Analytical analysis of ultimate ground pressure on tunnel support system

R. Rahmannejad[1*], A.R Kargar[1], V. Maazallahi[2] and E. Ghotbi-Ravandi[3]

1. Mining Engineering Department, Shahid Bahonar University of Kerman, Kerman, Iran
2. School of Mining Engineering, College of Engineering, University of Tehran, Tehran, Iran
3. Mineral Industries Research Center, Shahid Bahonar University of Kerman, Kerman, Iran

Abstract

The ground reaction curve (GRC) is a vital component of the convergence-confinement method, which possesses many applications in the underground space designs. It defines a relation between the tunnel wall deformations and the ground pressure acting on the tunnel walls. Generally, GRC includes descending and ascending branches. According to many researchers, the descending branch trend for the ground pressure stops after the critical deformation, and thus the ground pressure on the support system increases due to the formation of a loosening zone and an ascending branch, and finally, the creation of an ultimate pressure on the support system. In this work, two relations are proposed to determine the ultimate ground pressure acting on a circular tunnel in a continuous medium. It is assumed that the rock mass obeys the elastic perfectly plastic model with a cohesionless behavior in the broken zone. This is accomplished by incorporating the Duncan-Fama solution and the two models of Yanssen-Kötter and Caquot rigid plastic. The ground pressure obtained by the Caquot model shows a better correlation with the Goel-Jethwa equation compared with the Yanssen-Kötter solution.

Keywords: *Ground Reaction Curve (GRC), Ground Pressure, Rigid Plastic Model, Convergence-Confinement Method (CCM), Critical Strain.*

1. Introduction

The convergence-confinement method (CCM) is a 2D method that is used to analyze the interaction between a rock mass and a support system. Different authors such as Pacher (1964), Sulem et al. (1987), Carranza-Torres (2000), and Lombardi (1980) have expanded this method [1-4]. One of the main components of CCM is the ground reaction curve (GRC) or the characteristic line. The characteristic line of a rock mass is a correlation between the ground pressure and the tunnel wall displacement. Depending on the rock mass conditions, GRC can demonstrate the elastic, plastic without perturbed zone, and plastic with perturbed zone behaviors. The latter case is characterized by two descending and ascending branches, which simulate a decrease and an increase in the rock pressure, respectively, with growth in the tunnel wall convergence (Figure 1). These two branches are separated from each other by a critical deformation. Tunnel convergences larger than the critical deformation are accompanied with increasing ground pressure due to a reduction in the intrinsic rock mass strength and the loosening phenomenon.
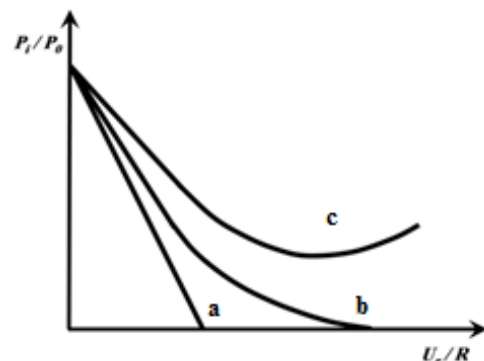

Figure 1. GRC curves for rock masses with (a) elastic, (b) plastic without perturbed zone, and (c) plastic with perturbed zone behaviors.

