Journal of Physical and Theoretical Chemistry
of Islamic Azad University of Iran, 11 (3) 105-110: Fall 2014
(J. Phys. Theor. Chem. IAU Iran)
ISSN 1735-2126

Entanglement in the XZ spin chain model in the external magnetic field

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Received June 2014; Accepted August 2014

ABSTRACT

The entanglement at zero-temperature is studied in a two-qubit XZ spin-1/2 chain model in an external magnetic field which is applied in the z direction. We have obtained an analytical relation for the effect of the external magnetic field on the concurrence. It is shown that the ground state of the system depends on the strength of exchange coupling. We have shown that the analytical relation of the concurrence is independent of Jz. It is shown that for Jz >0 the concurrence is depended on the critical value of the external magnetic field and it is changed at the critical external magnetic field. Also we have shown that for h <hc the concurrence is maximum (C=1) or states of the system are maximally entangled. For values of the field h>hc, the concurrence decreases by increasing the external magnetic field. For Jz <0, the concurrence decreases by increasing the external magnetic field.

Keywords: Entanglement; XZ spin-1/2 chain model; External magnetic field; Critical external magnetic field; Density operator.

INTRODUCTION

Entanglement plays important role in physical chemistry [1-7] quantum physics [8] because it has attractive non classical nature such as quantum information [9] quantum teleportation [10], quantum cryptography [11, 12] quantum phase transition in condense matter [13], superconductivity [14, 15], quantum dots[16,17]. The Entanglement in quantum many body system prepare in low temperature or zero temperature. To give information about the other subsystem and provides possibilities of manipulating, must be a measurement on one subsystem. The single qubit gate is unable to generate entanglement thus to prepare an entanglement state must using two qubit system. Concurrence [18, 19], discord [20], and negativity [21] used for measurement of entanglement. The concurrence and discord used for measure of entanglement of spin- ½ systems, while the negativity used for measure of entanglement for any spin systems. The important model of spin chain is Heisenberg spin model.

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The Heisenberg spin chains have been used to quantum computer [22] and perform quantum computation [23]. The quantum entanglement of the Ising spin chain model and Heisenberg spin chain model have been studied [24-25]. The quantum entanglement of two qubits in such models have been studied. The isotropic Heisenberg spin chain XXX model in the antiferromagnetic case in low temperature are entangled while for ferromagnetic case are unentangled.[26] The other special models of Heisenberg, XXZ model[27], XX[28] model have been studied and shown that the entanglement of the system depends on the coupling constant and the temperature. The effect of magnetic field on the entanglement of the spin chain at zero temperature and in the temperature T have been studied and is shown that the entanglement of the state depends on the magnetic field.[29-32] Also it is shown that decreasing the entanglement of the state with increasing magnetic field.

In this paper we study the concurrence of the one dimensional two qubit XZ model in a magnetic field in the z direction. First we review of the concurrence, second we obtain the eigenvalues and eigenstates of the model, and third we calculated the concurrence of the model at the zero temperature. Finally we summarized and conclusion of the report.

REVIEW OF COCURRENCE

At zero temperature the density operator is given by:

$$\rho = |GS\rangle\langle GS| \qquad (1)$$

where is the ground state of the system. For measure of entanglement we used the concurrence. The concurrence is defined as [18-19]

$$C = \max\{\lambda_1 - \lambda_2 - \lambda_3 - \lambda_4, 0\} \qquad (2)$$

where $\lambda_i (i = 1,2,3,4; \lambda_1 \rangle \lambda_2 \rangle \lambda_3 \rangle \lambda_4)$ are the eigenvalues of the function R:

$$R = \sqrt{\rho\tilde{\rho}} \qquad (3)$$

where $\tilde{\rho} = (\sigma_y \otimes \sigma_y)\rho^*(\sigma_y \otimes \sigma_y)$ and σ_y is the y component of Pauli's matrix. The ranges of the concurrence vary between zero for unentangled states and one for maximally entangled states. Then the concurrence is bounded function.

EIGENVALUES AND EIGENSTATES OF THE TWO QUBIT XZ SPIN CHAIN MODEL IN THE EXTERNAL MAGNETIC FIELD

The Hamiltonian of the XZ spin chain model in the external magnetic field in the z direction for N qubit is written as:

$$H = \sum_{j=1}^{N}(J_x S_j^x S_{j+1}^x + J_z S_j^z S_{j+1}^z) + h\sum_{j=1}^{N} S_j^z \qquad (4)$$

where $J_i (i = x, z)$ are real exchange coupling constant coefficient,   is spin-1/2 operator on the j-th site and h is the magnetic field in the z direction. The two qubit of Hamiltonian (4) is:

