{p}_{rih} \qquad \forall m\in M, i\in I, l\in L \tag{13}$$
$$\textstyle\sum_{j\in J}\sum_{k\in K} X^{r}_{mljko}r_{ml} \le \sum_{h\in H} cap^{x}_{roh}Y^{x}_{oh} \qquad \forall m\in M, i\in I, l\in L \tag{14}$$
$$\textstyle\sum_{j\in J}\sum_{k\in K} X^{r}_{mljkr}r_{ml} \le \sum_{h\in H} cap^{e}_{rh}Y^{e}_{rh} \qquad \forall m\in M, i\in I, l\in L \tag{15}$$
$$0 \le X^{f}_{mijl}, X^{f}_{mkjl}, X^{r}_{mljki}, X^{r}_{mljk}, X^{r}_{mljko}, U_{ml}, X_{ml} \le 1 \quad \forall m\in M, i\in I, j\in J, k\in K, r\in R, l\in L \tag{16}$$
$$Y^{p}_{i}, Y^{d}_{j}, Y^{r}_{k}, Y^{x}_{o}, Y^{e}_{r} \in \{0,1\} \quad \forall m\in M, i\in I, j\in J, k\in K, r\in R, l\in L \tag{17}$$

The introduced model is a modified one which is based on the past researches, except that all constraints related to the capacity options shape the contribution of our work. On the other hand, traditional models are improved to contain the contributions.

In this model constraint (1) shows the objective function which is minimizing the total cost of the logistics network. Constraint (2) ensures that all of the repaired products are used to meet the demands of the customers.

Constraints (3) and (4) indicate that all of the demands and returns of the customers are either met or remain unanswered. Constraint (5) ensures that for each manufacturing/remanufacturing plant the total output flows are at least as big as the total input flows. Constraint (6) indicates the minimum percentage of disposal for the reverse flow. Constraint (7) indicates the maximum percentage of repairable products. Constraint (8) indicates the maximum percentage of recyclable products. Constraints (9) through (15) ensure that the capacity constraints of the facilities are observed. Constraints (16) and (17) are non-negativity and binary constraints.

3. The Proposed Benders Decomposition Method

In this section a solution method based on benders decomposition is proposed to solve the problem described in the previous section. Following Boschetti and Maniezzo (2009), benders decomposition is preferable to the meta-heuritics when the problem can be solved within an acceptable time duration. That is why this paper uses benders algorithm.

Benders decomposition involves decomposing a mixed integer programming problem into a master problem and a sub-problem; these problems are iteratively solved to obtain an optimal solution for the main problem (Benders, 1962). The sub-problem involves continuous variables and their related constraints and the master problem includes integer variables and one continuous variable that links the two problems. An optimal solution for the master problem provides a lower bound for the solution of the main problem. Using the solution obtained by the master problem and fixing the integer variables as an input for the sub-problem, the dual sub-problem is solved and the result can be used to obtain an upper bound. In the next iteration, a cut is added to the master problem and the master problem is solved again with the additional constraint to obtain a new lower bound. This new lower bound is guaranteed to be lower than or equal to the previous lower bound. This procedure is followed until the difference between the lower bound and the upper bound is low enough. Benders decomposition method reaches the optimum solution in finite number of iterations. Before developing the master problem and sub-problem, the main problem is adjusted to facilitate the process. This problem can be represented as:

$Z_p = \min \sum_{i\in I}\sum_{h\in H} f_{ih}^{p} y_{ih}^{p} + \sum_{j\in J}\sum_{h\in H} f_{jh}^{d} y_{jh}^{d} + \sum_{k\in K}\sum_{h\in H} f_{kh}^{r} y_{kh}^{r} + \sum_{o\in O}\sum_{h\in H} f_{oh}^{x} y_{oh}^{x} + \sum_{r\in R}\sum_{h\in H} f_{rh}^{e} y_{rh}^{e} +$
$BSP(X_{mijl}^{f}, X_{mkjl}^{f}, X_{mljk}^{r}, X_{mljki}^{r}, X_{mljko}^{r}, X_{mljkr}^{r}, U_{ml}, W_{ml} | y_{ih}^{p}, y_{jh}^{d}, y_{kh}^{r}, y_{oh}^{x}, y_{rh}^{e},)$ *Subject to:*
Equations (16) through (20)
Or:

$$Z_p = \min \sum_{i\in I}\sum_{h\in H} f_{ih}^{p} y_{ih}^{p} + \sum_{j\in J}\sum_{h\in H} f_{jh}^{d} y_{jh}^{d} + \sum_{k\in K}\sum_{h\in H} f_{kh}^{r} y_{kh}^{r} + \sum_{o\in O}\sum_{h\in H} f_{oh}^{x} y_{oh}^{x} + \sum_{r\in R}\sum_{h\in H} f_{rh}^{e} y_{rh}^{e} + BSP(X_{mijl}^{f}, X_{mkjl}^{f}, X_{mljk}^{r}, X_{mljki}^{r}, X_{mljko}^{r}, X_{mljkr}^{r}, U_{ml}, W_{ml} | y_{ih}^{p}, y_{jh}^{d}, y_{kh}^{r}, y_{oh}^{x}, y_{rh}^{e})$$

Subject to:

$$\sum_{h} y_{ih}^{p} \le 1 \ \forall i$$
$$\sum_{h} y_{jh}^{d} \le 1 \ \forall j$$
$$\sum_{h} y_{kh}^{r} \le 1 \ \forall k$$
$$\sum_{h} y_{oh}^{x} \le 1 \ \forall o$$
$$\sum_{h} y_{rh}^{e} \le 1 \ \forall r$$

In which $BSP\left(\begin{matrix} X_{mijl}^{f}, X_{mkjl}^{f}, X_{mljk}^{r}, X_{mljki}^{r}, \\ X_{mljko}^{r}, X_{mljkr}^{r}, U_{ml}, W_{ml} \end{matrix} \middle| y_{ih}^{p}, y_{jh}^{d}, y_{kh}^{r}, y_{oh}^{x}, y_{rh}^{e}\right)$ is the benders sub-problem which is developed in the following section.

