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Production model with Selling Price dependent demand and Partial Backlogging under inflation

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Abstract. We developed an inventory model for ecaying items with selling price dependent demand in inflationary environment. Deterioration rate is taken as two parameter Weibull distribution. Shortages in inventory are allowed with partial backlogging. Backlogging rate is taken as exponential decreasing function of time. Profit maximization technique is used in this study.

Keywords: Selling price demand, partial backlogging, Weibull deterioration.

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1. Introduction

As inventory represents a very important part of the company's financial assets, it is very much affected by the market's response to various situations, especially inflation. In present times, inflation is a global phenomenon. Inflation can be defined as that state of disequilibrium in which an expansion of purchasing power tends to cause or is the effect of an increase in the price level. A period of prolonged, persistent and continuous inflation results in the economic, political, social and moral disruption of society. Inflation and time value of money have also attracted attention of researchers. Taking these two factors into consideration is of vital importance. With the integration of the global economy, the economic relationships among countries are closer and the mutual influences are deep. Currency's purchasing power will change from time to time and inflation should not be neglected.

Buzacott (1975) developed the first EOQ model taking inflationary effects into account. In this model, a uniform inflation was assumed for all the associated costs

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and an expression for the EOQ was derived by minimizing the average annual cost. An economic order quantity inventory model for deteriorating items was developed by Bose et al. (1995). The effects of inflation and time-value of money were incorporated into the model. A generalized dynamic programming model for inventory items with Weibull distributed deterioration was proposed by Chen (1998). The effects of inflation and time-value of money on an economic order quantity model have been discussed by Moon and Lee (2000). Chung et al. (2001) followed the discounted cash flow (DCF) approach to investigate inventory replenishment problem for deteriorating items with static demand taking account of time value of money over a fixed planning horizon. The demand was assumed to be time-proportional, and the effects of inflation and time-value of money were taken into consideration. Models for ameliorating/deteriorating items with time-varying demand pattern over a finite planning horizon were proposed by Moon et al. (2005). The effects of inflation and time value of money were also taken into account.

Jaggi et al. (2006) presented the optimal inventory replenishment policy of deteriorating items under inflationary conditions using a discounted cash flow (DCF) approach over a finite planning horizon. Jaggi et al. (2007) presented the optimal inventory replenishment policy for deteriorating items under inflationary conditions using a discounted cash flow (DCF) approach over a finite time horizon. Chern, M. S. et al. (2008) developed an inventory lot-size model for deteriorating items with partial backlogging and time value of money. Roy, A., Pal, S. and Maiti, M.K. (2009) considered a production inventory model with inflation and time value of money and demand of the item is displayed stock dependent and lifetime of the product is random in nature and follows exponential distribution with a known mean. Yang, H.L., Teng, J.T. and Chern, M.S. (2010) developed an economic order quantity model under inflation with shortages and partial backlogging. This paper deals with production system for time dependent decaying items and selling price dependent demand. The effect of inflation is considered in this study. Shortages are permitted with time dependent partial backlogging rate. Profit maximization technique is used in this study. Numerical example is given to illustrate the solution procedure for the mathematical model.

2. Assumption and Notations

To develop the proposed inventory model, the assumptions and notations are discussed:

ASSUMPTIONS

(1) Time horizon of the inventory system is finite.
(2) The demand rate, $d(s)$, is any non-negative, continuous, convex, decreasing function of the selling price is $[0, s]$.
(3) Two parameter Weibull distribution deterioration rate is considered
(4) Shortages in inventory are allowed and partially backordered.

NOTATIONS

$$
\begin{aligned}
P &= \text{Production rate} \\
d(s) &= \text{Demand rate, } d(s) = as^{-b} \\
\theta &= \text{Deterioration rate, } \theta = \alpha\beta t^{\beta-1} \\
e^{-\delta t} &= \text{Backlogging rate, } 0 < \delta < 1 \\
r &= \text{A constant that represents the difference between} \\
&\quad \text{the discount rate (cost of capital) and inflation rate.} \\
C_1 &= \text{Holding cost per unit per unit time} \\
C_2 &= \text{Backorder cost per unit per unit time} \\
C_3 &= \text{Lost sale cost per unit} \\
C_4 &= \text{Setup cost per production cycle at } t = 0 \\
T &= \text{Length of the inventory cycle} \\
s &= \text{Selling price per unit} \\
NP &= \text{Net Profit per unit time}
\end{aligned}
$$

