Journal of Physical and Theoretical Chemistry
of Islamic Azad University of Iran, 10 (2) 143-149: Summer 2013
(J. Phys. Theor. Chem. IAU Iran)
ISSN 1735-2126

Thermal negativity in a two qubit XXX and XX spin chain model in an external magnetic field

M. Ghasemloo, N. Nikdel, M. R. Soltani*, M. Firoozi and F. Soheilian

Department of Physics, College of Science, Shahre- Rey Branch, Islamic Azad University, Tehran, Iran

Received February 2014; Accepted February 2014

ABSTRACT

In this paper we studied the thermal negativity in a two-qubit XX spin ½ chain model and XXX spin 1/2 chain model(isotropic Heisenberg model)spin-1/2 chain subjected to an external magnetic field in z direction. We calculate analytical relation for the thermal negativity for two qubit XX and XXX spin chain models in the external magnetic field. Effects of the magnetic field and temperature on the negativity are shown. We have also plotted two kind of behavior in the thermal negativity which scales by magnetic field and temperature. It is shown that when the magnetic field along in axis z is considerable, thermal negativity can be decrease for higher temperature. We found the critical temperature of negativity for XX spin chain model and XXX spin chain model with added external magnetic field where there are independent from magnetic field. We also have shown in both of two models after $T=T_C$ the negativity is zero. We find the critical temperature for this models and show that the critical temperature is independent of magnetic field. We found that the negativity with increasing the temperature T, at first the quantum correlations in the system increase, smoothly reaches the maximum, and then after the critical temperature turn into zero.

Keywords: Thermal negativity; XXX spin chain model; XX spin chain model; External magnetic field; Critical temperature

INTRODUCTION

In recent year, entanglement has been studied by physicist and chemist. It is a useful concept in quantum phase transition in physical chemistry[1-3] and condense matter physics.[4-8] The entanglement can be observed in low temperature quantum many body systems. The single qubit gates are not able to generate entanglement in an N-qubit, because for single qubit is separable state and starting from separable state we will obtain another separable state and for to prepare an entanglement state must using two qubit. The Heisenberg magnetic spin chain is a very important interaction in magnetic material. This model have been studied in very experimental and theoretical research.[9] For understanding quantum entanglement behavior of many body system have been

*Corresponding author: soltani@iausr.ac.ir

studied in two or more qubits consist of spin-half, spin – integer and mixed – spin of Heisenberg spin systems or another spin model. For measure of quantum entanglement used the concurrence [10,11] and negativity .[12-17] For study thermal entanglement of a system consist of the higher spins used the negativity.[18] The quantum entanglement of two qubit in such models has been studied.[19] many of papers studied the entanglement of Heisenberg spin chain model with use concurrence. The Heisenberg model, including XXX (isotropic Heisenberg spin model) model [20], XXZ model [21], XYZ model [22], XX model [23], XY model [24,25], studied in thermal entanglement by concurrence. Thermal negativity of the two qubit XYZ Heisenberg model with inhomogeneous magnetic field have been studied.[26] They shown that induce entanglement have been existence. The effects of the staggered magnetic field on the thermal entanglement have been have been studied and it is found that the thermal entanglement depends on the anisotropy parameter and the staggered magnetic field. [27] The bound entanglement in the XY model is also studied. [28]

This paper is prepared as follows. We review the concept of the negativity. Then we study the thermal negativity for two-qubit in XX and XXX models with added external magnetic field and we find that critical temperature and we show that the critical temperature is independent of magnetic field. Finally, we conclude and summarize our results.

NEGATIVITY OF TYPICAL PURE STATES

If ρ is the density matrix of a two qubit system and is the partial transpose with respect to system A then the negativity for this system is defined as [29]:

$$N(\rho) = \frac{\left\|\rho^{T_A}\right\| - 1}{2} \tag{1}$$

where ||...|| denotes the trace norm of ρ^{T_A}. ρ^{T_A} is the partial transpose of density operator with respect to the A subsystem. The partial transpose is defined as:

$$\left\langle e_i^{(1)} e_j^{(2)} \middle| \rho^{T_A} \middle| e_k^{(1)} e_l^{(2)} \right\rangle = \left\langle e_i^{(1)} e_l^{(2)} \middle| \rho \middle| e_k^{(1)} e_j^{(2)} \right\rangle$$

where $e_i^{(1)}, e_j^{(2)}$ are the bases of Hilbert space.