3.1 Benders sub-problem

$BSP\left(\begin{matrix} X_{mijl}^{f}, X_{mkjl}^{f}, X_{mljk}^{r}, X_{mljki}^{r}, \\ X_{mljko}^{r}, X_{mljkr}^{r}, U_{ml}, W_{ml} \end{matrix} \middle| y_{ih}^{p}, y_{jh}^{d}, y_{kh}^{r}, y_{oh}^{x}, y_{rh}^{e}\right)$ is a minimization problem that finds the optimum values for the continuous variables ($X_{mijl}^{f}, X_{mkjl}^{f}, X_{mljk}^{r}, X_{mljki}^{r},$

$X^r_{mljko}, X^r_{mljkr}, U_{ml}, W_{ml}$) for the fixed values of $(\hat{y}^p_{ih}, \hat{y}^d_{jh}, \hat{y}^r_{kh}, \hat{y}^x_{oh}, \hat{y}^e_{rh})$. This problem can be developed as follows:

$$\min \sum_{m\in M}\sum_{i\in I}\sum_{j\in J}\sum_{l\in L} C^f_{mijl} d_{ml} X^f_{mijl} + \sum_{m\in M}\sum_{k\in K}\sum_{j\in J}\sum_{l\in L} C^f_{mkjl} d_{ml} X^f_{mkjl} + \sum_{m\in M}\sum_{l\in L}\sum_{j\in J}\sum_{k\in K} C^r_{mljk} r_{ml} X^r_{mljk} + \sum_{m\in M}\sum_{l\in L}\sum_{j\in J}\sum_{k\in K}\sum_{i\in I} C^r_{mljki} r_{ml} X^r_{mljki} + \sum_{m\in M}\sum_{l\in L}\sum_{j\in J}\sum_{k\in K}\sum_{o\in O} C^r_{mljko} r_{ml} X^r_{mljko} + \sum_{m\in M}\sum_{l\in L}\sum_{j\in J}\sum_{k\in K}\sum_{r\in R} C^r_{mljkr} r_{ml} X^r_{mljkr} + \sum_{m\in M}\sum_{l\in L} C^u_{ml} d_{ml} U_{ml} + \sum_{m\in M}\sum_{l\in L} C^w_{ml} r_{ml} W_{ml} \tag{18}$$

$$\sum_{l\in L}\sum_{j\in J} X^r_{mljk} r_{ml} \le \sum_{l\in L}\sum_{j\in J} X^f_{mkjl} d_{ml} \qquad \forall m \in M, k \in K \tag{19}$$

$$\sum_{l\in L}\sum_{j\in J} X^r_{mljk} r_{ml} \ge \sum_{l\in L}\sum_{j\in J} X^f_{mkjl} d_{ml} \qquad \forall m \in M, k \in K \tag{20}$$

$$\sum_{i\in I}\sum_{j\in J} X^f_{mijl} + \sum_{k\in K}\sum_{j\in J} X^f_{mkjl} + U_{ml} \le 1 \qquad \forall m \in M, l \in L \tag{21}$$

$$\sum_{i\in I}\sum_{j\in J} X^f_{mijl} + \sum_{k\in K}\sum_{j\in J} X^f_{mkjl} + U_{ml} \ge 1 \qquad \forall m \in M, l \in L \tag{22}$$

$$\sum_{j\in J}\sum_{k\in K}\Big(\sum_{i\in I} X^r_{mljki} + \sum_{o\in O} X^r_{mljko} + \sum_{r\in R} X^r_{mljkr} + X^r_{mljk}\Big) + W_{ml} \le 1 \quad \forall m \in M, l \in L \tag{23}$$

$$\sum_{j\in J}\sum_{k\in K}\Big(\sum_{i\in I} X^r_{mljki} + \sum_{o\in O} X^r_{mljko} + \sum_{r\in R} X^r_{mljkr} + X^r_{mljk}\Big) + W_{ml} \ge 1 \quad \forall m \in M, l \in L \tag{24}$$

$$\sum_{j\in J}\sum_{k\in K}\sum_{l\in L} X^r_{mljki} r_{ml} \le \sum_{j\in J}\sum_{l\in L} X^f_{mijl} d_{ml} \qquad \forall m \in M, i \in I \tag{25}$$

$$\gamma_m\left(X^r_{mljk} + \sum_{i\in I} X^r_{mljki} + \sum_{o\in O} X^r_{mljko} + \sum_{r\in R} X^r_{mljkr}\right) \le \sum_{o\in O} X^r_{mljko} \quad \forall m \in M, l \in L, j \in J, k \in K \tag{26}$$

$$\beta_m\left(X^r_{mljk} + \sum_{i\in I} X^r_{mljki} + \sum_{o\in O} X^r_{mljko} + \sum_{r\in R} X^r_{mljkr}\right) \ge X^r_{mljk} \quad \forall m \in M, l \in L, j \in J, k \in K \tag{27}$$

$$\eta_m\left(X^r_{mljk} + \sum_{i\in I} X^r_{mljki} + \sum_{o\in O} X^r_{mljko} + \sum_{r\in R} X^r_{mljkr}\right) \ge \sum_{r\in R} X^r_{mljkr} \quad \forall m \in M, l \in L, j \in J, k \in K \tag{28}$$

$$\sum_{j\in J} X^f_{mijl} d_{ml} \le \sum_{h\in H} cap^p_{ih} \hat{Y}^p_{ih} \qquad \forall m \in M, l \in L, i \in I, \tag{29}$$

$$\sum_{i\in I} X^f_{mijl} d_{ml} + \sum_{k\in k} X^f_{mkjl} d_{ml} \le \sum_{h\in H} cap^d_{jh} \hat{Y}^d_{jh} \qquad \forall m \in M, l \in L, j \in J \tag{30}$$

$$\sum_{j\in J} X^r_{mljk} r_{ml} + \sum_{j\in J}\sum_{i\in I} X^r_{mljki} r_{ml} + \sum_{j\in J}\sum_{o\in O} X^r_{mljko} r_{ml} + \sum_{j\in J}\sum_{r\in R} X^r_{mljkr} r_{ml} \le \sum_{h\in H} cap^r_{kh} \hat{Y}^r_{kh} \qquad \forall m \in M, k \in K, l \in L \tag{31}$$

$$\sum_{k\in K} X^r_{mljk} r_{ml} + \sum_{k\in K}\sum_{i\in I} X^r_{mljki} r_{ml} + \sum_{k\in K}\sum_{o\in O} X^r_{mljko} r_{ml} + \sum_{k\in K}\sum_{r\in R} X^r_{mljkr} r_{ml} \le \sum_{h\in H} cap^d_{rjh} \hat{Y}^d_{jh} \qquad \forall m \in M, j \in J, l \in L \tag{32}$$

$$\sum_{j\in J}\sum_{k\in K} X^r_{mljki} r_{ml} \le \sum_{h\in H} cap^p_{rih} \hat{Y}^p_{rih} \qquad \forall m \in M, i \in I, l \in L \tag{33}$$

$$\sum_{j\in J}\sum_{k\in K} X^r_{mljko} r_{ml} \le \sum_{h\in H} cap^x_{oh} \hat{Y}^x_{oh} \qquad \forall m \in M, o \in O, l \in L \tag{34}$$

$$\sum_{j\in J}\sum_{k\in K} X^r_{mljkr} r_{ml} \le \sum_{h\in H} cap^e_{rh} \hat{Y}^e_{rh} \qquad \forall m \in M, r \in R, l \in L \tag{35}$$

$$0 \le X^f_{mijl}, X^f_{mkjl}, X^r_{mljki}, X^r_{mljk}, X^r_{mljko}, U_{ml}, X_{ml} \le 1 \quad \forall m \in M, i \in I, j \in J, k \in K, r \in R, l \in L \tag{36}$$

In this model the constraints that were in the form of equality have been replaced with two inequality constraints to facilitate the process of developing the dual sub-problem.

