Probabilistic analysis of stability of chain pillars in Tabas coal mine in Iran using Monte Carlo simulation

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Abstract

Performing a probabilistic study rather than a determinist one is a relatively easy way to quantify the uncertainty in an engineering design. Due to the complexity and poor accuracy of the statistical moment methods, the Monte Carlo simulation (MCS) method is wildly used in an engineering design. In this work, an MCS-based reliability analysis was carried out for the stability of the chain pillars in the Tabas coal mine, located in Iran. For this purpose, the chain pillar strengths were calculated using the Madden formula, the vertical stress on the chain pillars was determined by an empirical method, and a numerical modeling was performed using the FLAC3D software. The results obtained for the probabilistic stability analysis of the chain pillars showed that the failure probability obtained for the designed pillars by applying the MCS method were approximately the same as that obtained by the advanced second moment (ASM) method, and the values obtained varied between 12 and 18 percent.

Keywords: *Reliability Analysis, Probabilistic Method, Monte Carlo Simulation, Statistical Moments Method.*

1. Introduction

To extract ores, most underground mining methods utilize the temporary or permanent pillars. The coal pillars in underground coal mines play a key role for providing support to the superincumbent strata [1]. In longwall mining, a system of individual pillars named chain pillars are designed to protect panel entries from the influence of panel extraction. One of the fundamental tasks involved in the analysis and design of the longwall chain pillars is the determination of an adequate chain pillar size for a given mining site, where the geological settings, materials, and mining methods are explored [2].

A number of conventional methods such as the ultimate strength and progressive failure methods have been developed and used in the analysis and design of mine pillars. Such conventional methods are based upon the use of a deterministic safety factor as an indicator of the safety of a pillar [2]. Coal pillars have traditionally been designed using a conventional approach based on the fact that the load applied to a pillar should be lower than its strength [1, 3]. While considering the safety factor is a way to take uncertainty into account, it is now well-recognized that the deterministic methods are intrinsically limited for handling the uncertainties in the material properties and non-regular geometries [4, 5]. For example, in a pillar design, the material properties of the pillar are variable in the panel, and the pillar height varies as a function of the seam thickness. Therefore, the probabilistic methods are more appropriate than the deterministic ones for designing coal pillars.

The advantages of a probabilistic analysis are two-fold. First, it handles the uncertainties in the input parameters, and particularly it determines the sensitivity of the uncertainties in various design variables. Secondly, while decisions are seldom clear and perfect, this approach provides a more rational decision-making basis than a purely deterministic analysis [6]. Due to these features,

the probabilistic analysis has been increasingly applied to mining engineering in the recent years. The study carried out by Coates at the Elliot Lake mine is probably one of the oldest probability applications to the pillar design. He found that the variation coefficient for the measured pillar stresses was 22% [7, 8]. Furthermore, he calculated the contribution to the total coefficient of variation using the irregularities in the stope geometry, effect of irregular mining boundaries, and variability in the rock stiffness. Peytel has applied the reliability methods to design the square pillars [9]. He illustrated a design procedure involving the failure probability by the numerical examples. Pine has described the application of risk analysis to the pillar design based on the displacement-discontinuity stress modeling and empirical strength assessment [10]. He presented a reliability analysis for the pillar design, where the normal probabilistic distributions were assumed for the random variables and safety margin. Zhang et al. have proposed an approach employing the fuzzy neural network (FNN) for designing a longwall chain pillar system. The results obtained showed a good agreement between the FNN and ALPS results [11]. Griffiths et al. have combined the random field theory with an elasto-plastic finite element algorithm in a Monte Carlo (MC) framework to estimate the stability of the pillars [12, 13]. Nikitin has proposed a method for the stability analysis and a failure prognosis by the Monte Carlo simulation (MCS). This method is applicable in different geological conditions, where the room-and-pillar mining system is used [14]. Carlisle and Jung have also used MCS for the design of the openings and pillars in hard-rock mines [15].

Van der Merwe has updated the traditional formulation by Salamon and Munro in the light of the statistical consideration on new data from several South African mines [16, 17]. Hutchinson et al. have proposed techniques for the stability assessment and crown pillar failure using the mechanistic, empirical, and numerical simulation techniques [18]. Carter and Miller have recommended the use of MC approach for the crown-pillar risk assessment [19]. They suggested some probabilistic approaches for determining the risk of crown-pillar failure by reference to the previous experiences.

Cauvin et al. have introduced a logical framework that can be used to incorporate the different kinds of uncertainties related to the data and models as well as to the specific expert's choices in the risk

analysis process of the pillar design [20]. Carlisle and Jung have used MCS for the design of openings and pillars in hard-rock mines [21]. Cauvin et al. have also described a way to use an MC probabilistic approach to assess the uncertainties in the mining pillar stability analysis [22]. They introduced a logical framework that can be used to incorporate the different kinds of uncertainties related to the data and models as well as to the specific expert's choices in the hazard or risk analysis process. Galvin et al. and Hill and Buddery have used probability to define the pillar design guidelines [23, 24]. Deng et al. have presented a pillar design based on MCS by combining the finite element methods, neural networks, and reliability analysis [2]. Ghasemi et al. have quantified the effects of the random variables on the pillar safety factor, and the probability of pillar failure was determined using MCS [25]. Zhou et al. have applied some statistical and soft computing methods such as the Fisher discriminant analysis (FDA) and the support vector machine (SVM) methodology to the determination of the pillar stability for the underground mines selected from various coal and stone mines [26]. Najafi et al. have studied the failure probability of the designed chain pillar of the Tabas coal mine by the first order second moment (FOSM) and advanced second moment (ASM) methods [27]. Finally, Guarascio and Oreste have proposed a probabilistic approach for the evaluation of the degree of safety of a pillar. This approach is based upon the exact evaluation of the stress state inside the pillar, and it takes into due consideration the typical uncertainty of the geomechanical parameters for the rock mass that makes up the pillars [28].

