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Determination of optimal bandwidth in upscaling process of reservoir data using kernel function bandwidth

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Abstract

Upscaling based on the bandwidth of the kernel function is a flexible approach to upscale the data because the cells will be coarse-based on variability. The intensity of the coarsening of cells in this method can be controlled with bandwidth. In a smooth variability region, a large number of cells will be merged, and vice versa, they will remain fine with severe variability. Bandwidth variation can be effective in upscaling results. Therefore, determining the optimal bandwidth in this method is essential. For each bandwidth, the upscaled model has a number of upscaled blocks and an upscaling error. Obviously, higher thresholds or bandwidths cause a lower number of upscaled blocks and a higher sum of squares error (SSE). On the other hand, using the smallest bandwidth, the upscaled model will remain in a fine scale, and there will be practically no upscaling. In this work, different approaches are used to determine the optimal bandwidth or threshold for upscaling. Investigation of SSE changes, the intersection of two charts, namely SSE and the number of upscaled block charts, and the changes of SSE values versus bandwidths, are among these approaches. In this particular case, if the goal of upscaling is to minimize the upscaling error, the intersection method will obtain a better result. Conversely, if the purpose of upscaling is computational cost reduction, the SSE variation approach will be more appropriate for the threshold setting.

1. Introduction

In porous media, because of a large volume of data, upscaling as a pre-processing stage can reduce the volume of computations, and consequently, decrease the cost of the computations in the later stages, especially in fluid flow simulation. New geologic modeling techniques produce reservoir models consisting of up to 10^8-10^{10} cells, each populated by different properties such as permeability, porosity, and fluid saturation. However, numerical reservoir simulations are usually performed with fewer than 100,000 cells, a factor of 10,000 down on the geologic grid [1]. Upscaling in reservoir simulation is a process that scales-up all properties at a fine-scale model to equivalent properties defined at a coarse-scale model such that the two models act as most similarly as possible [2].

Many upscaling techniques have already been introduced in the literature such as analytical methods, single-phase upscaling methods, and two-phase upscaling methods. The analytical methods (e.g. arithmetic and geometric averaging) are very simple to apply, and they are attractive methods for upscaling but for performance, they are not suitable for complex reservoirs. For single-phase flow upscaling, the only parameters to be upscaled are the absolute permeability or transmissibility and porosity. In two-phase methods, in addition to these parameters, relative

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permeability can also be upscaled. Upscaling results, especially in complex reservoirs, are strongly dependent on boundary conditions [3, 4]. Single-phase upscaling methods have been classified by Durlofsky (2003), and according to the selection of the appropriate boundary conditions, are set to local, extended-local, global, and quasi global (also called local-global) [5].

The upscaling method has been used by substituting a heterogeneous region consisting of fine-scale cells into a homogenous region comprising of coarse-scale cells to decrease the calculation and computation time. The properties of the coarse-scale cells are calculated from the average values of the fine-scale cells [6-8]. Wavelet transform and its extensions are one of the most important methods used in oil and gas reservoirs for upscaling reservoir data [9-11]. Chen *et al*. (2018) compared a novel upscaling method called multiple boundary method in three-dimensional fractured porous rocks with the commonly used Oda upscaling method, and also with the volume averaging method. The results computed by the multiple boundary method are comparable with those of the other two methods and fit best the analytical solution for a set of fractures. The errors in flow rate of the equivalent fracture model decrease when the multiple boundary method [12] is used.

The coarse model result may be inaccurate due to heterogeneity loss, connectivity distortion, and numerical dispersion [13]. The accuracy of the upscaling method depends on the approach of grid scaling; otherwise, all methods that are applied for upscaling are based on averaging. If the gridding and coarsening processes are carried out based on variability, we can expect that the accuracy of the upscaling method will be increased. The new upscaling method presented in this paper is referred to as the kernel function method with adaptive bandwidth. Given that in the primary model the upscaling is related to the cell variability, the kernel function bandwidth can be considered as a function of variability. On the other hand, due to the varying intensity of variability in different regions, it uses a variable bandwidth approach and the bandwidth of the variable in the kernel method that represents the system variability, and it is the essence of upscaling.

In areas with high variability, by choosing a small bandwidth, we will have the smallest upscaling; therefore, these areas will remain fine-scale. Conversely, in areas with a low or smooth variability, by choosing a high bandwidth, most blocks will be merged and then coarsened. In this method, by determining a bandwidth or threshold, which is a function of cell variability, we can control the number of upscaled blocks and the computational error. Clearly, in an upscaling process, using the kernel function bandwidth, an upscaled model will be obtained for each bandwidth. An optimal bandwidth should be determined to determine the optimal upscale model. If one can choose the optimal bandwidth, s/he can expect the least amount of data to be wasted in the upscaling process. Thus determination of the optimal threshold in the upscaling algorithm based on the kernel function bandwidth is a major challenge in the kernel-based upscaling process. The threshold is actually a function of the system variability. Naturally, with a higher threshold, the number of cells reduces from a fine scale to a coarse scale. However, in this case, the amount of computational error also increases.