3.1.1 Dual sub-problem

In order to generate cuts to add to the master problem, the dual of the sub-problem is used.

In order to obtain the dual of sub-problem dual variables π^{1}_{mk} ، π^{2}_{mk} ، π^{3}_{ml} ، π^{4}_{ml} ، π^{5}_{ml} ، π^{6}_{ml} ، π^{7}_{mi} ، π^{8}_{mljk} ، π^{9}_{mljk} ، π^{10}_{mljk} ، π^{11}_{mli} ، π^{12}_{mlj} ، π^{13}_{mkl} ، π^{14}_{mjl} ، π^{15}_{mil} ، π^{16}_{mol} ، π^{17}_{mrl} for constraints (2) to (18) are introduced. Using these dual variables, the dual of the sub-problem, $DBSP(X^{f}_{mijl}, X^{f}_{mkjl}, X^{r}_{mljk}, X^{r}_{mljki}, X^{r}_{mljko}, X^{r}_{mljkr}, U_{ml}, W_{ml} | y^{p}_{ih}, y^{d}_{jh}, y^{r}_{kh}, y^{x}_{oh}, y^{e}_{rh})$ is developed as:

$$
\begin{aligned}
\max - &\sum_{m\in M}\sum_{l\in L}\pi^{3}_{ml} + \sum_{m\in M}\sum_{l\in L}\pi^{4}_{ml} - \sum_{m\in M}\sum_{l\in L}\pi^{5}_{ml} + \sum_{m\in M}\sum_{l\in L}\pi^{6}_{ml} \\
&- \sum_{m\in M}\sum_{l\in L}\sum_{i\in I}\sum_{h\in H} cap^{p}_{ih}\hat{Y}^{p}_{ih}\,\pi^{11}_{mli} - \sum_{m\in M}\sum_{l\in L}\sum_{j\in J}\sum_{h\in H} cap^{d}_{jh}\hat{Y}^{d}_{jh}\,\pi^{12}_{mlj} \\
&- \sum_{m\in M}\sum_{l\in L}\sum_{k\in K}\sum_{h\in H} cap^{r}_{kh}\hat{Y}^{r}_{kh}\,\pi^{13}_{mkl} - \sum_{m\in M}\sum_{l\in L}\sum_{j\in J}\sum_{h\in H} cap^{d}_{rjh}\hat{Y}^{d}_{jh}\,\pi^{14}_{mjl} \\
&- \sum_{m\in M}\sum_{l\in L}\sum_{i\in I}\sum_{h\in H} cap^{p}_{rih}\hat{Y}^{p}_{rih}\,\pi^{15}_{mil} - \sum_{m\in M}\sum_{l\in L}\sum_{o\in O}\sum_{h\in H} cap^{x}_{oh}\hat{Y}^{x}_{oh}\,\pi^{16}_{mol} \\
&- \sum_{m\in M}\sum_{l\in L}\sum_{r\in R}\sum_{h\in H} cap^{e}_{rh}\hat{Y}^{e}_{rh}\,\pi^{17}_{mrl}
\end{aligned}
\tag{37}
$$

$$-\pi^{3}_{ml} + \pi^{4}_{ml} + d_{ml}\pi^{7}_{mi} - d_{ml}\pi^{11}_{mli} - d_{ml}\pi^{12}_{mlj} \le C^{f}_{mijl}d_{ml} \quad \forall m,i,j,l \tag{38}$$

$$d_{ml}\pi^{1}_{mk} - d_{ml}\pi^{2}_{mk} - \pi^{3}_{ml} + \pi^{4}_{ml} - d_{ml}\pi^{12}_{mlj} \le C^{f}_{mkjl}d_{ml} \quad \forall m,k,j,l \tag{39}$$

$$
\begin{aligned}
-r_{ml}\pi^{1}_{mk} + r_{ml}\pi^{2}_{mk} - \pi^{5}_{ml} + \pi^{6}_{ml} - \gamma_m\pi^{8}_{mljk} + (\beta_m - 1)\pi^{9}_{mljk} + \eta_m\pi^{10}_{mljk} - r_{ml}\pi^{13}_{mkl} - r_{ml}\pi^{14}_{mjl} \\
\le C^{r}_{mljk}r_{ml} \quad \forall m,l,j,k
\end{aligned}
\tag{40}
$$

$$
\begin{aligned}
-\pi^{5}_{ml} + \pi^{6}_{ml} - r_{ml}\pi^{7}_{mi} - \gamma_m\pi^{8}_{mljk} + \beta_m\pi^{9}_{mljk} + \eta_m\pi^{10}_{mljk} - r_{ml}\pi^{13}_{mkl} - r_{ml}\pi^{14}_{mjl} - r_{ml}\pi^{15}_{mil} \\
\le C^{r}_{mljki}r_{ml} \quad \forall m,l,j,k,i
\end{aligned}
\tag{41}
$$

$$
\begin{aligned}
-\pi^{5}_{ml} + \pi^{6}_{ml} - (\gamma_m - 1)\pi^{8}_{mljk} + \beta_m\pi^{9}_{mljk} + \eta_m\pi^{10}_{mljk} - r_{ml}\pi^{13}_{mkl} - r_{ml}\pi^{14}_{mjl} - r_{ml}\pi^{16}_{mol} \\
\le C^{r}_{mljko}r_{ml} \quad \forall m,l,j,k,o
\end{aligned}
\tag{42}
$$