3. Formulation of the Model

We considered a production inventory system in which demand in market is met by its produced items. Production starts at $t = 0$ and continuous up to T_1 and inventory level reaches at highest level. At $t = T_2$ inventory level reaches at zero due to depletion from the combined effects of demand and deterioration. At $t = T_3$ shortages occurs and partially backlogged. At $t = T_3$ production starts again and backlog is cleared at $t = T$. The cycle repeats itself after time T. The inventory system with respect to time is describing by the following equations:

$$I'(t) + \alpha\beta t^{\beta-1} I(t) = P - as^{-b}, \qquad 0 \leqslant t \leqslant T_1 \tag{1}$$

$$I'(t) + \alpha\beta t^{\beta-1} I(t) = as^{-b}, \qquad 0 \leqslant t \leqslant T_2 \tag{2}$$

$$I'(t) = -e{-}\delta t a s^{-b}, \qquad 0 \leqslant t \leqslant T_3 \tag{3}$$

$$I'(t) = P - as^{-b}, \qquad 0 \leqslant t \leqslant T_4. \tag{4}$$

With boundary conditions $I(0) = 0,\ I(T_1) = S,\ I(T_2) = 0,\ I(T_3) = -R$ and $I(T) = 0$.

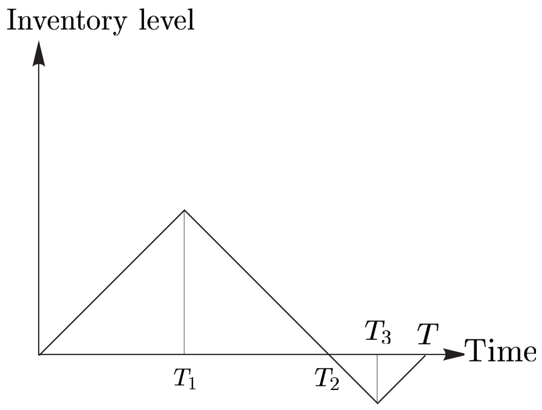

Figure 1.

Solutions of the above equations are:

$$I_1(t) = \left(P - as^{-b}\right)e^{-\alpha t^{\beta}}\left(t + \frac{\alpha t^{\beta+1}}{\beta+1}\right) \quad (5)$$

$$I_2(t) = e^{-\alpha t^{\beta}} as^{-b}\left(T_2 + \frac{\alpha T_2{}^{\beta+1}}{\beta+1} - t - \frac{\alpha t^{\beta+1}}{\beta+1}\right) \quad (6)$$

$$I_3(t) = as^{-b}\left(\frac{e^{-\delta t} - e^{-\delta T_2}}{\delta}\right) \quad (7)$$

$$I_4(t) = \left(P - as^{-b}\right)(t - T) \quad (8)$$

Now at $t = T_1$ from equation (5) and (6), we have

$$T_1 \cong \frac{as^{-b}}{P}\left(T_2 + \frac{\alpha T_2{}^{\beta+1}}{\beta+1}\right) \quad (9)$$

At $t = T_3$, from equation (7) and (8), we have

$$T_3 \cong T + \frac{P - as^{-b}}{as^{-b}}(T_2 - T) \quad (10)$$

Present Worth Holding Cost- Inventory occurs during T_1 and T_2 time period therefore, present worth the holding cost is given by

$$\begin{aligned} H.C &= C_1\left[\int_0^{T_1} I_1(t)e^{-rt}\,dt + \int_0^{T_2} e^{-r(T_1+t)} I_2(t)\,dt\right] \\ &\cong C_1\Big[(P - as^{-b})\left(\frac{T_1^2}{2} - \frac{rT_1^3}{3} - \frac{\alpha\beta T_1^{\beta+2}}{(\beta+1)(\beta+2)}\right) \\ &\quad + as^{-b}\left(\frac{(1 - rT_1)T_2^2}{2} - \frac{rT_2^3}{6} + \frac{\alpha\beta T_2^{\beta+2}}{(\beta+1)(\beta+2)}\right)\Big] \end{aligned} \quad (11)$$

Present Worth Shortage Cost- Shortage occurs during T_3 and T_4 time periods therefore present worth the shortage cost is given by

$$\begin{aligned} S.C &= C_2\Big[\int_0^{T_3} -I_3(t)e^{-r(T_1+T_2+t)}dt + \int_0^{T} -I_4(t)e^{-r(T_1+T_2+T_3+t)}dt\Big] \\ &\cong C_2\Big[\frac{as^{-b}}{\delta}\Big(\frac{e^{-\delta T_2-r(T_1+T_2+T_3)}}{-r} + \frac{e^{-\delta T_2-r(T_1+T_2)}}{r} + \frac{e^{-\delta T_3-r(T_1+T_2+T_3)}}{\delta+r} - \frac{e^{-r(T_1+T_2)}}{\delta+r}\Big) \\ &\quad + (P-as^{-b})\Big(\frac{e^{-r(T_1+T_2+T_3+T)}}{r^2} - \frac{e^{-r(T_1+T_2+T_3)}}{r^2} + \frac{Te^{-r(T_1+T_2+T_3)}}{r}\Big)\Big] \end{aligned} \tag{12}$$