In this paper, we first focus on the kernel adaptive bandwidth method as a new upscaling method. In this regard, we consider the kernel bandwidth-based upscaling. Then the effect of threshold value on the upscaling results will be examined. In the next step, the optimal threshold in different upscaling approaches will be determined. Three upscaling approaches that will be examined in this paper are the SSE differential, intersection of SSE, and number of upscaled block charts and the SSE changes versus thresholds.

2. Research data

In order to investigate the upscaling algorithm based on the bandwidth of the kernel function in one dimension, we used a well dataset with 1613 data. The data used in this work is a density log with a resolution of about 15 cm (Figures 1). The number of samples is 1613. The depth of the studied well varies from 2605 to 2847 m in length. In order to simplify the process, the data of the density log in this work is presented as a signal. The mean and the variance of the data are 2.7070 and 0.0572, respectively. The purpose of upscaling the data is to convert the scale of data from cell to block or from fine-scale to coarse-scale, while retaining the main characteristics of the data. As it is clear from the data, the initial section of well is highly variable, and in the middle and the end, the variability is smooth.

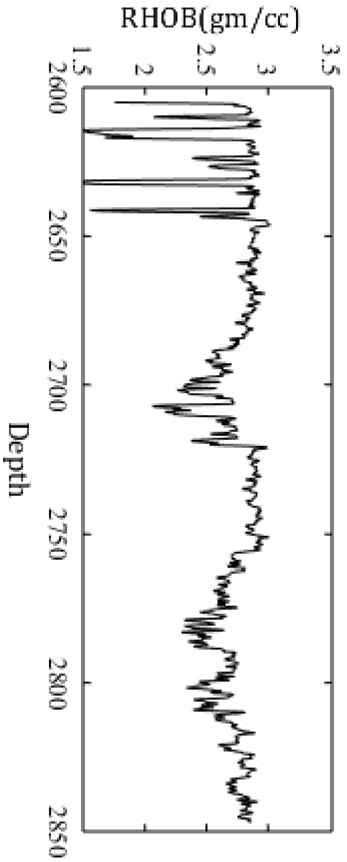


Figure 1. Density log data.

3. Research method

Kernel density estimation (KDE) is a very important statistical technique with many practical applications. It has been applied successfully to both the univariate and multivariate problems. There exists extensive literature on this issue including several classical monographs; see [14-16]. There are two main computational problems related to KDE: (a) the fast evaluation of the kernel density estimates, $\hat{f}$ and (b) the fast estimation (under certain criteria) of the optimal bandwidth matrix H (or scalar h in the univariate case) [17].

The bandwidth of the kernel function can be defined as a variability parameter. In an upscaling process, if cells are merged based on cell variability, an appropriate pattern will be obtained on coarse-scale. Therefore, since upscaling is related to the block variability, the bandwidth of the kernel method can be considered as a function of the variability of the reservoir property. In areas with high variability, by considering a small bandwidth, we will have the smallest upscaling, and these areas will remain fine as much as possible. Conversely, in areas with low and smooth variabilities, with selection of a high bandwidth, most cells will be merged and become coarse. Considering the variability, the variable bandwidth approach is used in the upscaling process. Therefore, due to the variability, the bandwidth will be determined for any property.

The variable bandwidth can directly indicate the degree of data variability. How to calculate and determine the optimal bandwidth can be challenging. In this paper, we propose a new method for upscaling based on data variability and also a dynamic method for determining the optimal bandwidth. Bandwidth in each area can control the upscaling. Also the threshold or bandwidth defined in this problem is controllable so that the rates of change as well as its maximum changes are determined by the data. The flowchart in Figure 2 shows the steps of the upscaling algorithm based on the kernel function bandwidth. Changes and maximum threshold or bandwidth will be determined from the data. For each threshold or bandwidth, the upscaling error, the number of upscaled blocks, and the calculation time will be different. It will be possible that by choosing a specific threshold, one can reach the specified number of upscaled blocks and the upscaling error.