Singh (1997) has analyzed the monitoring results obtained for the support pressure and convergences of various Indian tunnels, and reported that the critical convergence (minimum ground pressure) is equal to 5 to 6 percent of the tunnel diameter (Figure 2a) [5]. As it can be seen in this figure, the unstable failure propagation begins for the convergences more than the critical convergence value, and the ground pressure increases up to the ultimate value. Singh correlated the increment in the ground pressure after the critical strain to the variation in the rock mass cohesion from its initial value to zero, as shown in Figure 3 [6]. Based on the data for the analysis of the tunnel convergence and ground pressure data obtained from a number of constructed tunnels in Japan within the same depth and similar geological conditions, Tanimoto and Myres-Bohlk (1983) have concluded that the ground pressure acting on the support reduces with increase in the tunnel convergence up to a maximum 2% of the tunnel diameter, and then starts to increase, as shown in Figure 2b [7].
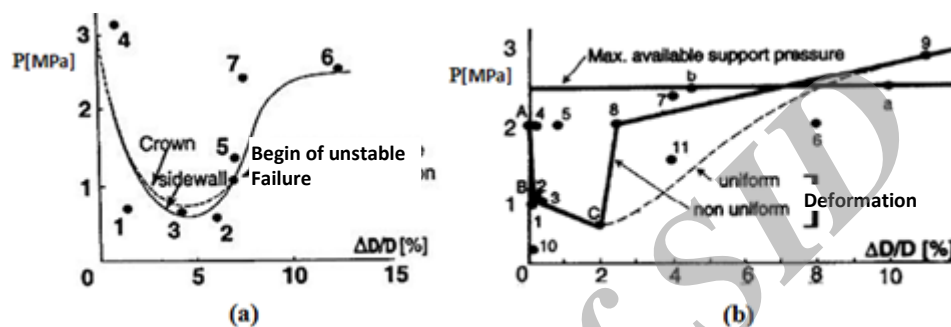

Figure 2. Measured support pressure and tunnel convergence obtained from a number of tunnels in (a) India and (b) Japan [6].
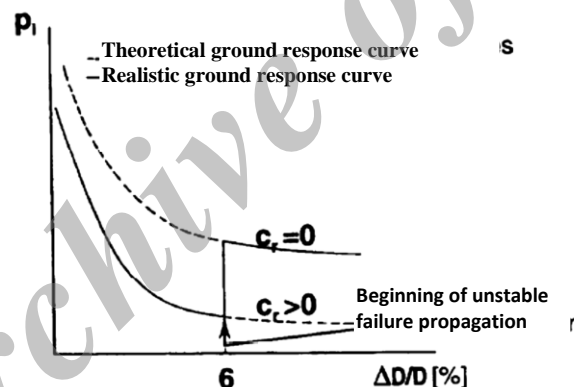

Figure 3. Jump of GRC curve after beginning of unstable failure propagation [6].

Maazallahi and Rahmannejad (2007) have proposed a method for the calculation of the increasing part of the ground reaction curve [8]. Further, based on the same procedure, Rahmannejad and Maazallahi (2010) have presented a method for the determination of the critical strain, and compared the results obtained with those obtained for the Sakurai critical strain [9, 10]. They used the ground reaction curve, based on the elasto-plastic solution of Carranza-Torres-Fairhurst (2000) and the rigid plastic model proposed by Bulychev (1992), to calculate the critical strain [11, 3]. The point of intersection of GRC and the rigid plastic curves has been assumed to be the critical strain. In other words, it has been supposed that the strength reduction of the rock, failure, and creation of the ascending branch begin from this point around the tunnel periphery. Using the same approach, the aim of this work is to determine the ultimate pressure on the roof of the support system of a circular tunnel.

2. Methodology
2.1. Assumptions
In this analytic solution, the following simplifications and assumptions were made.

A circular tunnel was driven in a weak or moderate quality rock mass with an elastic plastic behavior. The surrounding rock mass was assumed to be homogeneous and continuous, and the joint effect was considered using the equivalent deformation module, E.

152

As a result of the tunnel driving elastic plastic deformation, the unstable failure propagation and falling of the surrounding rocks would be observed.

The two constitutive models of elastic perfectly plastic and rigid plastic were used in this work to simulate the elastic plastic deformation and unstable failure propagation, respectively.

2.2. Rigid plastic model

Figure 4. represents the schematic rigid plastic model. The plastic strains begin to develop immediately, without producing significant elastic strains (lines 1 and 2 in Figure 4). According to this model, adjusted to rocks with low cohesions, two broken and undisturbed zones (1 and 2 in Figure 5), respectively, exist around the tunnel.

An example of the rigid plastic model is the well-known loosened arch of Protodiakonov [11].
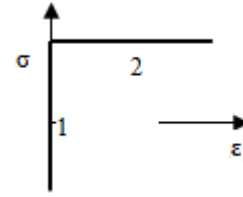

Figure. 4. Stress-strain curve for rigid plastic model.
1- Non-significant elastic strain branch 2- Plastic branch [11].