According to the above-mentioned knowledge, it is clear that there has been no unique study on the probabilistic evaluation of the chain pillar safety factor using MCS.

The aim of the present study was to propose a new probabilistic method in order to estimate the failure probability of the chain pillars in longwall minings using MCS. For this aim, the chain pillar strength was calculated using the Madden formula, and the vertical stress on the chain pillars was subsequently determined using an empirical method and a numerical modeling using the FLAC3D software. Finally, the results obtained were compared with the existing references in the literature.

2. Case Study

The Tabas coal mine is located approximately 75 km south of the city of Tabas in Iran (Figure 1). It has three minable seams (C_1, B_1, and B_2). The C_1 seam, which is located in the Tabas coal mine-field #1, is mined by a mechanized longwall retreat mining method. The thickness and dip of the C_1 seam mostly vary from 1.8 to 2 meters and from 11 to 26 degrees, respectively. Intermittently low-strength sandstone and siltstone layers have been formed in the hanging wall of the coal seam. Its footwall consists of siltstone and mudstone seams (Figure 2) [29].

In this mine, a three-entry system is used to serve a 220 m long longwall face. There are two rows of chain pillars between two adjacent panels. Considering the dip of the coal seam, the first set of chain pillars is 200 m deep, and the last one, located along the lowest panel, is approximately 700 m deep. The entries to the panels are 2.8 m high and 4.6 m wide [29].

The present work concerns the first set of chain pillars located between the first two panels. In this case, the depth of chain pillars is approximately equal to 200 m, while their designed width and length are 28 m and 100 m, respectively. The location of the entries and panels in the mine-field #1 are shown in Figure 3.
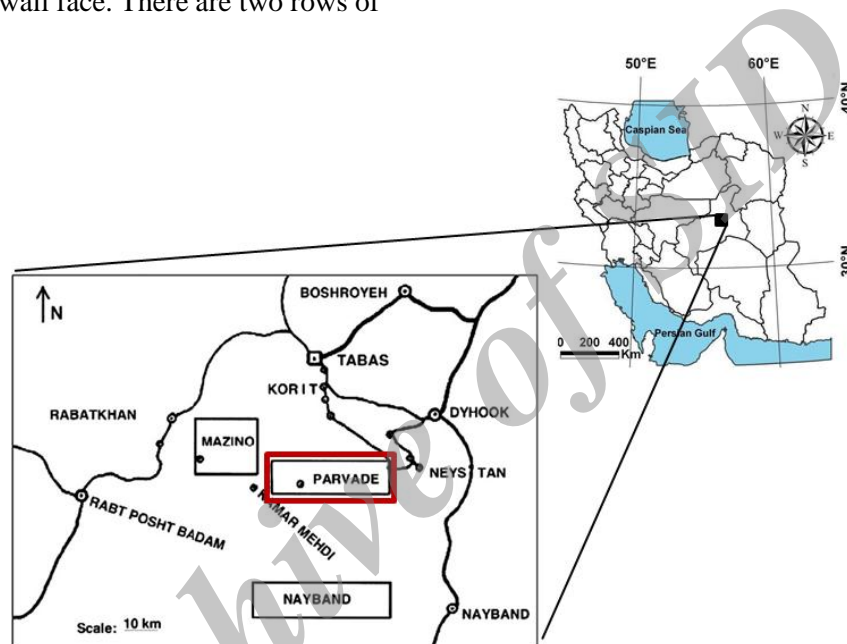

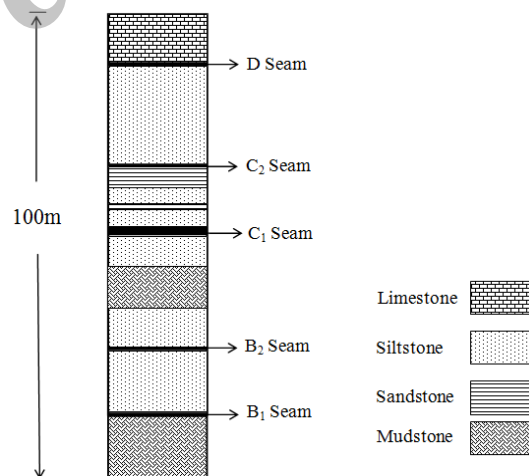**Figure 1. Location map showing Tabas coal mine [30].**

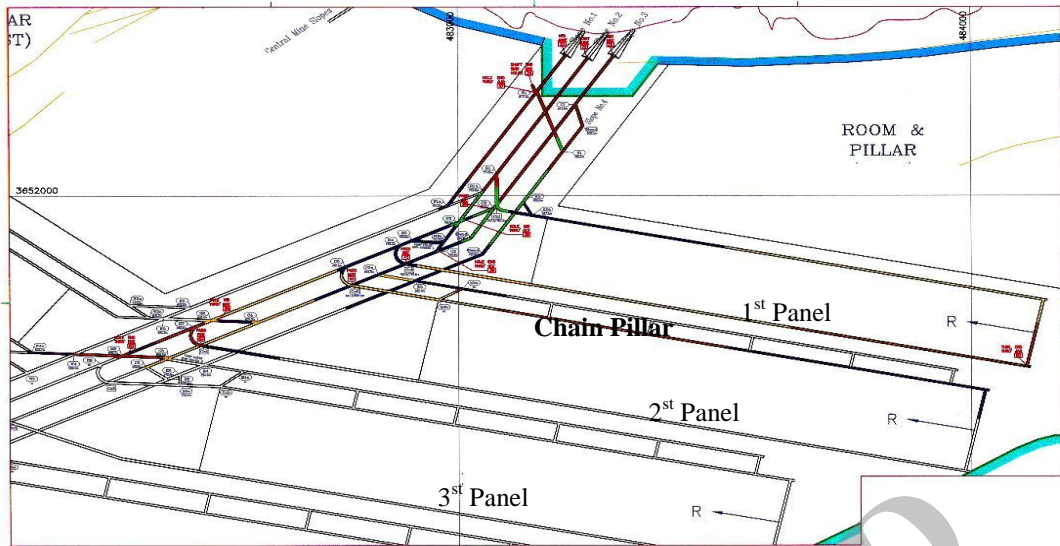**Figure 2. Generalized stratigraphic column at Tabas coal mine.**

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Figure 3. Location of entries and panels in mine-field #1 of Tabas coal mine.