The stages of the kernel-based upscaling algorithm are as follow:

1. Calculate the difference between two consecutive data and compare it with the threshold or bandwidth value defined in each step.
2. If the difference between the two values of the first and second cells is less than the threshold, the first and third data are compared.
3. If the difference between the two data is more than the threshold, then the upscaling will be carried out and all the previous cells of this stage are merged together. Therefore, an equivalent value for these cells is calculated and will be allocated to the entire block.
4. The square of the difference between the actual value and the obtained value in each block is considered as the error of the block and the sum of these squared differences is equivalent to SSE. For all the data, this operation is performed and the upscaling process will be completed.
5. Then the threshold is increased by a defined step, and steps 1 through 4 will be executed for the new threshold.
6. The operation is performed for all the cells, and at the end, for each threshold, a SSE value and the number of upscaled blocks are calculated.
7. Finally, the graph of the error variation against the threshold and also the graph of the

error variation versus the number of upscaled blocks will be plotted.

It should be noted that in the kernel-based upscaling, the defined threshold controls the computational efficiency. Moreover, in this upscaling method, the threshold can be simulated as a function of the number of required cells, and the number of cells will be as a function of time. By changing the threshold, the error value and the number of blocks will be changed accordingly. The main idea behind using kernel is to run a simulator that takes a reasonable time with the highest accuracy. Increasing the accuracy requires an increase in the number of cells, thus when the sizes or dimensions of the cells are kept small, what happens on the other side is that the large number of cells will greatly reduce the computational efficiency. In fact, computational efficiency depends entirely on the number of cells.

In the process of upscaling using the kernel bandwidth, we can see the results based on the number of upscaled blocks. If the goal of upscaling is to obtain a cell number that can manage the variance of the fine scale model and also provide the appropriative precision, it can also be achieved through the process of upscaling based on the bandwidth of the kernel function. In this regard, by considering the kernel evaluation criteria such as the mean square error (MSE), sum of square error, (SSE), and threshold defined, the number of upscaled blocks can be controlled. With this process, the variability of the considered feature in the reservoir is a function of the threshold, and by controlling this threshold, the number of final blocks of model is determined. This is the main difference between the kernel and the other upscaling methods such as wavelet transformation.