Two solid plastic models of falling rock column of Yanssen-Kötter and disturbed rock zone of Caquot were used in this study. These models are illustrated in Figure 5.
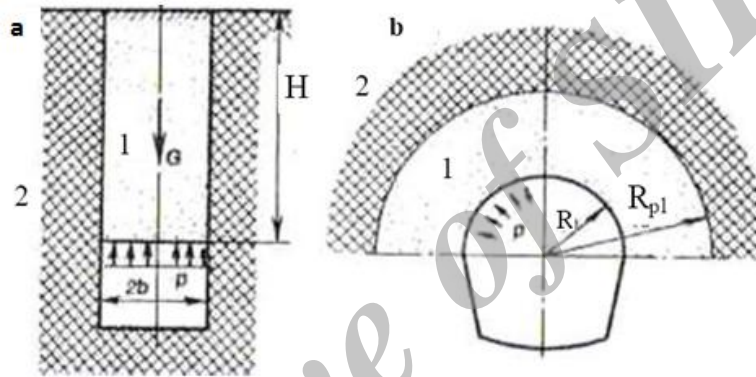

Figure 5. Yanssen-Kötter (a) and Caquot (b) models of rigid plastic [10].
1- Plastic zone 2- Undisturbed zone.

According to the Yanssen-Kötter model, the ground pressure acting on the support system, due to the overburden weight, was obtained as follows [11]:

$$P = \frac{\gamma.b - c}{\lambda \tan\varphi} \left[1 - \exp\left(-\frac{\zeta H}{b} \tan\varphi \right) \right] \quad (1)$$

$$\zeta = \frac{1 - \sin\varphi}{1 + \sin\varphi} \quad (2)$$

where b and H are the span and depth of the tunnel, respectively. C, γ, and φ stand for the cohesion, unit weight, and friction angle of the rock mass.

Based on the Caquot model, the ground pressure acting on the support system, due to the disturbed rock zone around the tunnel periphery, was derived as follows [11]:

$$P = \frac{\gamma.R}{\alpha - 1}\left[1 - \left(\frac{R}{R_{pl}}\right)^{\alpha-1} \right] - c.\left[1 - \left(\frac{R}{R_{pl}}\right)^{\alpha} \right] \cot\phi \quad (3)$$

$$\alpha = \frac{2\sin\varphi}{1 - \sin\varphi} \quad (4)$$

where R and R_{pl} are the radius of the tunnel and the radius of the plastic zone, respectively.

2.3. Duncan-Fama solution

In the Duncan-Fama solution, it has been assumed that a circular tunnel of radius r_0 is excavated under the external hydrostatic field stress p_0 and the internal support pressure p_i (Figure 6). The surrounding rock mass was supposed to obey the Mohr-Coulumb failure criterion. The critical pressure P_{cr}' for transition of the rock mass from the elastic to the plastic behavior, and the elastic radial displacement at the tunnel wall can be obtained as follow [12]:

$$P_{cr}' = \frac{2p_0 - \sigma_{cm}}{1 + k} \quad (5)$$

153

$$U_{ie} = \frac{r_o(1+v)(P_o - P_i)}{E_m} \qquad (6)$$

$$k = \frac{1}{\zeta} \qquad (7)$$

where σ_{cm}, P_0, and E_m are the rock mass compressive strength, in situ stress, and elastic modulus of the rock mass, respectively, and v is the Poisson's ratio. The parameter ξ can be defined by relation 2.
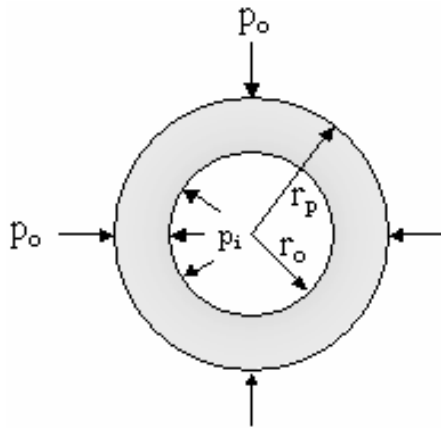

Figure. 6. Plastic zone around tunnel [12].