$$
\begin{aligned}
-\pi^{5}_{ml} + \pi^{6}_{ml} - \gamma_m\pi^{8}_{mljk} + \beta_m\pi^{9}_{mljk} + (\eta_m - 1)\pi^{10}_{mljk} - r_{ml}\pi^{13}_{mkl} - r_{ml}\pi^{14}_{mjl} - r_{ml}\pi^{17}_{mrl} \\
\le C^{r}_{mljkr}r_{ml} \quad \forall m,l,j,k,r
\end{aligned}
\tag{43}
$$

$$-\pi^{3}_{ml} + \pi^{4}_{ml} \le C^{u}_{ml}d_{ml} \quad \forall m,l \tag{44}$$

$$-\pi^{5}_{ml} + \pi^{6}_{ml} \le C^{w}_{ml}r_{ml} \quad \forall m,l \tag{45}$$

1.2 Benders master problem

The benders master problem is obtained as follows:

$$\min_{y^{p}_{ih}, y^{d}_{jh}, y^{r}_{kh}, y^{x}_{oh}, y^{e}_{rh}} z \tag{46}$$

Subject to: (47)

$$
\begin{aligned}
z \ge &\sum_{i\in I}\sum_{h\in H} f^{p}_{ih}y^{p}_{ih} + \sum_{j\in J}\sum_{h\in H} f^{d}_{jh}y^{d}_{jh} + \sum_{k\in K}\sum_{h\in H} f^{r}_{kh}y^{r}_{kh} + \sum_{o\in O}\sum_{h\in H} f^{x}_{oh}y^{x}_{oh} + \sum_{r\in R}\sum_{h\in H} f^{e}_{rh}y^{e}_{rh} - \sum_{m\in M}\sum_{l\in L}\hat{\pi}^{3k'}_{ml} \\
&+ \sum_{m\in M}\sum_{l\in L}\hat{\pi}^{4k'}_{ml} - \sum_{m\in M}\sum_{l\in L}\hat{\pi}^{5k'}_{ml} + \sum_{m\in M}\sum_{l\in L}\hat{\pi}^{6k'}_{ml} \\
&- \sum_{m\in M}\sum_{l\in L}\sum_{i\in I}\sum_{h\in H} cap^{p}_{ih}\hat{Y}^{p}_{ih}\,\hat{\pi}^{11k'}_{mli} - \sum_{m\in M}\sum_{l\in L}\sum_{j\in J}\sum_{h\in H} cap^{d}_{jh}\hat{Y}^{d}_{jh}\,\hat{\pi}^{12k'}_{mlj} \\
&- \sum_{m\in M}\sum_{l\in L}\sum_{k\in K}\sum_{h\in H} cap^{r}_{kh}\hat{Y}^{r}_{kh}\,\hat{\pi}^{13k'}_{mkl} - \sum_{m\in M}\sum_{l\in L}\sum_{j\in J}\sum_{h\in H} cap^{d}_{rjh}\hat{Y}^{d}_{kh}\,\hat{\pi}^{14k'}_{mjl} \\
&- \sum_{m\in M}\sum_{l\in L}\sum_{i\in I}\sum_{h\in H} cap^{p}_{rih}\hat{Y}^{p}_{ih}\,\hat{\pi}^{15k'}_{mil} - \sum_{m\in M}\sum_{l\in L}\sum_{o\in O}\sum_{h\in H} cap^{x}_{oh}\hat{Y}^{x}_{oh}\,\hat{\pi}^{16k'}_{mol} \\
&- \sum_{m\in M}\sum_{l\in L}\sum_{r\in R}\sum_{h\in H} cap^{e}_{rh}\hat{Y}^{e}_{rh}\,\hat{\pi}^{17k'}_{mrl} \quad \forall k' = 1, \dots, \acute{K}
\end{aligned}
\tag{48}
$$

$$
\begin{aligned}
-\sum_{m\in M}\sum_{l\in L}\hat{\pi}_{ml}^{3l'} + \sum_{m\in M}\sum_{l\in L}\hat{\pi}_{ml}^{4l'} - \sum_{m\in M}\sum_{l\in L}\hat{\pi}_{ml}^{5l'} + \sum_{m\in M}\sum_{l\in L}\hat{\pi}_{ml}^{6l'} \\
- \sum_{m\in M}\sum_{l\in L}\sum_{i\in I}\sum_{h\in H} cap_{ih}^{p}\hat{Y}_{ih}^{p}\hat{\pi}_{mli}^{11l'} - \sum_{m\in M}\sum_{l\in L}\sum_{j\in J}\sum_{h\in H} cap_{jh}^{d}\hat{Y}_{jh}^{d}\hat{\pi}_{mlj}^{12l'} \\
- \sum_{m\in M}\sum_{l\in L}\sum_{k\in K}\sum_{h\in H} cap_{kh}^{r}\hat{Y}_{kh}^{r}\hat{\pi}_{mkl}^{13l'} - \sum_{m\in M}\sum_{l\in L}\sum_{j\in J}\sum_{h\in H} cap_{rjh}^{d}\hat{Y}_{jh}^{d}\hat{\pi}_{mjl}^{14l'} \\
- \sum_{m\in M}\sum_{l\in L}\sum_{i\in I}\sum_{h\in H} cap_{rih}^{p}\hat{Y}_{ih}^{p}\hat{\pi}_{mil}^{15l'} - \sum_{m\in M}\sum_{l\in L}\sum_{o\in O}\sum_{h\in H} cap_{oh}^{x}\hat{Y}_{oh}^{x}\hat{\pi}_{mol}^{16l'} \\
- \sum_{m\in M}\sum_{l\in L}\sum_{r\in R}\sum_{h\in H} cap_{rh}^{e}\hat{Y}_{rh}^{e}\hat{\pi}_{mrl}^{17l'} \le 0 \quad \forall l' = 1,\dots,\hat{L}
\end{aligned} \tag{49}
$$

$$\sum_{h} y_{ih}^{p} \le 1 \quad \forall i \tag{50}$$

$$\sum_{h} y_{jh}^{d} \le 1 \quad \forall j \tag{51}$$

$$\sum_{h} y_{kh}^{r} \le 1 \quad \forall k \tag{52}$$

$$\sum_{h} y_{oh}^{x} \le 1 \quad \forall o \tag{53}$$

$$\sum_{h} y_{rh}^{e} \le 1 \quad \forall r \tag{54}$$

In this model constraint (46) is the objective function of benders master problem. Constraint (47) is the optimality cut which is introduced to the master problem if the sub-problem is solved to optimality. Parameters $\hat{\pi}_{mk}^{1k'}$ ‘$\hat{\pi}_{mk}^{2k'}$ ‘ $\hat{\pi}_{ml}^{3k'}$ ‘$\hat{\pi}_{ml}^{4k'}$ ‘$\hat{\pi}_{ml}^{5k'}$ ‘$\hat{\pi}_{ml}^{6k'}$ ‘$\hat{\pi}_{mi}^{7k'}$ ‘$\hat{\pi}_{mljk}^{8k'}$ ‘$\hat{\pi}_{mljk}^{9k'}$ ‘$\hat{\pi}_{mljk}^{10k'}$ ‘$\hat{\pi}_{mli}^{11k'}$ ‘ $\hat{\pi}_{mlj}^{12k'}$ ‘$\hat{\pi}_{mkl}^{13k'}$ ‘$\hat{\pi}_{mjl}^{14k'}$ ‘$\hat{\pi}_{mil}^{15k'}$ ‘$\hat{\pi}_{mol}^{16k'}$ ‘$\hat{\pi}_{mrl}^{17k'}$ are the values of dual variables obtained by solving benders dual sub-problem. Constraint (48) is the feasibility cut which is added to the master problem, that is the sub-problem, which is infeasible.