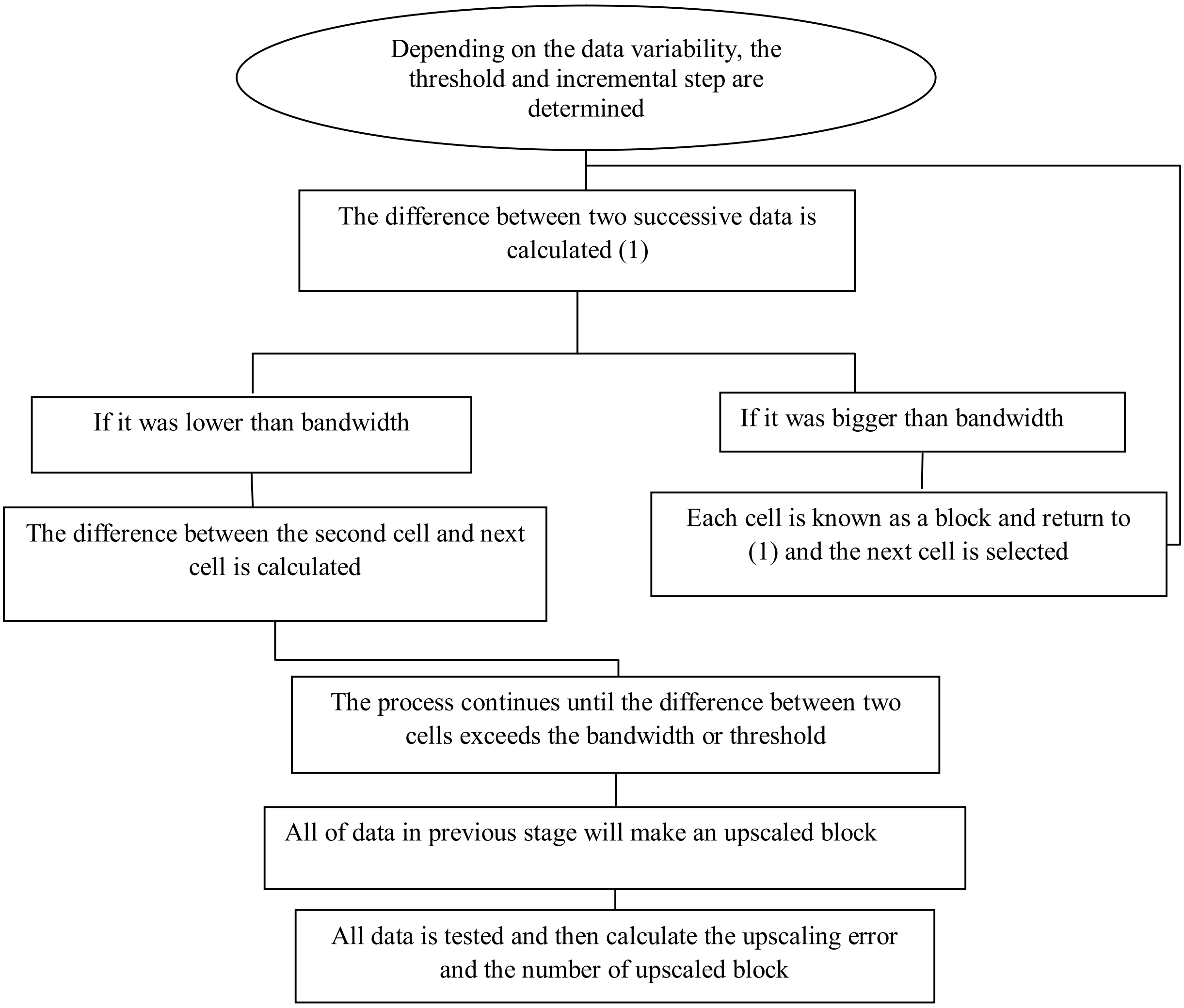


Figure 2. Kernel-based upscaling process.

4. Upscaling based on kernel function bandwidth

For the data described in this research work and based on the upscaling algorithm introduced in this paper, the threshold or adaptive bandwidth was changed from 0 to the maximum value of 0.17 with an incremental step of 0.0044. Naturally, in the minimum threshold value, the number of upscaled blocks will be equivalent to the cells of the fine scale model, and the error rate will be zero. Figures 3 and 4, respectively, show the number of bands (upscaled blocks) and the error rate for each threshold value.

As one can see in Figures 3 and 4, with decrease in the number of bands, the bandwidth increases and the upscaling error rate, namely SSE, will increase accordingly. It is obvious that in the first step, the minimum error will be obtained with the highest number of bands; we will have the lowest bandwidth and the least error. However, in the final steps, the highest error rate will be obtained with the least number of bands. In this process, it is possible to determine the number of upscaled blocks based on the error rate in the simulator model. For example, if the error is assumed to be equal to 0.1, based on the information given in Figures 3 and 4, this error rate will be reached at a threshold of approximately 0.07, which means that the final block number of the upscaled model will be 239. In fact, by accepting an error up to 0.1 for the simulator model, we can reduce the number of fine scale model cells from 1613 to 239 blocks in a coarse scale model. According to the opposite of this view, if the goal of upscaling is to reach the number of final blocks 150, in this case, the bandwidth of the kernel function will be 0.11 and the error rate obtained for this value will be about 0.661. For each one of these two states, the upscaling results are shown in Figure 5. This Figure shows the upscaling signal with bandwidths of 0.022, 0.044, 0.088, and 0.132, and the upscaled blocks in these states are 691, 398, 186, and 122, respectively.

Clearly, for each bandwidth, the simulator or coarse scale model will be different. In order to reach a unique answer to scale up the model, we need to determine the bandwidth or threshold optimal. Here are some approaches to determine the optimal bandwidth parameter.

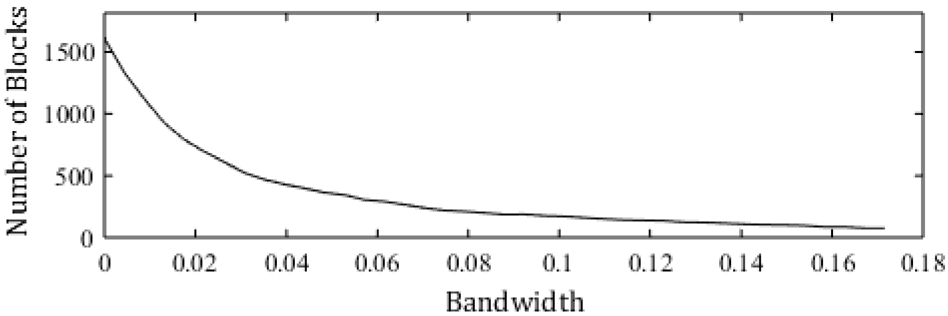


Figure 3. Variation in the number of blocks versus bandwidth.