In equation (1) $\left\|\rho^{T_A}\right\|$ is:

$$\left\|\rho^{T_A}\right\| = \sum_i |\lambda_i|$$

where λ_i are eigenvalues of ρ^{T_A}. If the system at the thermal equilibrium the density operator is:

$$\rho(T) = \frac{\exp(-\beta H)}{Z}$$

where $Z = tr(\exp(-\beta H))$ is the partition function of the system.($\beta = \frac{1}{k_B T}, k_B = 1$)

The negativity is a quantitative measure of entanglement for any system of any dimension. The negativity are ranged from zero(quantum states are unentangled) to one(quantum states are maximally entangled). The states are unentangled if and only if $\rho_{11}\rho_{44} \geq |\rho_{23}|^2$ and the states are entangled if and only if $\rho_{11}\rho_{44} \langle |\rho_{23}|^2$. [30] also, we can find the critical temperature for thermal negativity, i.e.

$T < T_C \;\; \rightarrow \;\; N \neq 0$

$T \geq T_C \rightarrow N = 0$

where T_C is the critical temperature.

If d is the smaller of the dimensions of the bipartite system we can define the negativity for higher dimension as: [31]

$$N(\rho) = \frac{\left\|\rho^{T_A}\right\| - 1}{d - 1} \tag{2}$$

NEGATIVITY OF TWO QUBIT XX MODEL WITH AN EXTERNAL MAGNETIC FIELD

In this section we study a system consisting of two qubit XX spin chain model the spins 1/2 with added magnetic field interaction. The Hamiltonian of this model is:

$$H = J\left(S_1^x S_2^x + S_1^y S_2^y\right) + \mathrm{h}(S_1^z + S_2^z) \quad (3)$$

where S_i^x, S_i^y, S_i^z are elements of spin for $S = \frac{1}{2}$, J is the exchange coupling and h is a magnetic field in the z direction. The eigenvalues and eigenstates of energy are

$$\varepsilon_1 = \frac{J}{2} \quad , \left|1\right\rangle = \frac{1}{\sqrt{2}}(\left|\uparrow\downarrow\right\rangle + \left|\downarrow\uparrow\right\rangle)$$

$$\varepsilon_2 = -\frac{J}{2} \quad , \left|2\right\rangle = \frac{1}{\sqrt{2}}(-\left|\uparrow\downarrow\right\rangle + \left|\downarrow\uparrow\right\rangle)$$

$$\varepsilon_3 = h \quad , \left|3\right\rangle = \left|\uparrow\uparrow\right\rangle$$

$$\varepsilon_4 = h \quad , \left|4\right\rangle = \left|\downarrow\downarrow\right\rangle$$

The thermal density matrix operator is $\rho(\mathrm{T}) = \frac{e^{-\beta H}}{Z}$, where $Z = Tr(e^{-\beta H})$ the partition function is ($\beta = \frac{1}{kT}$). The elements of the density matrix of Hamiltonian (3) are:

$$\rho_{11} = \frac{1}{Z}e^{-\beta \mathrm{h}}, \quad \rho_{44} = \frac{1}{Z}e^{\beta \mathrm{h}} \quad (4)$$

$$\rho_{22} = \rho_{33} = \frac{1}{Z}\cosh\frac{\beta J}{2}$$

$$\rho_{23} = \rho_{32} = -\frac{1}{Z}\sinh\frac{\beta J}{2}$$

where:

$$Z = 2\cosh\frac{\beta J}{2} + 2\cosh\beta h \quad (5)$$

By using of partial transpose we calculate the eigenvalues of ρ^{T_A} as:

$$\lambda_{1,2} = \rho_{22}$$

$$\lambda_{3,4} = \frac{1}{2}[(\rho_{11} + \rho_{22}) \pm \sqrt{(\rho_{11} - \rho_{22})^2 + 4|\rho_{23}|^2}\,] \quad (6)$$

By using the eq. (1) we found the thermal negativity as:

$$N = \frac{1}{2Z}[-\cosh\beta h + \sqrt{\sinh^2\beta h + \sinh^2\frac{\beta J}{2}} + \left|\cosh\beta h - \sqrt{\sinh^2\beta h + \sinh^2\frac{\beta J}{2}}\right|] \quad (7)$$

The critical temperature (T_C) for this model is found that:

$$T_C = \frac{J}{2Ln(1+\sqrt{2})}$$

In figure 1, we have plotted the negativity as a function of the magnetic field h for different values of the temperature T=0.2, 0.3, 0.4, 0.5. As is seen, in the absence of the magnetic field the system is entangled. By increasing the magnetic field, entanglement decreases and will be zero for sufficient values of the magnetic field. In addition, in figure 2, we have plotted the negativity as a function of the temperature, T, for different values of the magnetic field h=0.5, 0.7, 0.9, 1.0. It is clear it seen, that at T=0, the system is entangled for some small values of the magnetic field. In this case, by increasing the temperature, entanglement decreases and will be zero at a critical temperature. For some values of the magnetic field, the entanglement first is zero and by increasing the temperature, system will be entangled. But by more increasing the temperature, the entanglement will be zero at a critical temperature due to the sufficient thermal.