3.1.2 Overall procedure of benders decomposition method

The pseudo code for the overall procedure of Benders decomposition is presented in Fig. 2.

As it is shown in this figure, the procedure starts with an initial feasible solution for the master problem. This can be done by solving the problem without any additional cuts. Then the obtained solution for the master problem is given to the sub-problem, if the sub-problem is infeasible, i.e. the dual sub-problem is unbounded, an unbounded ray is used to generate an infeasibility cut to add to the master problem for the next iteration. If the sub-problem is feasible and solved to optimality, using the optimal solution obtained, an optimality cut is generated and added to the master problem for the next iteration. If the obtained solution provides a better upper bound, the upper bound is updated. Then the master problem is solved again. This process is repeated until the gap between the lower bound and upper bound is lower than a specified value.

This algorithm is developed in GAMS 23.1 and used to solve a numerical instance of the problem and the result is compared to the mathematical model, which is also solved using GAMS 23.1. The results are presented in Table 1. According to this table solving the mathematical model directly require 83.04 seconds to obtain the optimal solution, but using the benders decomposition method, this time can be reduced to 17.428 seconds. Considering these times, one can conclude that the differences between them are not admissible. However, increasing the size of the problem will lead to a greater gap for times and in such situation, the proposed benders will outperform the traditional solver.

Fig. 3 shows the upper bound and lower bounds obtained by Benders decomposition method in different iterations. As you can see in this figure, the algorithm reaches the optimal solution after 11 iterations.

4. Computational Results

In this section several instances with different number of products and hybrid distribution-collection centers are solved using the benders decomposition method. Then managerial insights are discussed. The considered instance is limited to 6 candidate locations for manufacturing/remanufacturing plants, 10 candidate locations for hybrid distribution-collection centers, 7 candidate locations for CRCs, 2 candidate locations for disposal centers and 3 candidate locations for recycling centers, 20 customer points and two products.

4.1 Single commodity network

In this section an instance of the problem with a single product is considered. The data for this instance is generated randomly and the problem is solved using the proposed benders decomposition method. After 28 iterations of the algorithms, the optimal cost of the network is obtained as 8589342671353 monetary units. The convergence of the algorithm for this instance is presented in Fig. 4. Moreover, in order to compare the results of the proposed method and mathematical model, they are presented in Table 2.

{initialization}
(U, V) = initial feasible integer solution
LB := -∞
UB := +∞
$L' = K' = 0$
while (UB – LB > ε) Do
{solve subproblem}
if (Subproblem is Unbounded) then
Get unbounded ray π
Add a feasibility cut to master problem
$L' := L' + 1;$
Else
Get extreme point π
Add an optimality cut to master problem
$K' := K' + 1;$

$$\text{UB} := \min\{\text{UB}, \sum_{i\in I}\sum_{h\in H} f_{ih}^{p} y_{ih}^{p} + \sum_{j\in J}\sum_{h\in H} f_{jh}^{d} y_{jh}^{d} + \sum_{k\in K}\sum_{h\in H} f_{kh}^{r} y_{kh}^{r} + \sum_{o\in O}\sum_{h\in H} f_{oh}^{x} y_{oh}^{x} + \sum_{r\in R}\sum_{h\in H} f_{rh}^{e} y_{rh}^{r}$$
$$- \sum_{m\in M}\sum_{l\in L} \hat{\pi}_{ml}^{3k'} + \sum_{m\in M}\sum_{l\in L} \hat{\pi}_{ml}^{4k'} - \sum_{m\in M}\sum_{l\in L} \hat{\pi}_{ml}^{5k'} + \sum_{m\in M}\sum_{l\in L} \hat{\pi}_{ml}^{6k'} - \sum_{m\in M}\sum_{l\in L}\sum_{i\in I}\sum_{h\in H} cap_{ih}^{p} \hat{Y}_{ih}^{p} \hat{\pi}_{mli}^{11k'}$$
$$- \sum_{m\in M}\sum_{l\in L}\sum_{j\in J}\sum_{h\in H} cap_{jh}^{d} \hat{Y}_{jh}^{d} \hat{\pi}_{mlj}^{12k'} - \sum_{m\in M}\sum_{l\in L}\sum_{k\in K}\sum_{h\in H} cap_{kh}^{r} \hat{Y}_{kh}^{d} \hat{\pi}_{mkl}^{13k'} - \sum_{m\in M}\sum_{l\in L}\sum_{j\in J}\sum_{h\in H} cap_{rjh}^{d} \hat{Y}_{kh}^{d} \hat{\pi}_{mjl}^{14k'}$$
$$- \sum_{m\in M}\sum_{l\in L}\sum_{i\in I}\sum_{h\in H} cap_{rih}^{p} \hat{Y}_{rih}^{p} \hat{\pi}_{mil}^{15k'} - \sum_{m\in M}\sum_{l\in L}\sum_{o\in O}\sum_{h\in H} cap_{oh}^{x} \hat{Y}_{oh}^{x} \hat{\pi}_{mol}^{16k'} - \sum_{m\in M}\sum_{l\in L}\sum_{r\in R}\sum_{h\in H} cap_{rh}^{e} \hat{Y}_{rh}^{e} \hat{\pi}_{mrl}^{17k'}\}$$

end if
{solve master problem}
LB := $\overline{z}$ //result of master problem
end while

Fig. 2. Procedure of Benders decomposition method

Table 1
Results of solving a numerical example

Solution method	Running time (s)	Objective function
Mathematical model	83.045	10066435179182
Benders Decomposition	17.428	10066435179182