3. Background

In this section, the results obtained for a preliminary reliability analysis of the chain pillars in the Tabas coal mine are presented.

The reliability analysis of the designed chain pillars in the Tabas coal mine has already been done to evaluate the failure probability (P_f) of the pillars using the "first order reliability method" (FORM).

FORM is considered to be one of the most reliable computational methods in the geotechnical engineering and structural reliability [31-33]. It encompasses both the FOSM method and the "advanced second moment" (ASM) method.

According to FOSM, the performance function $g_{(X)}$ was approximated by a Taylor polynomial expansion into a linear expression, from which the mean and standard deviations of $g_{(X)}$ may be easily calculated using the first two moments of the basic variables.

The values for the first order approximated mean, μ_g, and variance, σ_g^2, for $g_{(X)}$ can be calculated as follow [34]:

$$\mu_g = g(\mu_{X1}, \mu_{X2}, ..., \mu_{Xn}) \tag{1}$$

$$\sigma_g^2 \approx (\sum_i^n \sum_j^n \frac{\partial g}{\partial X_i}\bigg|_{\mu_{Xi}} \frac{\partial g}{\partial X_j}\bigg|_{\mu_{Xj}} Cov[X_i, X_j]) \tag{2}$$

where $Cov[X_i, X_j]$ is the covariance of X_i and X_j. The partial derivatives of $g_{(X)}$ were evaluated at the mean values of all parameters.

The reliability index defines the risk level, and can be expressed as a function of the first two moments, as follows:

$$\beta = \frac{\mu_g - F_0}{\sigma_g} \tag{3}$$

where β is the reliability index, F_0 is the minimum standard safety factor (usually equals 1), and (μ_g) and (σ_g) are the mean and standard deviations of $g_{(X)}$, respectively.

Furthermore, the resulting performance function distribution can be reasonably considered as a normal distribution according to the central limit theorem [35]. In MCS, the failure probability can be estimated by the following counting equation:

$$P_f = P[g_{(X)} < 0] \tag{4}$$

If the performance function is normally distributed, P_f and β have the following relationship:

$$P_f = \Phi(-\beta) \tag{5}$$

where Φ is the cumulative distribution function of a standardized normal random variable [6].

According to ASM, the reliability index, β, is defined as the shortest distance to the failure surface from the origin in the reduced coordinate system (i.e. system of transformed variables). The ASM approach may also be effectively employed numerically using the following matrix [36, 37]:

$$\beta* = \min_{X \in F} [(X - \mu)^T [Cov]^{-1} (X - \mu)]^{1/2} \tag{6}$$

where [*Cov*] is the covariance matrix, and *F* is the failure surface.

FORM demands the values and partial derivatives of the performance function with respect to the design random variables. Such calculations could be time-consuming or cumbersome when the performance functions are implicit.

However, based upon the discussed reliability analysis of the pillars, the reliability index and

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failure probability for the chain pillars in the Tabas coal mine have been previously calculated by the use of both FOSM and ASM for the safety factors between 1 and 1.5 [27]. The results obtained for the reliability analysis are shown in Table 1.

It has been well-recognized that the FORM methods have some intrinsic limitations such as complexity, where there are a large number of

input variables and a low accuracy when the performance function is non-linear [32, 38, 39]. Since the performance function used here is non-linear and because the variables involved in this function concern both the geomechanical characteristics of rock media and geometry, it is therefore necessary to validate the pillar reliability analysis using the methods such as MCS, which can cover the FORM limitations.

Table 1. Results for chain pillar reliability analysis using FOSM and ASM [27].

No.	Parameter	Symbol	Value FOSM	ASM	Unit
1	Mean of safety factor	μ_g	1.71	-	-
2	Reliability index	β	1.18	0.98	-
3	Failure probability	P_f (F<1)	11.9	16.35	%
4	Failure probability	P_f (F<1.5)	35.62	39.3	%

4. Reliability analysis of chain pillars using MCS

MCS is a powerful and accurate method, which can be applied instead of FORM. The MC techniques are commonly applied to a wide variety of problems involving the random behaviors in geotechnical engineering.

To make use of this method, the distribution function of each stochastic variable must be known. From each distribution, a parameter value is randomly sampled, and the performance function value is calculated for each set of random samples. The repetition of this process allows a distribution of the performance function to be established, although it requires the calculation of thousands of performance function values [40]. The failure probability can then be calculated as a ratio of the number of failed cases to the total number of simulations. Alternatively, the mean and standard deviations of the performance function distribution can be calculated in order to yield the reliability index, which then serves to determine the failure probability from the values tabulated for the standardized normal distribution [6]. The typical flow chart for MCS is shown in Figure 4.

To perform the reliability analysis of the chain pillar design, the failure criteria should be identified by means of the performance function $g_{(X)}$, which is traditionally defined as:

$$g_{(X)} = SF - 1 \tag{7}$$

where SF is the calculated safety factor, as R/S, in which R and S are the resulting resisting and driving forces, respectively [6]. R is the strength of the chain pillars, and S is the vertical stresses on the chain pillars [27].

In this research work, the chain pillar strength was calculated using the Madden formula (for the numerator of the 1st term in Equation 8) [41]. This formula was developed for use when the width to height ratio of the pillars was greater than five, as is the case for the chain pillars in the Tabas coal mine.