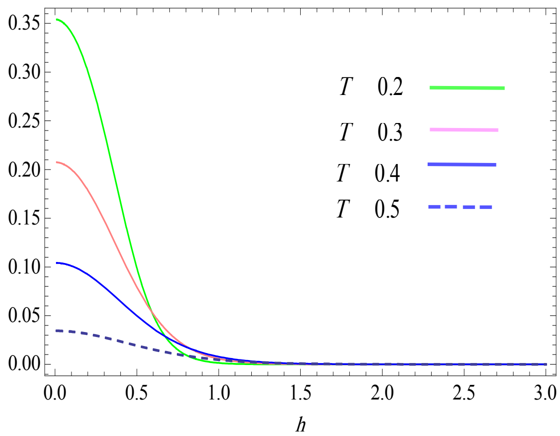

Fig.1. The negativity for ant ferromagnetic case J =1 as a function of magnetic field for a XX model with added magnetic field.

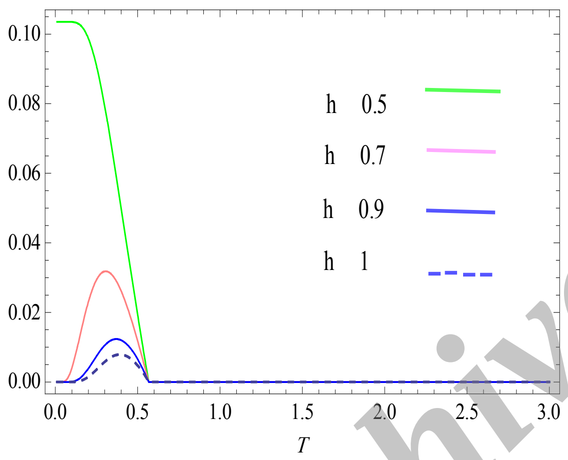

Fig.2. The negativity for ant ferromagnetic case J =1 as a function of temperature for a XX model with added magnetic field.

NEGATIVITY OF TWO QUBIT XXX MODEL WITH AN EXTERNAL MAGNETIC FIELD

In this section, we study thermal negativity of two qubit in a XXX spin chain model with an external magnetic field. The Hamiltonian of this model is

$$H = J(S_1^x S_2^x + S_1^y S_2^y + S_1^z S_2^z) + h(S_1^z + S_2^z) \quad (8)$$

The eigenvalues and eigenstates of energy are:

$$\varepsilon_1 = -\frac{3J}{4} \qquad ,|1\rangle = \frac{1}{\sqrt{2}}(-|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)$$

$$\varepsilon_2 = \frac{J}{4} \qquad ,|2\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)$$

$$\varepsilon_3 = \frac{J}{4} + h \qquad ,|3\rangle = |\uparrow\uparrow\rangle$$

$$\varepsilon_4 = \frac{J}{4} - h \qquad ,|4\rangle = |\downarrow\downarrow\rangle$$

The elements of the density matrix of Hamiltonian (8) are:

$$\rho_{11} = \frac{1}{Z} e^{-\beta(\mathrm{h}+\frac{\mathrm{J}}{4})} \quad ,\rho_{44} = \frac{1}{Z} e^{-\beta(-\mathrm{h}+\frac{\mathrm{J}}{4})}$$

$$\rho_{22} = \rho_{33} = \frac{1}{Z} e^{\frac{\beta \mathrm{J}}{4}} \,\mathrm{Cosh}\, \frac{\beta J}{2}$$

$$\rho_{23} = \rho_{32} = -\tfrac{1}{Z}\, e^{\frac{\beta \mathrm{J}}{4}} \,\mathrm{Sin\,h}\, \tfrac{\beta J}{2} \quad (8)$$

where:

$$Z = 2e^{\frac{\beta \mathrm{J}}{4}} \mathrm{Cosh} \frac{\beta \mathrm{J}}{2} + 2e^{\frac{-\beta \mathrm{J}}{4}} \mathrm{Cosh}\, \beta \mathrm{h} \qquad (9)$$