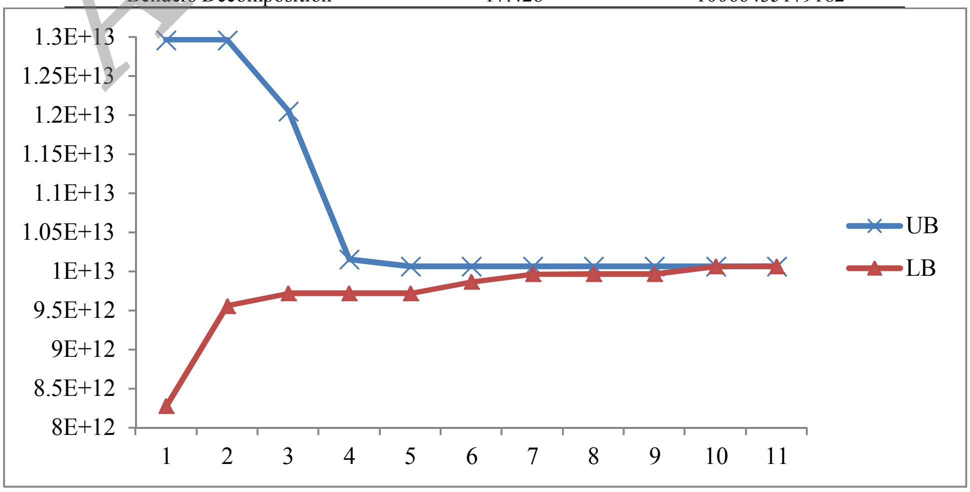

Fig. 3. Convergence of the benders decomposition method

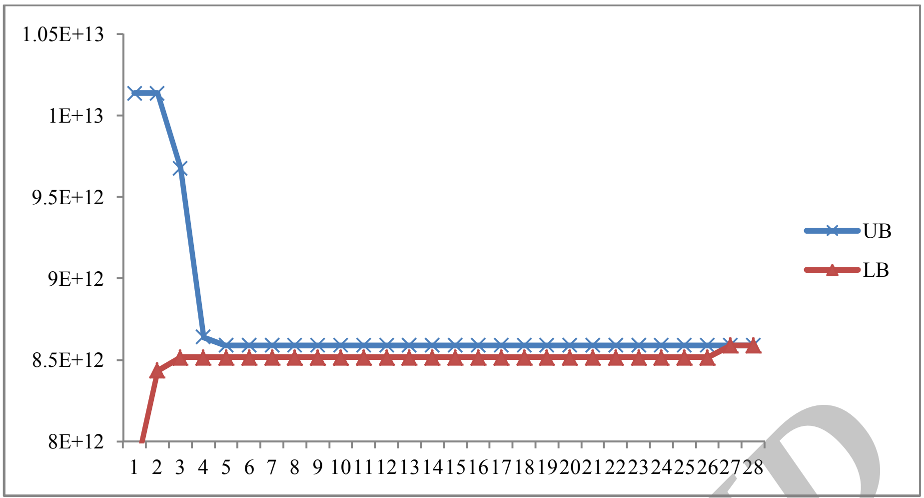

Fig. 4. Convergence of the algorithm for the single commodity network

Table 2
Results for the single commodity network

Solution method	Running time (s)	Objective function
Mathematical model	51.141	8589342671353
Benders Decomposition	33.075	8589342671353

The percentage of demands of the customers that are satisfied by different manufacturing plants and CRCs is presented in Fig. 5. In this figure the horizontal axis shows the customers while the vertical axis shows the percentage of demand of each customer that is satisfied by a manufacturer or CRC. The percentage of returns retrieved by different manufacturing plants, disposal centers, recycling centers and CRCs is presented in Fig. 6.

4.2 Multi-commodity network

In this section an instance of the problem with multiple products is considered. The data for this instance is generated randomly and the problem is solved using the proposed benders decomposition method. After 17 iterations of the algorithms, the optimal cost of the network is obtained as 17178626108393 monetary units. The convergence of the algorithm for this instance is presented in Fig. 7. Moreover, in order to compare the results of the proposed method and mathematical model, their results for this instance is presented in Table 3.

In order to further investigation, the distribution of the satisfied demand and returns, the percentages of the demands and returns for different products are presented in Figures 8 through 11.