In the case where the internal support pressure is lower than the critical support pressure ($P_i < P_{cr}'$), the radius of the plastic zone, r_p, is calculated by [12]:

$$r_p = r_\circ \left[\frac{2(P_o(k-1) + \sigma_{cm})}{(1+k)((k-1)P_i + \sigma_{cm})} \right]^{(1/(k-1))} \qquad (8)$$

The total radial displacement of the tunnel wall, due to the plastic failure, can also be determined using the following equation [12]:

$$U_{ip} = r_o \left(\frac{1+v}{E}\right) \left[2(1-v)(P_o - P_{cr})((\frac{r_p}{r_o})^2) \right.$$
$$\left. -(1-2v)(P_o - P_i) \right] \qquad (9)$$

2.4. Analytical solution for ascending branch

The framework for the proposed analytical method is such that the elastic plastic descending branch is initially plotted by the Duncan-Fama solution (Duncan and Fama 1993) using the relations 5-9, after which, the rigid plastic curve would be plotted from the beginning of the plastic strain of the GRC curve (point U_e/R in Figure 7)

[13]. The intersection point of these two curves is given as the critical strain. From this point, the ascending branch is formed, and the ground pressure increases. The ultimate pressure is the final pressure predicted by the ascending branch.

The Yanssen-Kötter and Caquot models were utilized to plot the ascending branch.

According to the Singh idea about the vanishing cohesion of the rock mass in the loosening zone [5], it can be assumed that the cohesion of the rock mass is zero for the Yanssen-Kötter model (Figure 3). Therefore, relation 1 changes to:

$$P = \frac{\gamma.b}{\zeta \tan\varphi} \left[1 - \exp\left(-\frac{\zeta R_{pl}}{R} \tan\varphi \right) \right] \qquad (10)$$

and, for the Caquot model, relation 6 can be used directly.

Then the values for R_{pl}/R, corresponding to the displacement, obtained from the descending ground reaction curve for the Duncan-Fama model, would be calculated in a range from the critical pressure (P_{cr}') to zero and substituted in the rigid plastic equations 3 or 10 to obtain the ground pressure for the Caquot or Yanssen-Kötter model, respectively. Finally, the ascending branch would be plotted by obtaining the ground pressure and the calculated displacement corresponding to the values for R_{pl}/R mentioned in the previous stage. The intersection point of the two curves for the elastic plastic and rigid plastic models is assumed as the critical strain, and the following branch of rigid plastic curve after the critical deformation (U_{cr}) is supposed to be the ascending branch, in which the intrinsic strength of the rock mass reduces, and the rocks would lead to failure (Figure 7).

As an example, the ground reaction curve for a circular tunnel of radius 7 m, driven at a depth of 200 m in a weak rock mass (e.g. RMR = 30), is illustrated in Figure 8. The elastic part has been omitted here.

The ultimate pressures for the ascending branch according to the Yanssen-Kötter and Caquot models are equal to 0.635 MPa ($\frac{P}{p_0} = 0.135$) and 0.319MPa ($\frac{P}{p_0} = 0.068$), respectively.
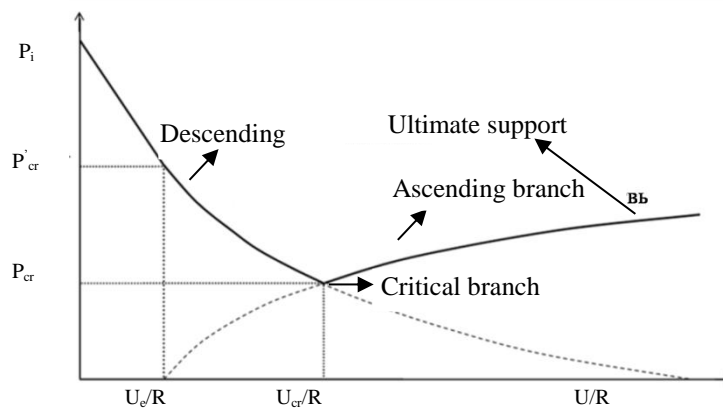
154