The vertical stress on the chain pillars (the denominator of the 1st term in Equation 8) was calculated using two different methods: an empirical method and a numerical modeling. In the empirical method, the Mark and Beniaweski formula was used to calculate the vertical and lateral stresses on the chain pillars. Therefore, the performance function is:

$$g_{(X),E} = \frac{K \times L^\beta \times \left[0.011 \times W^\alpha \times H^\lambda + 1.98 \times W^\beta \times H^\beta\right]}{0.00981 \times \gamma \times \left[2 \times W \times h + B \times h + h^2 \tan\phi\right] \times \left[\frac{(L+B)}{2 \times W \times L}\right]} - 1 \tag{8}$$

where $\alpha = 2.43$, $\beta = -0.0677$, and $\lambda = -2.566$ are the constants in this study, W is the width of the chain pillar (m), h is the overburden thickness (m), γ is the overburden unit weight (ton/m³), φ is the average shear angle of the overburden, B is the entry width (m), L is the length of the chain pillars (m), K is the in situ coal strength (MPa), and H is the pillar height (m).

In the alternative method, the mean and standard deviations of the imposed vertical stresses on a chain pillar are calculated using the 3D numerical analysis. The FLAC³ᴰ software [42] was used to model and simulate the longwall mining in the Tabas coal mine. Figure 5 illustrates the geometry of the modeled area.

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Start

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Create probability
distribution functions

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Stochastic sampling

↓

Model evaluation

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Calculate/update statistical
indexes

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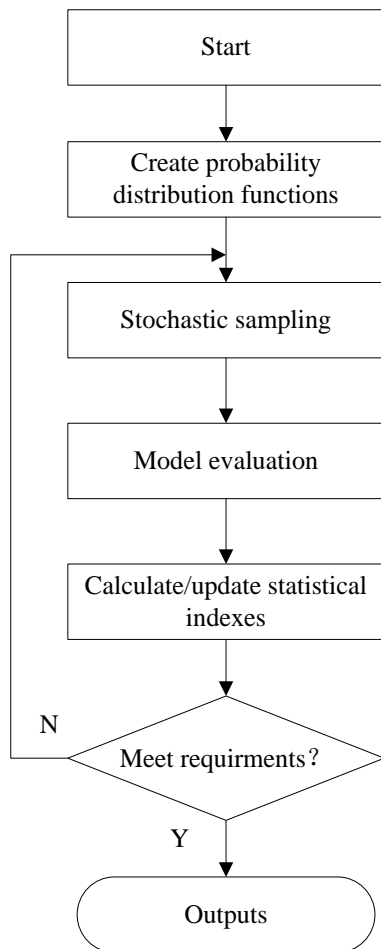N ← Meet requirments?

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Outputs

Figure 4. Flow chart of MCS [40].

In this mine, the longwall mining started at the first panel, and then continued to the second panel. Therefore, to simulate the longwall mining, each panel was divided into one hundred cuts along the face advancing. Considering the length of each panel (1000 m), this means that the width of each cut was 10 m. In each analysis step, the cut behind the longwall face was changed to the gob material, and the model was run to equilibrium before creating the next cut. In order to evaluate the stress distribution on the chain pillars, some observation points were placed on a chain pillar located in the middle of the panel, as shown on Figure 6.

The chain pillars in the longwall mining were subjected to the field stress and various stages of mining loading. As the face advances from right to left, and then to the next panel, as indicated in Figure 6, the chain pillars undergo five loading stages. By considering the five stages of loading on the chain pillars in the numerical modeling, the stress distribution on the chain pillars was

obtained. The numerical analysis results indicate a 8.02 MPa mean stress and a 1.93 MPa standard deviation of the vertical stress (S) on the selected chain pillars in the Tabas coal mine. This value was then used in the performance function, as shown below:

$$g_{(X),N} = \frac{K \times L^{\beta} \times \left[0.011 \times W^{\alpha} \times H^{\lambda} + 1.98 \times W^{\beta} \times H^{\beta} \right]}{S} - 1 \quad (9)$$

Considering the two described performance functions, the mean and standard deviations of each variable in the mentioned functions should be determined. There are eight parameters assumed as the variables with a normal distribution function. The means and standard deviations of these eight variables are shown in Table 2.

The mean and standard deviations of the pillar height and width were measured considering the variations in the width and height of the entries, respectively. The variation in the overburden thickness was also measured in regard to the surface topography. The variation in the unit weight was obtained using the existing literature and reports on the Tabas coal mine. It should be noted that in MCS, when the distribution of a parameter is not clear, the probabilistic methods allow the designer to assume it as normal [43]. Therefore, due to the limited number of data on the Tabas coal mine, the distributions of K and ϕ could not be clarified, and hence their distributions were assumed to be normal. Considering the values mentioned for the mean and standard deviations, stress variation was simulated according to the random real normal distribution. Ten thousand simulations were carried out for the stress variation using the Mathematica[TM] (ver. 6) software [44]. Then the mean value, standard deviation, and reliability index for each performance function were calculated. It should be noted that the stress over a pillar, as calculated with the empirical method, was considered to be uniform. Therefore, the standard deviation from it was higher than the standard deviation obtained using the numerical modeling.