$$H = J_x S_1^x S_2^x + J_z S_1^z S_2^z + h(S_1^z + S_2^z) \qquad (5)$$

By a straightforward calculation we can obtain the eigenvalues of (5):

$$\varepsilon_1 = -\frac{J_x + J_z}{4}$$
$$\varepsilon_2 = \frac{J_x - J_z}{4}$$
$$\varepsilon_3 = \frac{J_z + x}{4} \qquad (6)$$
$$\varepsilon_4 = \frac{J_z - x}{4}$$

with corresponding eigenstates:

$$
\begin{aligned}
|1\rangle &= \frac{1}{\sqrt{2}}(-|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle) \\
|2\rangle &= \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle) \\
|3\rangle &= \frac{1}{\sqrt{1+\alpha_+^{\,2}}}(\alpha_+|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle) \\
|4\rangle &= \frac{1}{\sqrt{1+\alpha_-^{\,2}}}(\alpha_+|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle)
\end{aligned}
\qquad (7)
$$

where $\alpha_\pm = \dfrac{4h \pm x}{J_x}$, $x = \sqrt{J_x^{\,2} + 16h^2}$.

CONCERNING OF THE XZ SPIN – ½ CHAIN MODEL IN THE EXTERNAL MAGNETIC FIELD AT ZERO TEMPERATURE

By using the eq.(2) we calculated the concurrence of the Hamiltonian(5) at zero temperature. We must obtain the density operator at zero temperature. The density operator at T=0 is used eq.(1). For calculation of density operator we must obtain the ground state of Hamiltonian (5).The ground state of Hamiltonian (5) depends on the J_x, J_z .

Case 1: $J_z \rangle 0$

In this case, the Hamiltonian (5) has a quantum phase transition at zero temperature. The critical value of the external magnetic field (h_c) obtain as:[33]

$$h_c = \frac{1}{2}\sqrt{J_z(J_z + J_x)} \qquad (6)$$

We selected the eigenstate of ground state as follow:

In the values of the field, $h \langle h_c$ the GS of the system depends on J_x, ie.

$$
\begin{aligned}
&J_x \rangle 0 : |GS\rangle = |1\rangle \\
&J_x \langle 0 : |GS\rangle = |2\rangle
\end{aligned}
\qquad (7)
$$

These states are Bell's states and the entanglement of the states for Bell's states are maximally i.e. C=1.

f $h \rangle h_c$ the GS is $|GS\rangle = |4\rangle$ and the elements of density matrix as:

$$\rho = \begin{pmatrix} \rho_{11} & 0 & 0 & \rho_{14} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \rho_{41} & 0 & 0 & \rho_{44} \end{pmatrix} \qquad (8)$$

where:

$$
\begin{aligned}
\rho_{11} &= \frac{(\sqrt{J_x^{\,2} + 16h^2} - 4h)^2}{J_x^{\,2} + (\sqrt{J_x^{\,2} + 16h^2} - 4h)^2} \\
\rho_{14} = \rho_{41} &= \frac{J_x(\sqrt{J_x^{\,2} + 16h^2} - 4h)}{J_x^{\,2} + (\sqrt{J_x^{\,2} + 16h^2} - 4h)^2} \\
\rho_{44} &= \frac{J_x^{\,2}}{J_x^{\,2} + (\sqrt{J_x^{\,2} + 16h^2} - 4h)^2}
\end{aligned}
\qquad (9)
$$

The concurrence is calculated by using the eq. (2) and we found that:

$$C = \frac{2|J_x|(\sqrt{J_x^{\,2} + 16h^2} - 4h)}{J_x^{\,2} + (\sqrt{J_x^{\,2} + 16h^2} - 4h)^2} \qquad (10)$$

Case 2: $J_z \rangle 0$

a) $J_z = 0.5$

By using the eq. (6), the critical value of the external magnetic field for $J_z = 0.5$ is $h_c = \dfrac{\sqrt{3}}{4}$. In figure 1 we have plotted the concurrence as a function of the external magnetic field for $J_z = 0.5$. For fields $h \langle h_c$ the entanglement is maximally or the entanglement of states is maximally. For the external magnetic field bigger than the critical value of external magnetic field, by increasing of external magnetic field the concurrence decreasing and for higher value of the external magnetic field the concurrence is limited to zero. We note that for critical of external magnetic field the concurrence is discontinues because the quantum phase transition occurred.