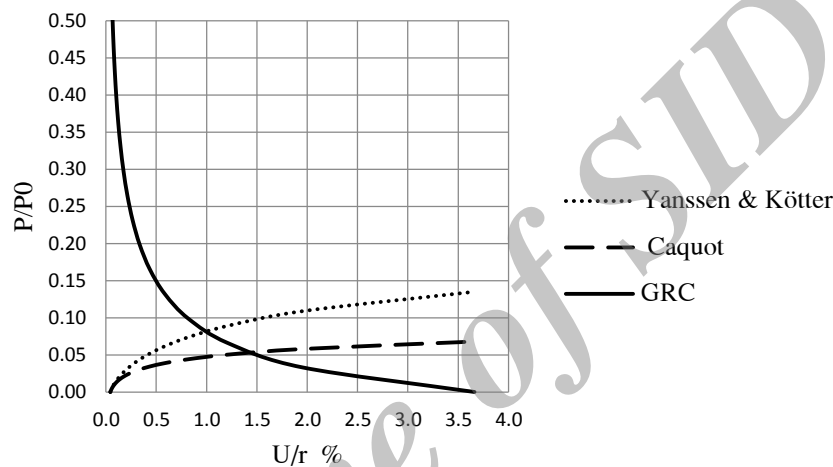**Figure 7. Calculation of ascending branch [8].**

Figure 8. An example of ground reaction curves with an ascending branch.

2.5. Model properties

Three types of rock masses were considered with the RMR values 30, 40, and 50, which describe the rock masses with weak to moderate qualities that obey the elastic perfectly plastic behavior [14]. The mechanical properties of the rock masses with respect to their RMR values as well as the geometric features of the model are given in Table 1.

3. Exploring ultimate pressure on support system

Goel and Jethwa (1991) have proposed the following relation for estimation of the vertical ground pressure [15]:

$$P_v = \frac{7.5B^{0.1}H^{0.5} - RMR}{20RMR}(MPa) \qquad (11)$$

where B is the tunnel span (m), H is the overburden or tunnel depth in meters (> 50 m), and P_v is the short-term roof support pressure in MPa.

This relation was extracted from the results obtained from monitoring 30 tunnels in India, preparing a useful base for comparison of the support pressures given by the empirical and proposed analytical methods.
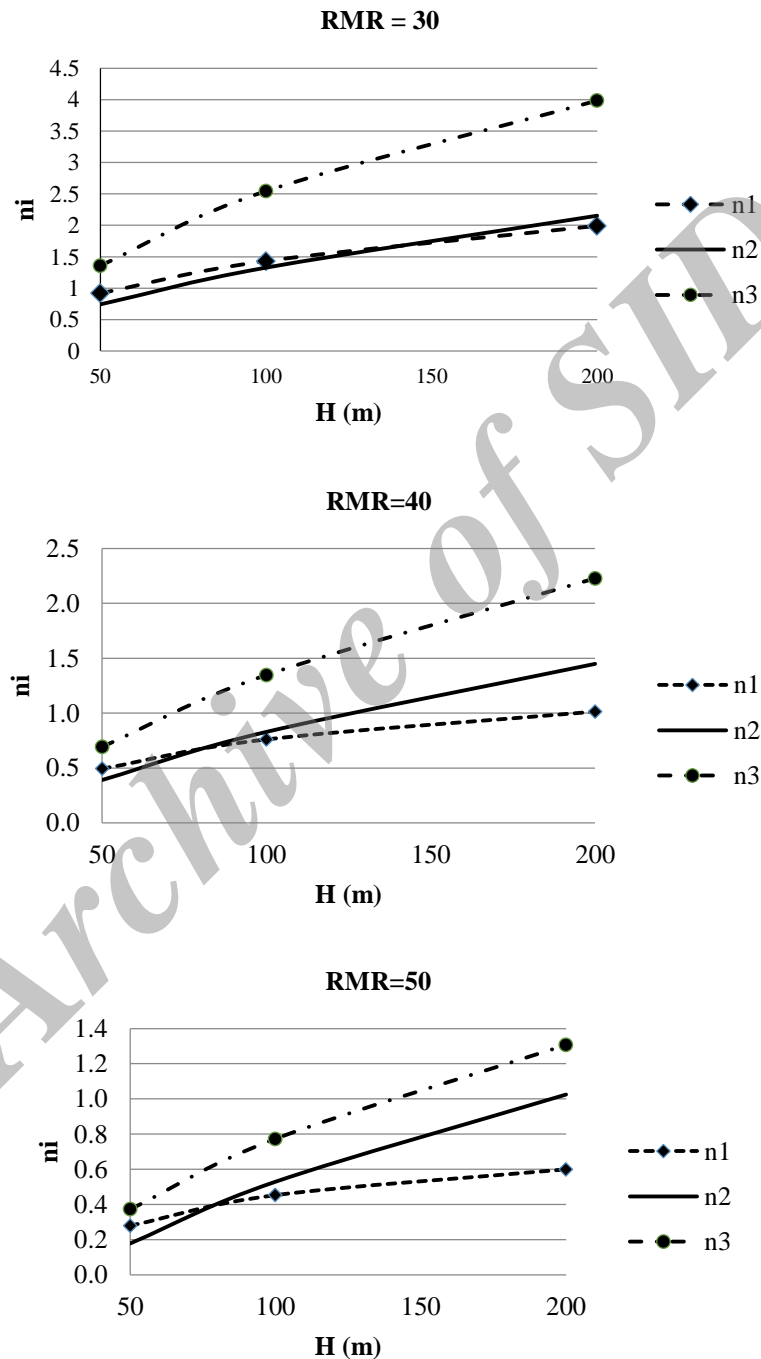
The research procedure was such that the descending and the ascending branches of the ground reaction curve were calculated for various hypothetic conditions. Then the maximum ground pressure for the ascending branch (P) was determined for the supposed conditions.