All of the parameters μ_g, P_f, β, and σ_g were then calculated (Table 3). The histogram of each performance function was found to follow a left-skewed normal distribution function, as illustrated in Figure 7.
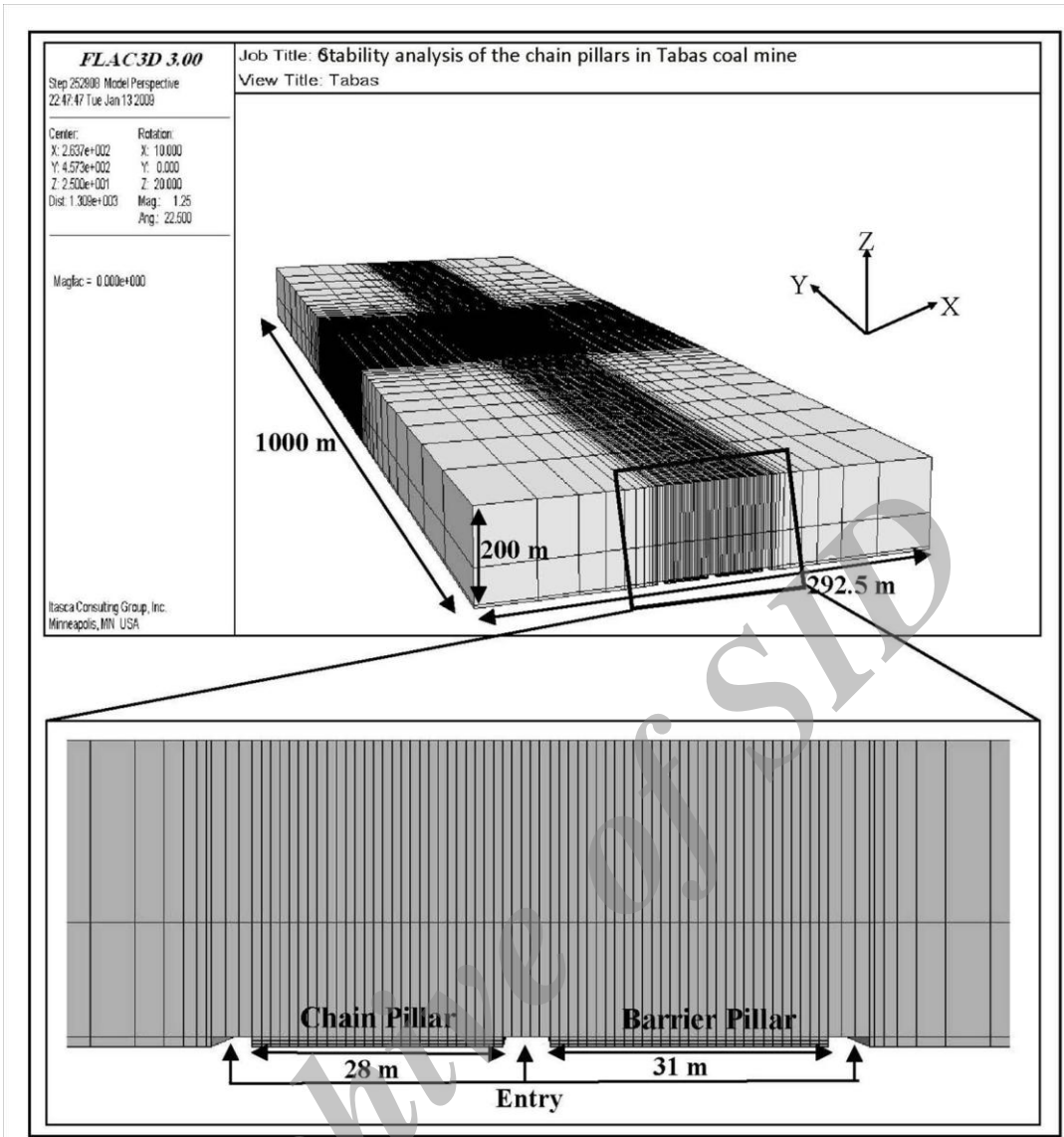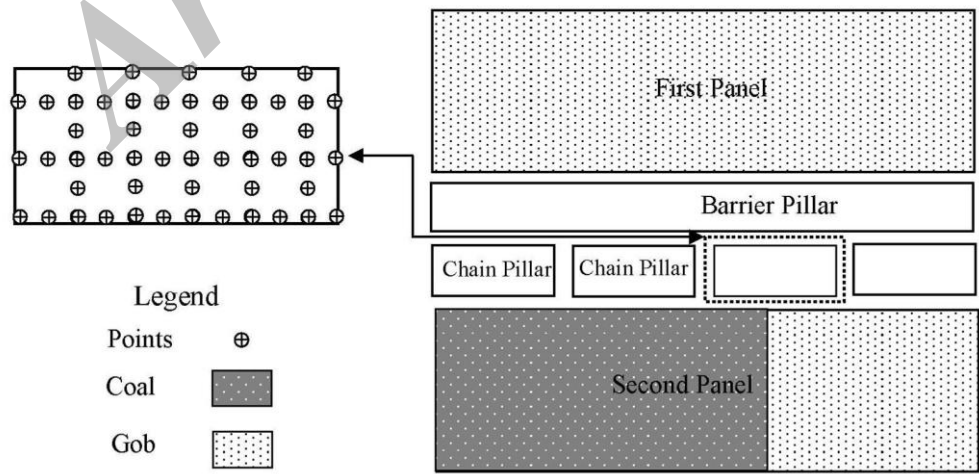
Figure 5. 3D model of longwall mining.

Figure 6. Arrangement of given points on chain pillar.

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Table 2. Statistical parameters for chain pillars in Tabas coal mine.

Variable	Unit	Mean	Standard deviation
W	m	28	0.3
H	m	2.82	0.53
L	m	102	7.13
K	MPa	6.72	0.5
γ	ton/m^3	2.63	0.068
B	m	4.6	0.3
h	m	200	7.45
φ	radian	0.34	0.075

Table 3. Results of reliability analysis applying MCS.