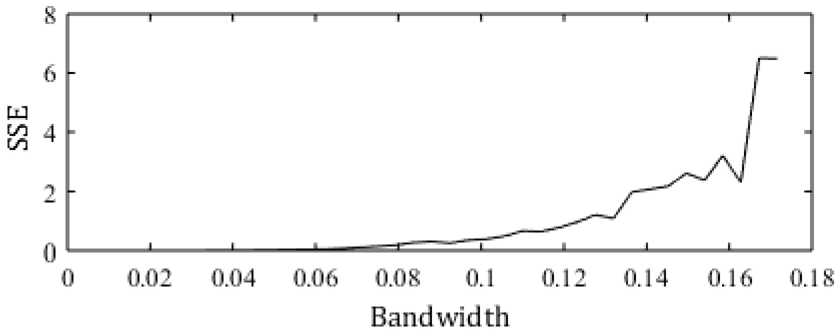
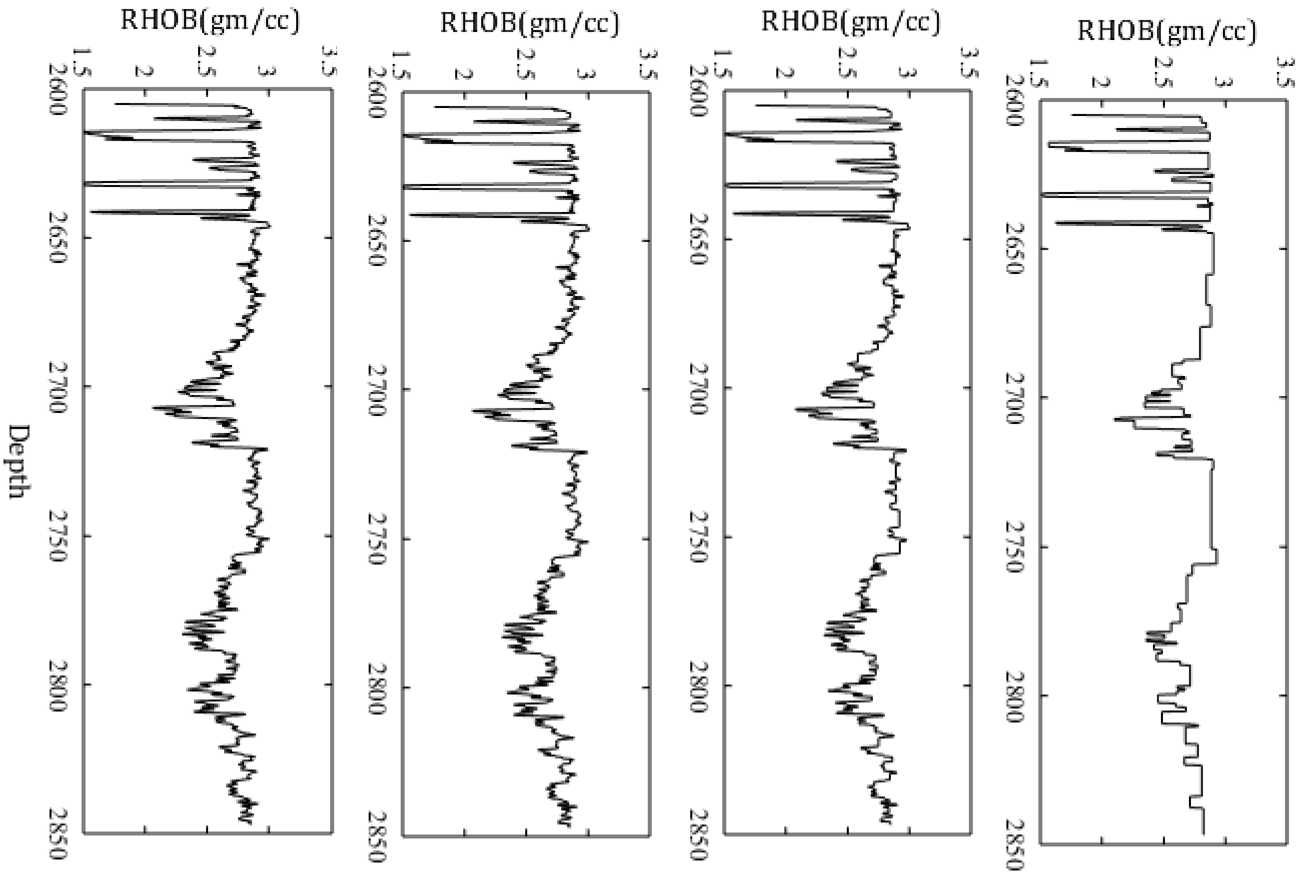


Figure 4. Variation in SSE versus bandwidth.



(a) (b) (c) (d)

Figure 5. Upscaled signals with a) threshold of 0.022, b) threshold of 0.044, c) threshold of 0.088, and d) threshold of 0.132.

5. Determination of optimal threshold

As mentioned earlier, for each threshold value, the number of upscaled blocks and SSE values can be obtained from Figures 3 and 4. Choosing the optimal threshold is a key parameter in the upscaling process. In the upscaling process based on the kernel bandwidth, there is an upscaled model for each threshold or bandwidth. To determine the coarse scale optimal model, it is necessary to determine the optimal threshold. One of the easiest methods available to select the optimal bandwidth is the number of upscaled blocks. With regard to what has been said, each bandwidth has its own upscaling parameters. Given the ability to select the number of blocks in this method, depending on the number of blocks, the optimal bandwidth can be selected, i.e. if the goal of upscaling is to convert the fine-scale model to a coarse-scale model with n upscaled blocks, then the appropriate bandwidth can be obtained based on this value. For example, if we want a coarse-scale model with 300 blocks, the bandwidth for this number will be 0.05. Furthermore in this work, three different approaches are used to determine the optimum threshold, referred to as below.