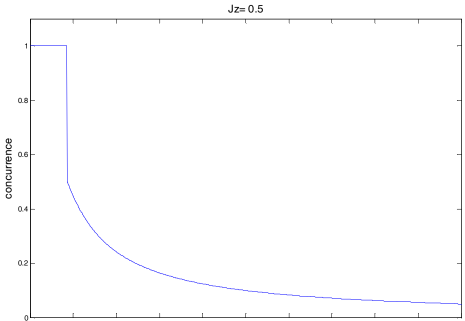

Fig. 1. The concurrence for $J_x = 1$ and $J_z = 0.5$ as a function of external magnetic field for two qubit XZ model in the external magnetic field.

b) $J_z = 1$

The critical value of the external magnetic field for $J_z = 1$ is $h_c = \frac{\sqrt{2}}{2}$. In this case the critical value of external magnetic field bigger than the critical value of external magnetic field for $J_z = 0.5$ .We plotted the concurrence as a function of external magnetic field in the figure 2. In this case we found that behavior of concurrence is same as the case $J_z = 0.5$. Note that the rang of external magnetic field of maximally entanglement for $J_z = 1$ is bigger than of $J_z = 0.5$.

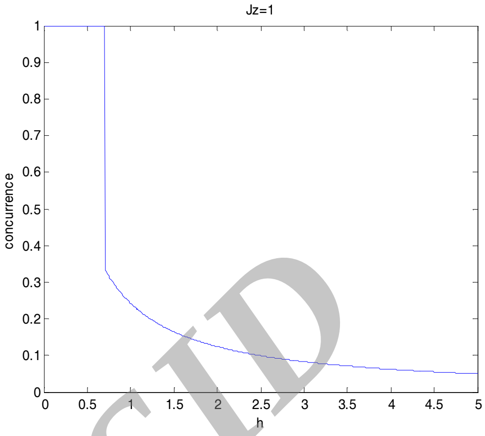

Fig. 2. The concurrence as a function of the external magnetic field for $J_x = 1$ and $J_z = 1$. It is seen that, up to h=1, the system is completely entangled.

c) $J_z = 2$

The critical value of the external magnetic field is $h_c = \frac{\sqrt{6}}{2}$ for $J_z = 2$. Figure 3 have been shown the concurrence as a function of the external magnetic field. The critical value of the external magnetic field in this case is bigger than of the critical value of the external magnetic field for $J_z = 1$.

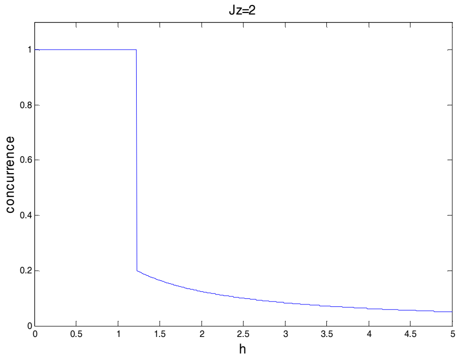

Fig. 3. The concurrence for $J_x = 1$ and $J_z = 2$ as a function of external magnetic field for two qubit XZ model in the external magnetic field.

Case 2: $J_z \langle 0$

The eigenstate of GS is $|4\rangle$ then the Hamiltonian (5) hasn't the quantum phase transition. By using the elements of density matrix (9) we calculated the concurrence and we found that the concurrence as the eq.(10). In the figure 4 we plotted the concurrence as a function of external magnetic field (h) for $J_x = 1$.

As seen in figure 4, in absence of magnetic field system is entangled maximally and with increasing of h, the concurrence decreases but it doesn't have critical magnetic field. In this case, concurrence doesn't depend on value of J_z and it shows the same behavior for all of the J_z value.

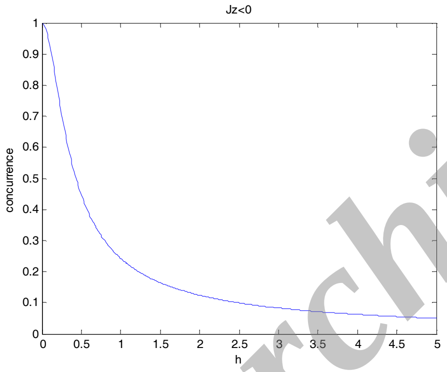

Fig. 4. The concurrence for $J_x = 1$ and any value of J_z as a function of external magnetic field for two qubit XZ model in the external magnetic field.

CONCLUSION

In this paper, we studied the Entanglement in a two qubit XZ spin-1/2 chain model in the external magnetic field in z direction at zero temperature. We find that for $J_z \rangle 0$ the system gains the quantum phase transition. It is shown that for $h \langle h_c$ the concurrence is constant(C=1) and for $h \rangle h_c$ by increasing the external magnetic field the concurrence decreasing. With increasing of J_z the quantum phase transition occurs in higher magnetic field. For $J_z \langle 0$ system doesn't represent quantum phase transition and by increasing the external magnetic field the concurrence is decreasing.

ACKNOWLEDGMENT

F. Soheilian gratefully acknowledge the financial and other support of this research, provided by the Islamic Azad University, Yadegar – e- Imam Khomeini (RAH) Branch, Tehran, Iran.

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