| No.	Parameter	Symbol	$g_{(X),E}$	$g_{(X),N}$	Unit
1	Mean of performance function	μ_g	1.72	2.92	-
2	Standard deviation of performance function	σ_g	0.82	1.66	-
3	Reliability index	β	0.88	1.15	-
4	Failure probability	P_f(F<1)	18.88	12.36	%
5	Failure probability	P_f(F<1.5)	39	19.5	%
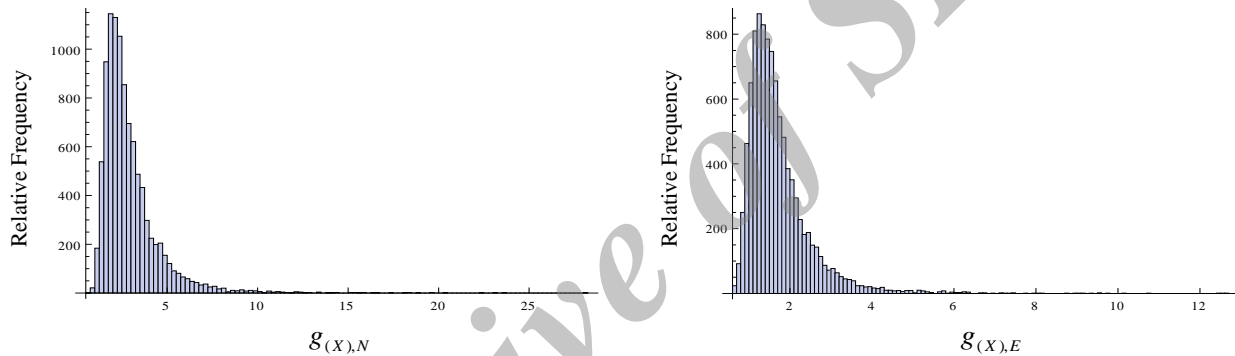
Figure 7. MCS results.

The cumulative distribution function of the safety factor for the chain pillars is shown in Figure 8. The most important results (Figure 8) can be summarized as follow:
- The failure probability should decrease by approaching the given safety factors of the chain pillars toward a more conservative design.

- The failure probability less than or equal to one (probability of SF < 1), is 19% for $g_{(X),E}$, and 12% for $g_{(X),N}$.
- For the safety factors greater than 0.5, the values for the cumulative failure probability reflected by $g_{(X),E}$ (MCS based on Equation 7) are greater than the values predicted by $g_{(X),N}$ (MCS based on Equation 8).
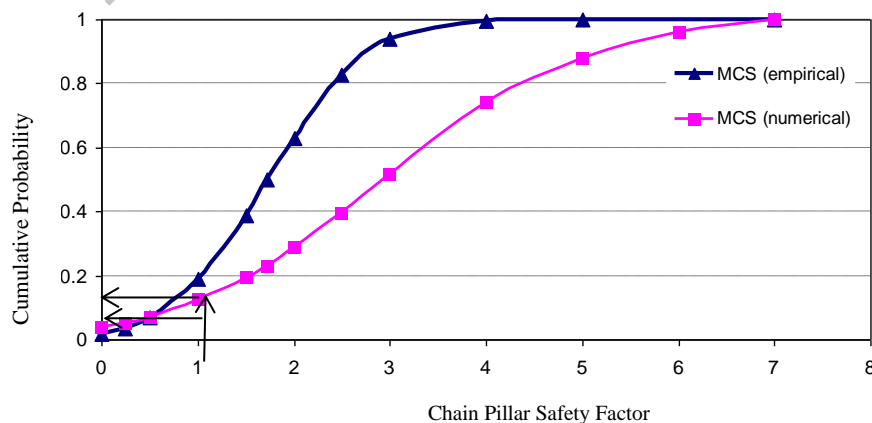
Figure 8. Cumulative distribution of chain pillar safety factor obtained by MCS.

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4. Discussions and Conclusions

A sensitivity analysis can provide a quantitative understanding of the parameters affecting the failure states of the system. However, it should be noted that the real failure probability cannot be achieved through a sensitivity analysis [6].

In this work, several analyses were performed to study the sensitivity of the reliability index generated by the MCS method to various uncertainty levels by changing the coefficient of variation in the material (where the mean is constant) and engineering parameters.

For the statistical results given by MCS to approach reality, at least 10,000 simulations should be performed. During each stage of an analysis, as the coefficient of variation for one parameter was increased, the coefficient of variation for the other parameters were held constant, and equal to the minimum value (Table 4). This feature serves to maximize the absolute value of the reliability index at the beginning (when all coefficients of variations are minimized).

Applying Equation 8, the results obtained for the sensitivity analysis are shown on Figure 9, where it can be seen that:

- The reliability index decreases, and consequently, the failure probability increases when the coefficient of variation for each parameter increases.
- The performance function is more sensitive to the parameter that has a higher standard deviation. The reliability index is more sensitive to the pillar height (H). Indeed, the reliability index varies from 1.6 to 0.8 dramatically, the coal thickness in the Tabas

coal mine has most variations in comparison with the other parameters.

- The results obtained for the sensitivity analysis show that the reliability index is also sensitive to the variation in the in situ strength of the coal and shear angle of overburden.

Generally speaking, the conventional deterministic methods are widely used to determine an adequate chain pillar size for coal mines. In these cases, the safety factors as well as all the input parameters are deterministic. Evidently, these parameters, which have probability characteristics, should be analyzed using the probabilistic methods.

In this research work, MCS was applied to determine the reliability index and failure probability of the chain pillars in the Tabas coal mine, and the results obtained were then compared with the statistical moment methods.

Since ASM enjoys a higher order statistical moment than FOSM, it is expected that the ASM results should be closer to the results obtained by MCS. According to the results shown in Tables 1 and 3, it is clear that the MCS results obtained by Equation 8 (i.e. g(X),E) and the ASM results are approximately the same. The results obtained for the probabilistic stability analysis of the chain pillars show that the failure probability for the designed pillars applying the MCS method vary from 12 to 18 percent. The results obtained for such an analysis can also be used as a basis for decision-making about the size of the chain pillars. However, it is necessary to do a cost trade-off to make the final decision. For further studies, it is recommended that using MCS with numerical modeling, the effect of the discontinuities on the probability failure of the chain pillars in longwall mining be considered.

Table 4. Pillar parameters in a sensitivity analysis.