By using the eq.(1) the thermal negativity found as:

$$N = \frac{e^{-\frac{\beta J}{4}}}{2Z} \left[-\cosh\beta h + \sqrt{\sinh^2\beta h + e^{\beta J}\sinh^2\frac{\beta J}{2}} + \left|\cosh\beta h - \sqrt{\sinh^2\beta h + e^{\beta J}\sinh^2\frac{\beta J}{2}}\right|\right] \quad (10)$$

The critical temperature for this model is found as:

$$T_C = \frac{J}{Ln3}$$

In figure 3 and figure 4, we have plotted the negativity as a function of the magnetic field h for different values of the temperature T=0.2, 0.3, 0.4, 0.5, the temperature, T, for different values of the magnetic field h=0.5, 0.7, 0.9, 1.0. As is seen, in the absence of the magnetic field the system is entangled. By increasing the magnetic field, entanglement decreases and will be zero for sufficient values of the magnetic field and by increasing the temperature, entanglement decreases and will be zero at a critical temperature. For some values of the magnetic field, the entanglement first is zero and by increasing the temperature, system will be entangled. But by more increasing the temperature, the entanglement will be zero at a critical temperature due to the sufficient thermal.

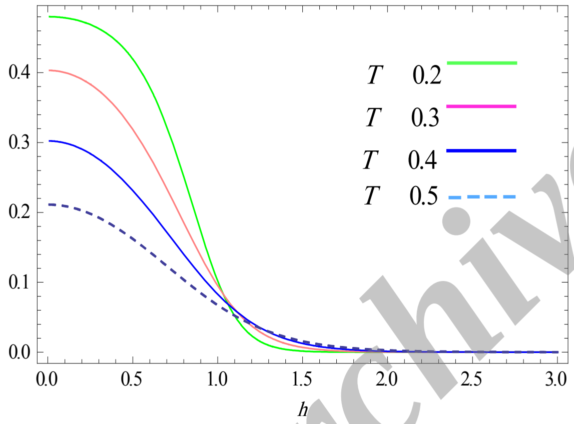

Fig.3. The negativity for ant ferromagnetic case J =1 as a function of magnetic field for a XXX model with added magnetic field.

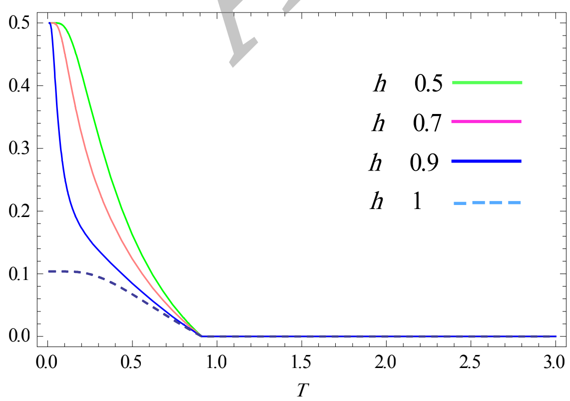

Fig.4. The negativity for ant ferromagnetic case J =1 as a function of temperature for a XXX model with added magnetic field.

CONCLUSION

We have investigated the effect of a magnetic field h on the thermal negativity of XXX and XX spin-1/2 chain model of two qubit with an external magnetic field. We have calculated thermal negativity for the antiferromagnetic case. Our calculations show that in the antiferromagnetic case that when the h magnetic field interaction along the z axis is considerable, thermal negativity can be decreased for higher temperatures. We found that the negativity with increasing the temperature T, smoothly reaches the maximum, and then turns into a zero. Also we found that the critical temperature (T_C) of negativity for XXX and XX spin-1/2 chain model of two qubit with an external magnetic field and we also plotted that the after critical temperature (T_C) the negativity is zero. We found that the critical temperatures are independent of magnetic field.

Acknowledgment

M.Ghasemloo and other authors gratefully acknowledge the financial and other support of this research, provided by the Islamic Azad University, Shahr-e Rey Branch, Tehran, Iran.

REFFERENCES

[1] Z. Huang, S. Kais, (2005), Entanglement as measure of electron–electron correlation in quantum chemistry calculations, Chem. Phys.Lett.413,1.
[2] H. Wang, S. Kais, (1997), Quantum teleportation in one-dimensional quantum dots system, Chem.l Phys. Lett. 421. 338.
[3] P. Gersdorf, W. John, J. P. Perdew, P. Ziesche, (1997), Correlation entropy of the H2 molecule, Int. J. Quantum. Chem. 61. 935.