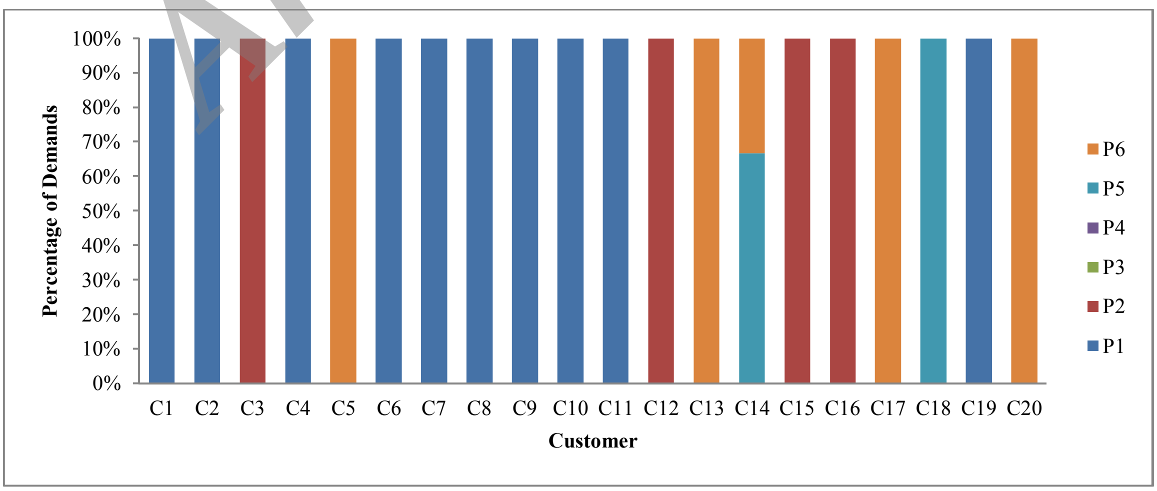

Fig. 5. Percentage of demands satisfied by manufacturing plant and CRCs

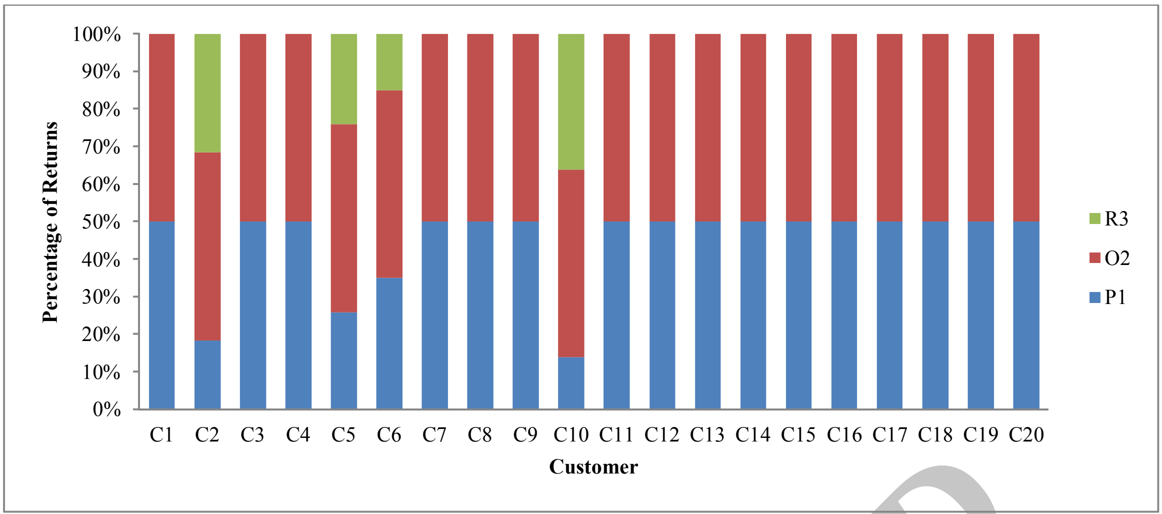

Fig. 6. Percentage of returns satisfied by different facilities

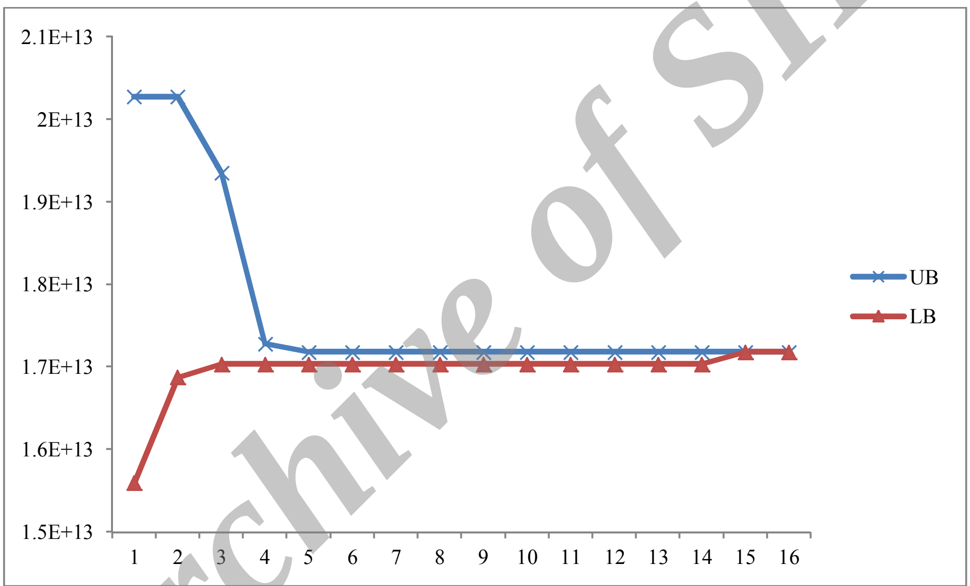

Fig. 7. Convergence of the algorithm for the multiple commodity network

Table 3
Results for the multiple commodity network

Solution method	Running time (s)	Objective function
Mathematical model	96.678	17178626108393
Benders Decomposition	29.167	17178626108393

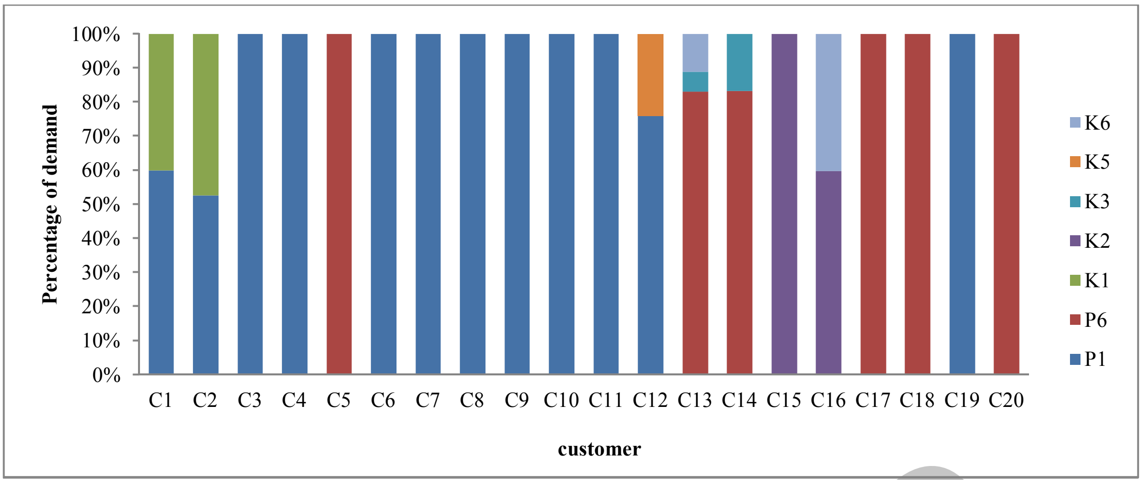

Fig. 8. Percentage of demands satisfied by manufacturing plant and CRCs for product 1