The results obtained were presented in term of the dimensionless ratio $n_i = P_i/\gamma R$, where the indices 1, 2, and 3 were used for the Caquot, Goel-Jethwa, and Yanssen-Kötter cases, respectively (Figure 9).

According to Figure 9, the Caquot model gave closer results, compared with the Yanssen-Kötter one, to the Goel-Jethwa relation. In all cases, pressure of the overburden weight according to the Yanssen-Kötter model was more than the disturbed rock zone in the Caquot model.

Table 1. Mechanical properties of rock masses.

Radius (m)	Depth (m)	Unit weight (MN/m³)	Poisson's ratio	Deformation modulus (GPa)	Angle of internal friction (°)	Cohesion (MPa)	RMR
3	50	0.025	0.27	10	30	0.25	50
5	100			6	25	0.2	40
	200			5	20	0.15	30

Figure 9. Comparison of ultimate pressures obtained from proposed analytical method and Goel-Jethwa relation.

156

Based on the different combinations of the RMR and tunnel depth values, new correlations are recommended for the dimensionless ratio of $n = p/\gamma R$ for the Caquot and Yanssen-Kötter models, as follow:

$$n_1 = 315.33 RMR^{-2.3} H^{0.52} (MPa) \qquad (12)$$

$$n_3 = 126.81 RMR^{-2.18} H^{0.75} (MPa) \qquad (13)$$

The root mean square error (RMSE) values for these relations were 0.23 and 0.84, respectively.

4. Conclusions

In this work, using an analytical method based on the combination of the elastic plastic and rigid plastic models, two relations were proposed to determine the ultimate ground pressure on the tunnel support system. The proposed relation, driven from the Caquot model, had a better correlation with the Goel-Jethwa empirical relation compared with the Yanssen-Kötter one. Based on the basic assumption made in the Yanssen-Kötter model, considering the overburden weight of the tunnel as the support pressure, the predicted ultimate pressure is always more than the Caquot one.

References

[1]. Pacher, F. (1964). Deformations messungen im Versuchs stollen als Mittel zur Erforschung des Gebirgs verhaltens und zur Bemessung des Ausbaus. Felsmech. u. Ing.Geol. 1: 149-161.

[2]. Sulem, J., Panet, M. and Guenot, A. (1987). An analytical solution for time-dependent displacements in circular tunnel.Int. J. Rock Mech. Min. Sci. &Geomech. Abstr. 24 (3): 155-164.

[3]. Carranza-Torres, C. and Fairhurst, C. (2000). Application of the convergence-confinement method of tunnel design to rock-masses that satisfy the Hoek-Brown failure criterion. Tunnelling Underground Space Technol. 15(2): 187-213.

[4]. Lombardi, G. (1980). Some Comments on the convergence-confinement method. Underground Space4 (4): 249-258.