The first approach is to show variation in the upscaling error rate versus bandwidth. Figure 6 shows the variation in bandwidth or threshold against SSE. According to this approach, the point at which the upscaling error increases suddenly is selected as the optimum. The break point in this graph occurs in the bandwidth of 0.132, which is shown in Figure 6. After this point, SSE varies more severely. In a threshold of 0.132, the computational error is 1.08, and in the next step, i.e. at a threshold of 0.164, the error rate increases to 1.98. This increase in error results from a change in the number of upscaled blocks from 122 blocks to 115 ones. The challenge ahead in this approach is the number of breakpoints. To solve this problem, we can use the SSE differential method. However, according to the first approach, the first breakpoint can be considered as the optimum threshold.

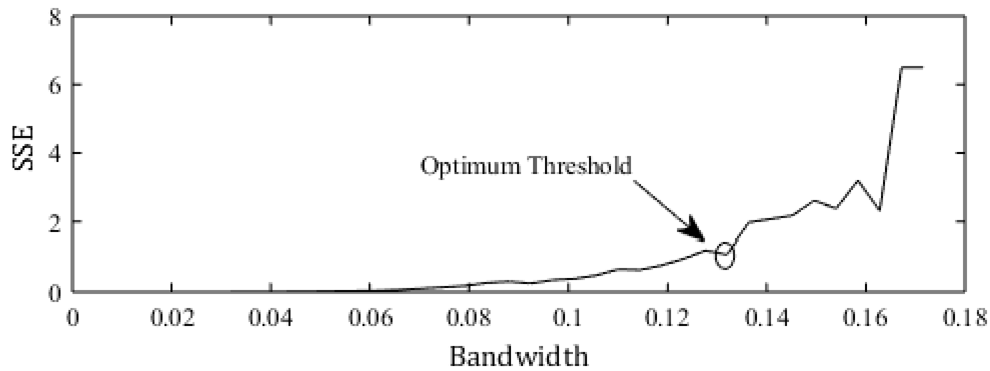


Figure 6. Determination of optimal threshold from the chart of SSE variation versus bandwidth.

When upscaling is carried out with a threshold of 0.132, the number of final upscaled blocks is 122 and the SSE value is equal to 1.08. Thus in this case, using the threshold value obtained from the proposed method, the number of 1613 cells in the fine-scale model becomes 122 blocks in the coarse-scale model and with only 7.5% of the initial data, and the simulation model can be presented from the main signal. In this case, in addition to maintaining the main features and variability in the log data, the computational time in the next steps (fluid flow simulation) will be much lower than the initial model with the number of initial cells.

The second approach is to examine the SSE differential versus the variable bandwidth. As noted, the SSE criterion is suitable to validate a coarse-scale model. If at each step the difference between the two successive SSE values is calculated and then these differences are plotted against the bandwidth, then we can get an idea of the optimal threshold, as shown in Figure 7. As it can be seen in this figure, at a threshold of 0.13, the SSE differential dramatically increases. At a point with a bandwidth of 0.13, the difference between two SSE values in two consecutive points is equal to 0.11 but in the next step of the bandwidth, this value increases to 0.9, which is a significant amount. The first extreme pick in the SSE differential can be considered as the optimum threshold. The results of the first and second approaches, presented above, are roughly the same but the SSE differential can provide a more appropriate pattern in the optimal bandwidth selection. Figure 7 shows the upscaled mode of the selected optimal threshold using the approach of SSE variation versus bandwidth.

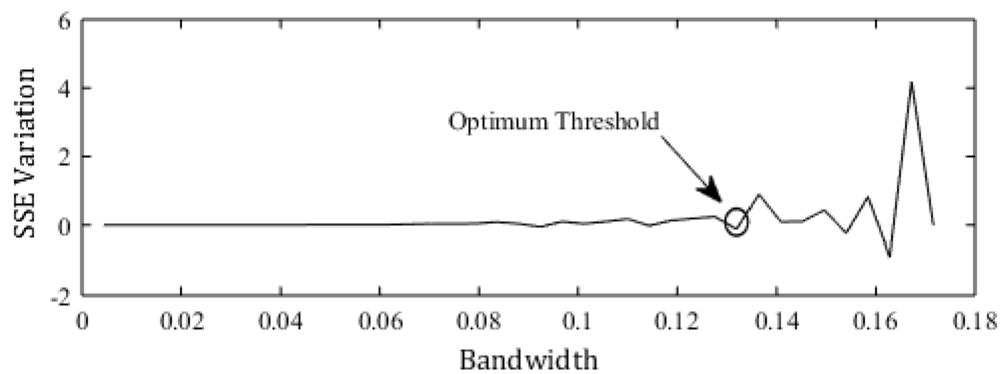


Figure 7. SSE differential versus bandwidth.