Present Worth Lost Sale Cost- Lost sale cost occurs during $(0, T_3)$. Therefore, present worth the lost sale cost is given by:

$$\begin{aligned} L.S.C. &= C_3\int_0^{T_3}(1-e^{-\delta t})e^{-r(T_1+T_2+t)}as^{-b}dt \\ &= C_3 e^{-r(T_1+T_2)}as^{-b}\Big[\frac{1-e^{-rT_3}}{r} + \frac{e^{-(r+\delta)T_3}-1}{r+\delta}\Big] \end{aligned} \tag{13}$$

Present Worth Set Up Cost- The first cycle has an initial production setup cost C4, at the start of the cycle and the second production set up cost occurs at $t = T_1 + T_2 + T_3$ when shortages at maximum level. So set up cost is

$$S.T.C. = C_4 + C_4(1 - r(T_1 + T_2 + T_3)) \tag{14}$$

Present Worth Sales Revenue- In a periodic cycle, the production in period T_1 is consumed by demand and deterioration during periods T_1 and T_2. During T_4, all production consumed by demand and backorders. With instantaneous cash transaction during sales. The present worth sale revenue is

$$S.R. = s\Big[\int_0^{T_1} as^{-b}e^{-rt}dt + \int_0^{T_2} as^{-b}e^{-r(T_1+t)}dt + \int_0^{T} Pe^{-r(T_1+T_2+T_3+t)}dt\Big] \tag{15}$$

There are actually, n production cycles during the planning horizon. However, since inventory starts and ends at zero. The last set up before t = TN is planned to satisfy all the backorders.

$$\begin{aligned} &\text{The total no. of set up} = (N+1)\text{times;} \\ &\text{The first production on lot size} = PT1; \\ &\text{The } 2^{nd}, 3^{rd}, \ldots, n^{th} \text{ production lot size} Q = P(T1+T4); \\ &\text{The last for}(N+1)\text{production lot size} = PT \end{aligned} \tag{16}$$

Present Worth Net Profit- The time value of money affects the net profit except the first set up cost, therefore the total net present worth profit for planning horizon is

$$
\begin{aligned}
NP(s, T_2) &= (S.R. - S.T.C - H.C. - S.C - L.S.C.) \sum_{i=0}^{N} e^{irT} - C_4 \\
&= (S.R. - S.T.C - H.C. - S.C - L.S.C.) \left(\frac{1 - e^{-rNT}}{1 - e^{-rT}} \right) - C_4
\end{aligned} \tag{17}
$$

For maximization of profit function, the optimum solution is obtained by solving the following equations simultaneously:

$$\frac{\partial}{\partial T_2} NP(s, T_2) = 0 \tag{18}$$

$$\frac{\partial}{ds} NP(s, T_2) = 0 \tag{19}$$

provided, they satisfy the following conditions

$$\frac{\partial^2 NP}{\partial s^2} < 0, \quad \frac{\partial^2 NP}{\partial T_2^2} < 0.$$

4. Numerical Example

The preceding theory can be illustrated by the following numerical example where the parameters are given as follows:

$\delta = 0.04$, $s = 35$, $r = 0.05$, $P = 100$, $T = 25$, $\alpha = 0.03$, $\beta = 1$, $C_2 = 8$, $C_1 = 5$, $C_3 = 15$, $C_4 = 100$. Therefore, the optimal solution of the model is $T_2 = 18.7812$, $NP = 320783$.

Table 1. Variation of selling price w.r.t. time and profit

S	T_2	NP
36	18.5678	315670
37	18.4312	312813
38	18.2445	306577
39	18.094	305069
40	17.9842	298456

Table 2. Variation of backlogging rate w.r.t. time and profit

δ	T_2	NP
0.05	18.9781	320123
0.06	19.1062	317629
0.07	19.5416	312361
0.08	19.8763	306789
0.09	20.5374	305216

5. Conclusion

This paper presents a deterministic inventory model with time dependent deteriorating items and partially backlogged shortages. In most of the inventory models unrealistically assume that during stock-out either all demand is backlogged or all is lost. In reality often some customers are willing to wait until replenishment, especially if the wait will be short, while others are more impatient and go elsewhere. The backlogging rate depends on the time to replenishment-the longer customers must wait, the greater the fraction of lost sales. Therefore, we have taken the time dependent backlogging rate.

From numerical illustration, it has been concluded that if selling price and backlogging rate are increases that profit is decreases. This model is very useful in practical life. For further study, stochastic demand can be considered.

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