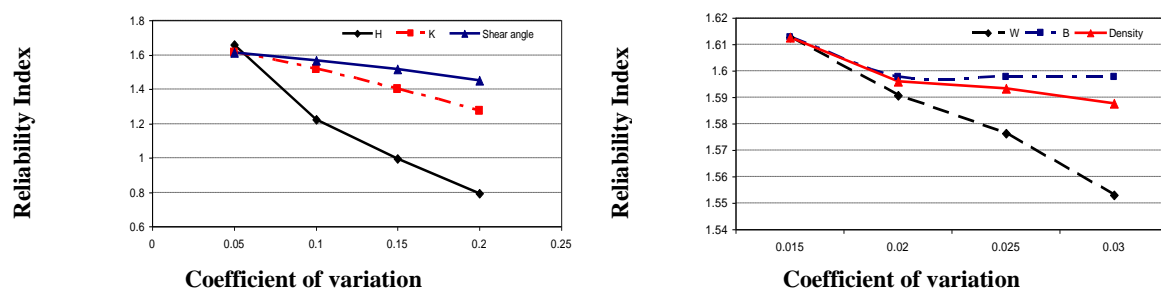| Variable	Unit	Mean	Coefficients of variations			
			CV$_1$	CV$_2$	CV$_3$	CV$_4$
W	m	28	0.005	0.01	0.02	0.03
H	m	2.82	0.05	0.1	0.15	0.2
K	MPa	6.72	0.05	0.1	0.15	0.2
φ	radian	0.34	0.05	0.1	0.15	0.2
γ	ton/m^3	2.63	0.015	0.02	0.25	0.03
B	m	4.6	0.005	0.01	0.02	0.03

Figure 9. Reliability index for chain pillars as a function of coefficient of variation.

References

[1]. Jaiswal, A. and Shrivastva, B.K. (2009). Numerical simulation of coal pillar strength. International Journal of Rock Mechanics and Mining Sciences. 46 (4): 779-788.

[2]. Deng, J., Yueb, Z.Q., Thamb, L.G. and Zhuc, H.H. (2003). Pillar design by combing neural networks and reliability: a case study of the Feng Huangshan copper mine China. International Journal of Rock Mechanics & Mining Sciences. 40: 585-599.

[3]. Ghasemi, E. and Shahriar, K. (2012). A new coal pillars design method in order to enhance safety of the retreat mining in room and pillar mines, Safety Science. 50: 579-585.

[4]. Harr, M.E. (1987). Reliability based design in civil engineering. New York: McGraw-Hill.

[5]. Melchers, R.E. (1999). Structural reliability analyses and predictions. 2nd ed. New York: Wiley.

[6]. Yarahmadi Bafghi, A.R. and Verdel, T. (2004) The probabilistic key-group method. International Journal for Numerical and Analytical Methods in Geomechanics. 28: 899-917.

[7]. Coates, D.F. (1973). Variance of pillar stresses at Elliot Lake. Int. J. Rock Mech. Min. Sci. 10 (6): 627-40.

[8]. Coates, D.F. (1974). Probability of pillar failure at Elliot Lake. Conference. Proc. Third Congress, Int. Soc. Rock Mech. Denver, V2, Part B. pp. 990-996.

[9]. Peytel, M.W. (1990). Reliability level III method in design of square pillar resting on weak floor stratum. Geotech. & Geol. Eng. 8: 149-162.

[10]. Pine, R.J. (1992). Risk analyses design applications in mining geomechanics. Trans Inst Min Metall, Section A 101: 49-58.

[11]. Zhang, Y.W., Chugh, Y.P. and Yang, G.P. (1995). Fuzzy neural network for chain pillar design in longwall coal mining. In The 35th US Symposium on Rock Mechanics (USRMS). American Rock Mechanics Association.

[12]. Griffiths, D.V., Fenton, G.A. and Lemons, C.B. (2002). Probabilistic analyses of underground pillar stability. International Journal for Numerical and Analytical Methods in Geomechanics. 26: 775-791.

[13]. Griffiths, D.V., Gordon, A., Fenton, A. and Lemons, C. (2007). The Random Finite Element Method (RFEM) in Mine Pillar Stability Analisis. In Probabilistic Methods in Geotechnical Engineering. 491: 271-294

[14]. Nikitin, O. (2003). Mining block stability prediction by the Monte Carlo method. 4th Intern. Conf. "Environment. Technology. Resources", Rezekne, Latvia. 26-28: 185-190.

[15]. Carlisle, S. and Jung, S.J. (2006). Probabilistic design of hard-rock openings and pillars. Golden Rocks 2006, The 41st U.S. Symposium on Rock Mechanics (USRMS), Golden, CO, USA.

[16]. Van der Merwe J.N. (2003). New Pillar Strength Formula for South African Coal. J. S.Afr. Inst. Min. Metall. pp. 281-290.

[17]. Salamon, M.D.G. and Munro, A.H. (1976). A study of the strength of coal pillars. J. S. Afr. Inst. Min. Metall.

[18]. Hutchinson, D.J., Phillips, C. and Cascante, G. (2002). Risk considerations for crown pillar stability assessment for mine closure planning. Geotech. & Geol. Eng. 20 (1): 41-64.

[19]. Carter, T.G. and Miller, R.I. (1995). Crown-pillar risk assessment-planning aid for cost-effective mine closure remediation. Int. J. Rock Mech. Min. Sci. & Geomech. Abst. 32 (8): 413A-413A.

[20]. Cauvin, M., Verdel, T. and Salmon, R. (2005). Modeling uncertainties in pillar stability analysis. Symposium Post mining 2005. Nancy, France. pp.NC. <ineris-00972516>.

[21]. Carlisle, S. and Jung, S.J. (2006). Probabilistic design of hard-rock openings and pillars. The 41st U.S. Symposium on Rock Mechanics (USRMS), Golden, CO, USA.

[22]. Cauvin, M., Verdel, T. and Salmon, R. (2009). Modelling uncertainties in pillar stability analyses. International Journal on Risk Analyses – Risk Analyses. 29 (10): 1371-1380.