The third approach for determination of the optimal threshold is to find the intersection between two graphs. For this, it is necessary to draw up the SSE changes and the number of upscaled block versus bandwidth. Since the scale of these two parameters is different, it must first perform a normalization operation and convert all the data (N and SSE values) to N [0, 1]. Then by plotting these two graphs simultaneously versus the bandwidth changes, the intersection is where (normalized SSE = normalized N) can be considered as a criterion for determination of the optimum threshold. The intersection point of the two graphs occurs at a bandwidth of 0.10. When the upscaling based on this threshold is made, it is observed that at this point the number of upscaled blocks is equal to 158 and the calculation error rate is 0.61.

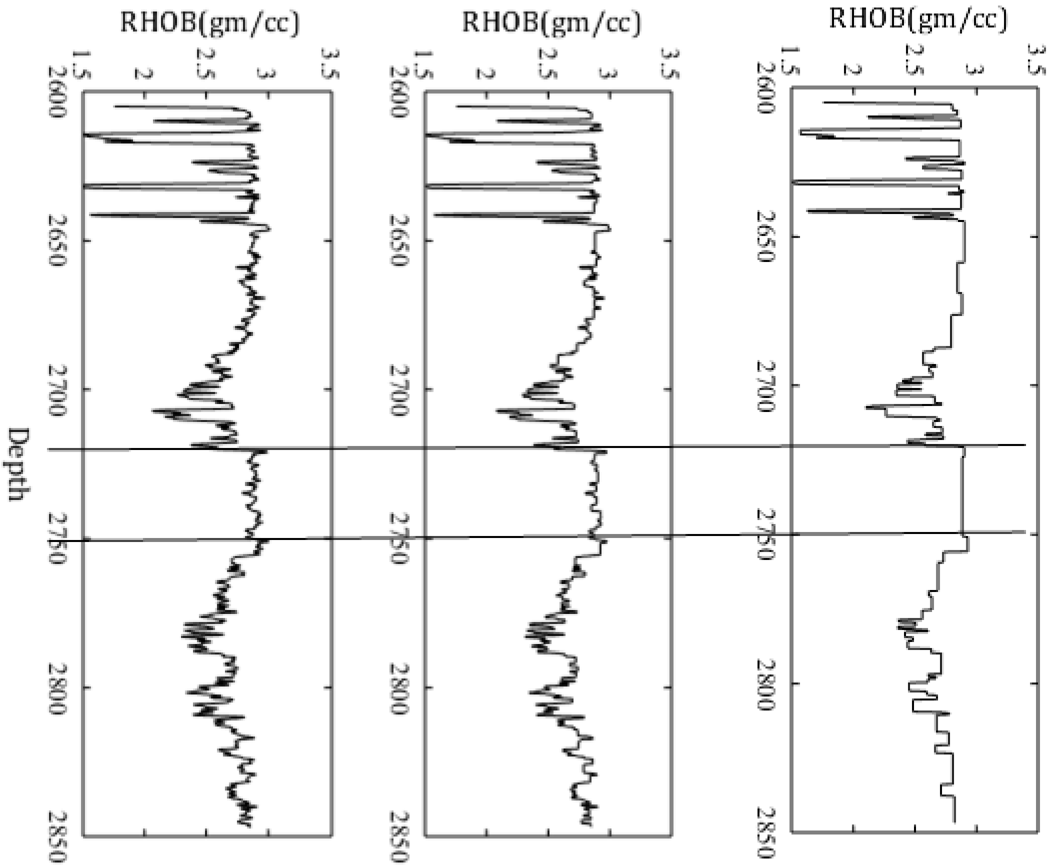
In summary, three methods have been used to determine the optimal threshold in the upscaling process based on the kernel function bandwidth. Based on different approaches, the upscaling results are presented and are shown in Table 1. Specifically, a higher threshold will result in a more upscaling error and a smaller number of upscaled blocks. In this particular case, if the goal of upscaling is to minimize the upscaling error, the intersection method will obtain a better result. Conversely, if the purpose of upscaling is computational cost reduction, the SSE variations approach will be more appropriate for the threshold setting.