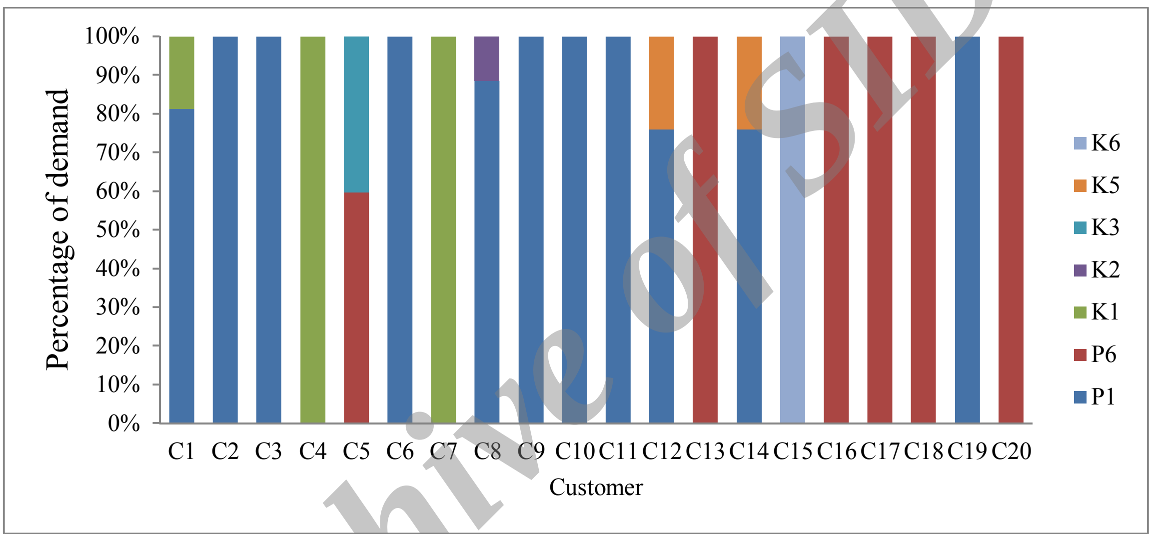

Fig. 9. Percentage of demands satisfied by manufacturing plant and CRCs for product 2

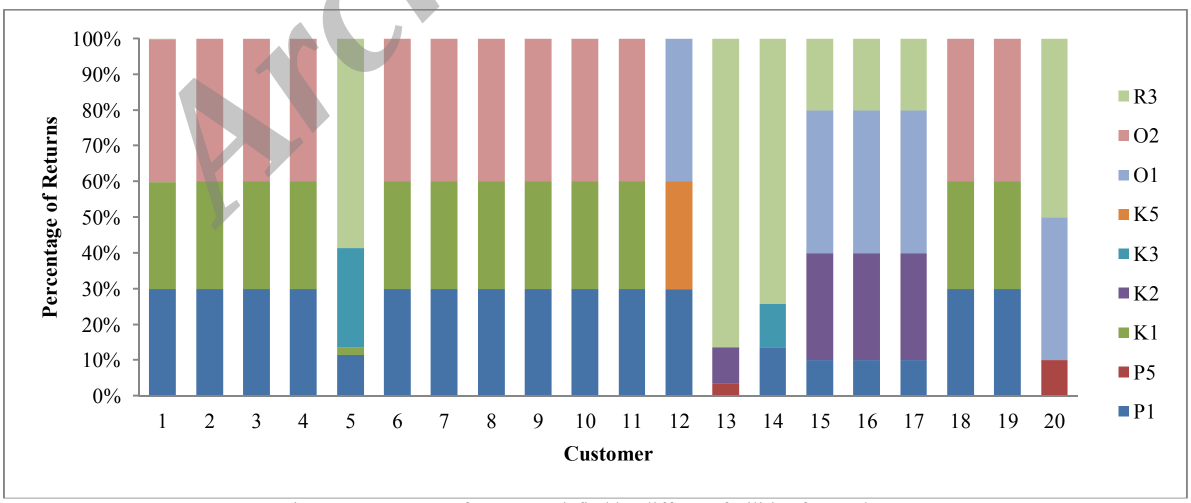

Fig. 10. Percentage of returns satisfied by different facilities for product 1

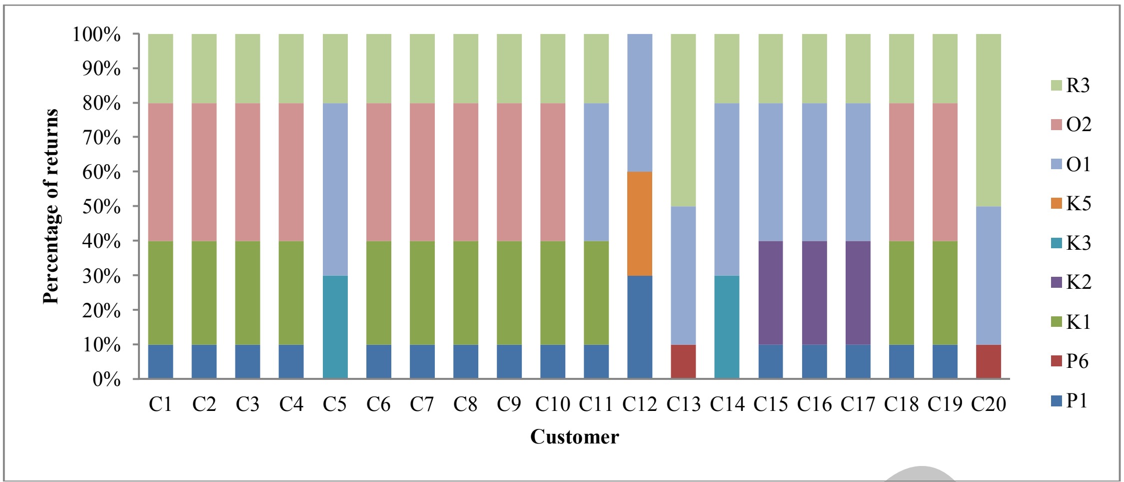

Fig. 11. Percentage of returns satisfied by different facilities for product 2

5. Conclusions and Suggestions for Future Research

In this paper a new model is presented for the supply chain network designing problem with multiple capacities for facilities and various options for product recovery. In most research papers in the field of logsitics network desiging problem, only a limited number of options for product recovery recied attention. Many of them consider only one option of remanufacturing. Moreover, many research papers assume that there is only one option for the capacity of the facilities in the network . However, this is not a realistic assumption because in most of the real world cases you always have many options for the capacity of the facilities in the network. In this paper a new model with several product recovery options and multiple options for product recovery is proposed and a methematical model is developed to solve this problem. Moreover, in order to efficiently solve this problem, a benders decomposition method is developed. The computational results show the efficiency of the proposed method.

In this paper the gap between the real world logistics and the research literature on logistics network designing problems is reduced by considering multiple options for capacity of the facilities and many product recovery options. However this research also has some limitations that call for further research in this area. One of the limitations of this study is negelecting the dynamic nature of the logistics network. In real world logistics, the demands and returns of the customers change throughout different periods. Therefore, in order to obtain a more realistic model it is suggested to consider a dynamic model for the problem considered in this paper. Moreover, considering special conditions for establishing different facilities is another suggestion for extending the current research. For example, if the demand assigned for a specific facility is lower than a predefined level, this facility should not be established. By adding this condition and other similar conditions more realistic models can be obtained.

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