[23]. Galvin, J.M., Hebblewhite, B.K. and Salamon, M.D.G. (1999). UNSW coal pillar strength determinations for Australian and South African mining conditions. Proc. 2nd Int. Workshop on Coal Pillar Mechanics and Design, Pittsburgh, NIOSH IC9448.

[24]. Hill, D.J. and Buddery, P.S. (2004). Coal pillar stability considerations for surface protection. Proceedings Sixth Triennial Conference on Subsidence Management Issues (Mine Subsidence Technological Society: Maitland, NSW).

[25]. Ghasemi, E., Shahriar, K., Sharifzadeh, M. and Hashemolhosseini, H. (2010). Quantifying the uncertainty of pillar safety factor by Monte Carlo simulation-a case study. Archives of Mining Sciences. 55: 623-635.

[26]. Zhou, J., Li, X.B., Shi, X.Z., Wei, W. and Wu, B.B. (2011). Predicting pillar stability for underground mine using Fisher discriminant analysis and SVM methods. Transactions of Nonferrous Metals Society of China. 21 (12): 2734-2743.

[27]. Najafi, M., Jalali, S.E., Yarahmadi, A.R. and Sereshki, F. (2011). Prediction of the Confidence

Interval for Stability Analyses of Chain Pillars in Coal Mines, Journal of Safety Science. 49 (5): 651-657.

[28]. Guarascio, M. and Oreste, P. (2012). Evaluation of the stability of underground rock pillars through a probabilistic approach. Am. J. Applied Sci. 9: 1273-1282.

[29]. Anon. Basic design of Tabas Coal Mine Project, Report-Mining. 2005, Vol 1 of 5.

[30]. Najafi, M., Jalali, S.E. and Kakaei, R. (2014). Thermal-Mechanical Numerical Analysis of Stress Distribution in the vicinity of Underground Coal Gasification (UCG) Panels. International Journal of Coal Geology. 134-135: 1-16.

[31]. Yarahmadi-Bafghi, A.R. and Verdel, T. (2005). Sarma-based key-group method for rock slope reliability analyses. Int. J. Numer. Anal. Meth. Geomech. 29: 1019-1043.

[32]. Chodhury, R.N., Tang, W.H. and Sidi, I. (1987). Probability model of progressive slope failure. Geotechnique; 37 (4): 467-481.

[33]. Deng, J., Gu, D., Li, X. and Yue, Z.Q. (2005). Structural reliability analyses for implicit performance functions using artificial neural network. Journal of Structural Safety. 27: 25-48.

[34]. Liang, R.Y., Nusier, O.K. and Malkawi, A.H. (1999). A reliability based approach for evaluating the slope stability of embankment dams. Journal of Engineering Geology. 54: 271-285.

[35]. Kreyszing, E. (1988). Advanced Engineering Mathematics (6th edn). Wiley: New York.

[36]. Low, B.K. and Tang, W.H. (1997). Reliability analyses of reinforced embankments on soft ground. Canadian Geotechnical Journal. 34: 672-685.

[37]. Christian, J.T. and Baecher, G.B. (1997). Probabilistic slope analyses using Jamb's generalized procedure of slices. Journal of Computers and Geotechnics. 22 (2):121-142.

[38]. Zhao, Y.G. and Ono, T. (2001). Moment methods for structural reliability. Structural Safety. 23: 47-75.

[39]. Baecher, G.B. (1988). Parameters and approximations in geotechnical reliability. Uncertainty Modeling in Civil Engineering, CRC Press LLC.

[40]. Wittwer, J.W. (2004). "Monte Carlo Simulation Basics" From Vertex42.com, On line at: http://www.vertex42.com/ExcelArticles/mc/MonteCarloSimulation.html.

[41]. Madden, B.J. (1988). The Performance of Coal Pillars Designed to them Squat Pillar Formula. Proceedings 29th U.S. Symposium on Rock Mechanics, Balkema, Rotterdam. pp. 699-708.

[42]. Itasca. User manual for FLAC3D, version.3.0.Minnesota:Itasca Consulting Group Inc. 2002.

[43]. Park, H. and West, T.R. (2001). Development of a probabilistic approach for rock wedge failure, Engineering Geology. 59: 233-251.

[44]. http://www.wolfram.com/products/mathematica/newin6/.

تحلیل احتمالاتی پایداری پایه‌های زنجیری در معدن زغال‌سنگ طبس با استفاده از شبیه‌سازی مونت‌کارلو

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چکیده:

تحلیل احتمالاتی نسبت به تحلیل قطعی روش بهتری برای بررسی تأثیر عدم قطعیت‌ها در طراحی مهندسی است. به دلیل پیچیدگی و دقت کم روش ممان‌هـای آماری، روش شبیه‌سازی مونت‌کارلو (Monte Carlo simulation (MCS)) امروزه کاربرد گسترده‌ای در طراحی مهندسی پیدا کرده است. بـرای ایـن منظـور در این تحقیق روش شبیه‌سازی مونت‌کارلو برای محاسبه اندیس قابلیت اعتماد طراحی پایه‌های زنجیری در معدن زغال‌سنگ طبس استفاده شـده اسـت. از ایـن‌رو در ابتدا مقاومت پایه‌های زنجیری با استفاده از رابطه مادن محاسبه شده است و برای محاسبه میزان تنش‌های وارد بر پایه از روش‌های تجربی و مدل‌سازی عـددی با استفاده از نرم‌افزار FLAC3D استفاده شده است. نتایج تحلیل احتمالاتی پایداری پایه‌های زنجیری با استفاده از روش شبیه‌سازی مونت‌کارلو نشـان داده اسـت که احتمال شکست محاسبه شده تقریباً مشابه روش پیشرفته دو ممان (Advanced Second Moment (ASM)) است و مقـدار آن بـین ۱۲ تـا ۱۸ درصـد است.

کلمات کلیدی: تحلیل قابلیت اعتماد، روش‌های احتمالاتی، شبیه‌سازی مونت‌کارلو، روش ممان‌های آماری.