[5]. Singh, B., Villadkar, M.N., Samadhiya, N.K. and Mehrotra, V.K. (1997). Rock mass strength parameters mobilized in tunnels. Tunnelling Underground Space Technol. 12 (1): 47-54.

[6]. Egger, P. (2000). Design and Construction Aspects of Deep Tunnels (with particular emphasis on strain softening rocks).Tunnelling Underground Space Technol. 15 (4): 403-408.

[7]. Tanimoto, C. and Myres-Bohlke, B. (1983). Allowable limit of convergence in tunnelling. In: Proceedings of the 24th U.S. Symposium on Rock Mechanics (USRMS), June 20- 23.

[8]. Maazallahi, V. and Rahmannejad, R. (2007). Suggestion of mathematical model of increasing part of ground reaction curve.In: Proceedings of the 3rd Iranian Rock Mechanic Conference, Amirkabir University of technology, Tehran, Iran.

[9]. Rahmannejad, R. and Maazallahi, V. (2010). Rigid-plastic modeling approach to critical strain. Min. Sci. 46(5): 35-42.

[10]. Sakurai, S. and Adachi, N. (1988). NATM in Urban tunneling, p7.Kajima shuppankai (in Japanese).

[11]. Bulychev, N.C. (1992). Mechanics of Underground Structures, Nedra, Moscow (In Russian).

[12]. Hoek, E. (2007). Practical Rock Engineering. www.rocscience.com.

[13]. Duncan Fama, M.E. (1993). Numerical modelling of yield zones in weak rocks. In Comprehensive rock engineering, (ed. J.A. Hudson) 2: 49-75. Oxford: Pergamon.

[14]. Barla, G. and Barla, M. (2000). Continuum and discontinuum modeling, Rud.-geol.-naft. z.b., Vol. 12, Zagreb.

[15]. Goel, R.K. and Jethwa, J.L. (1991). Prediction of Support Pressure using RMR Classification, Proc. Indian Getech. Conf, Surat, India, pp. 203-205.

آنالیز تحلیلی فشار نهایی زمین روی سیستم نگهداری تونل

رضا رحمان‌نژاد*[۱]، علی رضا کارگر[۱]، وحید معاذ الهی[۲] و ابراهیم قطبی راوندی[۳]

۱- بخش مهندسی معدن، دانشگاه شهید باهنر کرمان، ایران

۲- دانشکده فنی و مهندسی، دانشکده معدن، دانشگاه تهران، ایران

۳- مرکز تحقیقات صنعتی معدنی، دانشگاه شهید باهنر کرمان، ایران

* نویسنده مسئول مکاتبات: sreza99@uk.ac.ir

چکیده:

منحنی واکنش زمین (GRC) مؤلفه حیاتی روش همگرایی-همجواری محسوب شده و کاربردهای فراوانی در طراحی فضاهای زیرزمینی دارد. این منحنی رابطه بین جابجایی دیوار تونل و فشار زمین اعمال‌شده بر آن را تعریف می‌کند. معمولاً، منحنی واکنش زمین شامل شاخه‌های صعودی و نزولی است. بر طبق نظر بسیاری از محققین، شاخه نزولی فشار زمین با رسیدن به جابجایی بحرانی متوقف‌شده و بعد از آن، فشار زمین روی نگهداری به علت تشکیل ناحیه سست شده و ایجاد شاخه صعودی تا رسیدن به فشار نهایی رو به افزایش می‌گذارد. در این تحقیق، دو رابطه برای تعیین فشار نهایی تونل دایروی روی نگهداری در محیط پیوسته ارائه‌شده است. فرض شده است که توده سنگ دارای مدل رفتاری الاستوپلاستیک کامل بوده که در ناحیه سست شده فاقد چسبندگی است. بدین منظور از راه‌حل دانکن-فاما و دو مدل رفتاری پلاستیک صلب یانسن-کوتر و کاکوت بهره گرفته‌شده است. بر طبق نتایج حاصله، فشار زمین بر طبق مدل کاکوت همبستگی بهتری با رابطه گوئل-جتوا نسبت به راه‌حل یانسن-کوتر نشان می‌دهد.

کلمات کلیدی: منحنی واکنش زمین (GRC)، فشار زمین، مدل پلاستیک صلب، روش همگرایی-همجواری (CCM)، کرنش بحرانی.