The results of the signal upscaling with optimal threshold obtained from the SSE method and the intersection of the two diagrams are shown in Figure 8. The sections b and c in Figure 8 represent an upscaled signal with thresholds 0.10 and 0.132, respectively.

As shown in Figures 8, the regions with high variability (the beginning of the well) with both bandwidths remain fine scale so that up to a height of 2650 m, the fine-scale model has 266 cells, and in the coarse-scale model based on optimum bandwidth preserved 250 cells. However, in the middle part of the well, due to the smooth variability, 200 cells in the fine-scale model without loss of too much information have become 2 cells in the coarse model with 0.132 bandwidth.

Table 1. Determination of threshold and upscaling results with three approaches.

Approach / **Parameter**	SSE versus bandwidth	SSE differential	Intersection between SSE and N
Optimum threshold	0.132	0.13	0.10
SSE	1.08	0.98	0.61
Upscled blocks	122	128	158



(a) (b) (c)

Figure 8. a) Original signal, b) Upscaled signal with threshold 0.11, and c) Upscaled signal with threshold 0.132.

6. Conclusions

In the upscaling process with the bandwidth of the kernel function, choosing the optimal bandwidth or optimum threshold is a key point so that for each threshold value, the parameters of the upscaled model will be different. There are two important parameters in each upscaling model: the number of upscaled blocks and the upscaling error. Three different approaches have been investigated in the path of choosing the optimal threshold. The first approach is based on SSE changes versus bandwidth. In this approach, the first breakpoint is considered as the optimal point. The second approach is based on the intersection of two diagrams of upscaled block changes and SSE changes versus bandwidth or threshold. In this method, the intersection point of the two graphs will be an optimal threshold. Finally, the SSE differential is proposed as the criterion for optimal threshold selection. In choosing the optimal threshold, if the purpose of upscaling is decreasing the computational cost, the differential SSE method is more appropriate, and if the upscaling goal is to increase the accuracy, the intersection method will be better. The optimum threshold obtained by these methods is the maximum threshold that we can have in the upsclae process. Any amount below this optimal limit can be used for upscaling. Sometimes the optimum threshold can be determined by changing the target. If the acceptable error rate is specific in the model, we can easily determine the optimal threshold for the upscaling process, and this is one of the most important capabilities of upscaling based on the bandwidth of the kernel function, which is especially important in simulating fluid flow in reservoirs.

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تعیین پهنای باند بهینه در فرآیند افزایش مقیاس داده‌های مخزن با استفاده از پهنای باند تابع کرنل

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چکیده:

افزایش مقیاس بر اساس پهنای باند تابع کرنل، یک روش منعطف برای افزایش مقیاس داده‌ها است، چرا که سلول‌ها بر اساس تغییرپذیری‌شان درشت خواهند شد. میزان افزایش مقیاس سلول‌ها در این روش، با پهنای باند تابع کرنل کنترل می‌شود. در نواحی با تغییرپذیری هموار، سلول‌های زیادی ادغام خواهند شد و برعکس در نواحی با تغییرپذیری شدید، سلول‌ها به صورت ریز باقی خواهند ماند. تغییرات پهنای باند می‌تواند بر نتایج پهنای باند تأثیرگذار باشد؛ بنابراین، تعیین پهنای باند بهینه در این روش امری ضروری است. برای هر پهنای باندی، مدل نهایی افزایش مقیاس، یک تعداد سلول افزایش مقیاس یافته و یک خطای افزایش مقیاس دارد. بدیهی است که حد آستانه یا پهنای باند بزرگ‌تر منجر به افزایش خطا و همچنین کاهش تعداد سلول‌های بزرگ‌مقیاس خواهد شد. از طرف دیگر، با استفاده از پهنای باند کوچک‌تر، مدل نهایی افزایش مقیاس در مقیاس ریز باقی‌مانده و در واقع افزایش مقیاس قابل‌توجهی انجام نخواهد شد. در این پژوهش، رویکردهای متفاوتی برای تعیین پهنای باند یا حد آستانه بهینه استفاده شده است. بررسی دیفرانسیل SSE، تقاطع دو نمودار SSE و تعداد بلوک‌های افزایش مقیاس در مقابل پهنای باند و همچنین تغییرات SSE در مقابل پهنای باند از جمله آن رویکردها هستند. در عمل، اگر هدف از افزایش مقیاس کمینه کردن خطای افزایش مقیاس باشد، روش تقاطع نتایج بهتری را ارائه می‌دهد. در مقابل اگر هدف از افزایش مقیاس، کاهش هزینه محاسباتی باشد، رویکرد تغییرپذیری SSE روش مناسب‌تری برای انتخاب حد آستانه خواهد بود.

کلمات کلیدی: افزایش مقیاس، حد آستانه بهینه، دیفرانسیل SSE، کرنل، پهنای باند.