[4] M. A. Nielsen, I. L. Chuang, (2000), Quantum Computation and Quantum Information, Cambridge Univ. Press.

[5] L. Masanes, (2006), All Bipartite Entangled States Are Useful for Information Processing Phys. Rev. Lett. **96**, 150501.

[6] M. Piani, J. Watrous, (2009,) All Entangled States are Useful for Channel Discrimination Phys. Rev. Lett. **102**, 250501.

[7] D. A. Meyer, (2000), Sophisticated Quantum Search Without Entanglement Phys. Rev. Lett. **85**, 2014.

[8] Datta, S. T. Flammia, Caves C. M., (2005), Entanglement and the power of one qubit Phys. Rev. A **72**, 042316.

[9] Datta, S. Gharibian, (2009), Signatures of nonclassicality in mixed-state quantum computation, Phys. Rev. A 79, 042325.

[10] S. Hill, W. K. Wootters,(1997), Entanglement of a Pair of Quantum Bits, Phys. Rev. Lett. 78, 5022.

[11] Datta, ,(2010), On the negativity of random pure states, Phys. Rev. A, 81, 052312.

[12] K. Zyczkowski, P. Horodecki, A. Sanpera., M. Lewenstein, (1998), Volume of the set of separable states, Phys. Rev. A58, 883.

[13] P. Calabrese, J. Cardy, E. Tonni, (2012), Entanglement negativity in quantum field theory, Phys. Rev. Lett. 109, 130502.

[14] G. Vidal, Werner R.,(2002), Computable measure of entanglement, Phys. Rev. A 65, 032314.

[15] Deschner, Sorensen, (2011), Impurity Entanglement in the J−J2−δ Quantum Spin Chain, J. Stat. Mech. P10023.

[16] S. Rai, J. R. Luthra, Negativity and Concurrence for two qutrits, quant-ph/0507263.

[17] G. Zhang , Y. Hou, A. Ji,(2011), Measurement-induced disturbance and thermal negativity of qutrit-qubit mixed spin chain, Solid State Communications, 151 790. Lee Y. A.,

[18] G. Vidal,(2013), Entanglement negativity and topological order, Phys. Rev. A 88, 042318.

[19] S. D.Han, E. Aydiner,(2013) Thermal entanglement and quantum phase transition in the mixed three spin XXZ model on a triangular cell, arxiv:1305.6230v1.

[20] G. L. Kamta , A. F. Starace, (2002), Anisotropy and Magnetic Field Effects on the Entanglement of a Two Qubit Heisenberg XY Chain, Phys. Rev. Lett.88,107901.

[21] M. Asoudeh, V. Karimipour, (2005), Thermal entanglement of spins in an inhomogeneous magnetic field, Phys. Rev. A 71, 022308.

[22] G. F. Zhang, S. S. Li, (2005), Thermal entanglement in a two-qubit Heisenberg XXZ spin chain under an inhomogeneous magnetic field, Phys. Rev. A 72, 034302.

[23] L. Zhou, H. S. Song, Y. Q. Guo, C. Li, (2003), Enhanced thermal entanglement in an anisotropic Heisenberg XYZ chain, Phys. Rev. A 68, 024301.

[24] F. Kheirandish, S. J. Akhtarshenas , H. Mohammadi, (20080, Effect of spin-orbit interaction on entanglement of two-qubit Heisenberg XYZ systems in an inhomogeneous magnetic field , Phys. Rev. A 77, 042309.

[25] K. Sengupta, D. Sen, (2009), Entanglement production due to quench dynamics of an anisotropic XY chain in a transverse field, Phys. Rev. A 80, 032304.

[26] Q. Meng, T. Dong-Ping, (2009), bipartite entanglement of a two-qubit system with anisotropic couplings

under nonuniform magnetic field, Chin. Phys.B, **18** (4), 1338-1341.

[27] K. Sengupta, D. Sen, (2009), Entanglement production due to quench dynamics of an anisotropic XY chain in a transverse field, Phys. Rev. A **80,** 032304.

[28] D. Patané, R. Fazio, L. Amico, (2007), Bound entanglement in the XY model, New J. Phys. **9,** 322.

[29] Peres, (1996), Separability Criterion for Density Matrices, Phys. Rev. Lett. 77, 1413.

[30] A.Sanpera, R. Tarrach, G. Vidal, (1998), Local description of quantum inseparability Phys. Rev. A 58, 826.

[31] S. Lee, D. P. Chi, S. D. Oh, J. Kim, (2003), Convex-roof extended negativity as an entanglement measure for bipartite quantum systems Phys. Rev. A, 68, 062304.