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A Cyclone Induced Storm Surge Forecasting Model for the Coast of Bangladesh with Application to the Cyclone 'SIDR'

M. Mizanur Rahman [a*], G. Chandra Paul [b], and A. Hoque [b]

[a] *Department of Mathematics, Shahjalal University of Science & Technology, Sylhet 3114, Bangladesh.*
[b] *Department of Mathematics, University of Rajshahi, Rajshahi-6205, Bangladesh .*

Abstract. The coast of Bangladesh has a specialty in terms of high bending and many offshore islands. Incorporation of the coastline and island boundaries properly in the numerical scheme is essential for accurate estimation of water levels due to surge. For that purpose a numerical scheme consisting of very fine mesh is required along the coastal belt, whereas this is unnecessary away from the coast. In this study, a fine mesh scheme covering the coastal belt and islands has been nested into a coarse mesh scheme covering up to 15 N latitude in the Bay of Bengal. For the existence of so many small and big islands and also for high bending of the coastline along the Meghna estuary, a very fine mesh scheme for the region between Barisal and Chittagong is again nested into the fine mesh scheme. A vertically integrated model is developed in Cartesian coordinate system to solve the shallow water equations using semi-implicit finite difference technique for computing surge associated with storms. The developed system is applied on a severe cyclonic storm 'SIDR' that hit Bangladesh on 15th November, 2007. The computed water levels are found to be in good agreement with those observed.

Keywords: Bay of Bengal, shallow water model, finite difference method, cyclone SIDR, water level.

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1. Introduction

A tropical cyclone has extreme consequences for the environment and ecology as well as having devastating, affects on human communities. Bangladesh is one of the most cyclone prone countries in the world. The southern coastal region in

*Corresponding author. Email: mizanmath_sust@yahoo.com

Bangladesh is where most of the cyclone affected areas are concentrated. Every year in pre-monsoon periods of April-May and in post-monsoon periods of November-December there is a high potential period for cyclone occurrence. In November, 2007, the cyclone SIDR with a wind speed 250 km/hr attacked the south coast of Bangladesh. The impact of the storm was severe: over 3000 people died, and extensive damage occurred to roads, bridges, houses, livestock, crops and so on (table 1).

Table 1. Damages by the SIDR

Category	Damage
People death	3,064
People missing	1,180
Families affected	16,11,139
People affected	68,51,147
Houses Damaged	1210685
Crop Damage	1705922 acre
Tree destroyed	3369366

During storms, the surge level along the coast of Bangladesh may be high even for a less intense storm. The factors responsible for this are extreme bending of the coastline, shallowness of water, off-shore islands, huge discharge through the Meghna and other rivers, low lying islands and coastal regions. Moreover, the head Bay of Bengal is a large tidal range (difference between high and low tides) area with the highest range around the Sandwip Island. Since the astronomical tidal oscillation is a continuous process in the sea, it becomes the initial dynamical condition in the tidal-surge interaction. The water level due to the tide-surge interaction becomes significantly high if a storm approaches the coast during a high tide period.

Many analyses on prediction of surge height due to tropical storm have been made all over the world.. Most of the works have been done for the Atlantic, North Sea and North-West European Continental Shelf, Australian region and Bay of Bangal. The SLOSH model of Jelesnianski et al. [2] is used as the operational surge forecasting model in USA. Thacker [3,4] incorporated the curving coast line and island boundaries with irregular grid finite difference technique as a substitute of the finite element method; but with less computational overheads. The studies on the development of storm surge models at the Institute of Oceanographic Sciences, Bidstone, England were documented by Heaps [5]. The studies of Arnold [6], Bills and Noye [7], and Tang and Grimshaw [8] for the Australian region are mainly based on investigation of various open sea boundary conditions. A considerable number of works on modeling have so far been done for the Bay of Bengal region covering the coast of Bangladesh and the East Coast of India. Out of them the models of Das et al.[9] and Johns et al. [13,14] are designed for the East Coast of India. But until now no attempt has been made to develop an efficient model suitable for operational forecasting purpose for the coast of Bangladesh. One of the major problems for numerical modeling of storm surge lies with the fact that fine resolution is required in the finite difference scheme near the coast in order to properly incorporate the bending of the coast line and off-shore islands whereas this is unnecessary away from the coast.

The present study attempts to develop a surge forecasting model based on the Cartesian coordinate system that incorporates the whole coastal belt and off-shore islands very accurately. For that purpose, we nested a fine mesh scheme (FMS) for the coastal belt of Bangladesh into the coarse mesh scheme (CMS) covering up to 150 N latitude. The region between Barisal and Chittagong is full of so many small and big islands and also the bending of the coastline is very high.

Considering those facts, a very fine mesh scheme (VFMS) for this region is nested into FMS. Vertically integrated shallow water equations in Cartesian coordinates are solved with the above mentioned doubly nested schemes. The model is verified using water-level data from the severe cyclonic storm 'SIDR' that hit south coast of Bangladesh on 15 November 2007. For analysis and verification of the results, 10 locations along the coastal belt are considered. The locations are Hiron Point, Patharghata, Kuakata, Char Madras (South Bhola), Chital Khali, Char Jabbar, Sandwip, Chittagong, Bashkhali and Cox's Bazar.

2. Mathematical Formulation

2.1 *Vertically Integrated Shallow Water Equations*

A system of rectangular Cartesian coordinates is used in which the origin, O, is in the undisturbed level of the sea surface as the xy-plane and Oz is directed vertically upwards. The displaced position of the free surface is considered as $z = \zeta(x, y, t)$ and position of the sea floor as $z = -h(x, y)$, so that, the total depth of the fluid layer is $\zeta + h$. Then the vertically integrated shallow water equations given by Roy [16] are

$$\frac{\partial \zeta}{\partial t} + \frac{\partial}{\partial x}[(\zeta + h)u] + \frac{\partial}{\partial y}[(\zeta + h)v] = 0 \tag{1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} - fv = -g\frac{\partial \zeta}{\partial x} + \frac{T_x}{\rho(\zeta + h)} - \frac{C_f\ u\sqrt{u^2 + v^2}}{(\zeta + h)} \tag{2}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} - fu = -g\frac{\partial \zeta}{\partial y} + \frac{T_y}{\rho(\zeta + h)} - \frac{C_f\ v\sqrt{u^2 + v^2}}{(\zeta + h)} \tag{3}$$

where

$u, v =$ velocity components of sea water in x and y directions, ms-1

$f =$ Coriolis parameter

$g =$ gravitational acceleration, ms-2

$T_x, T_y =$ components of wind stress, Nm-2

$h =$ ocean depth from the mean sea level, m

$C_f =$ friction coefficient

$\rho =$ water density, kgm-3

Also, in the above equations u and v are the vertically integrated components given by

$$(u, v) = \frac{1}{\zeta + h}\int_{-h}^{\zeta} (\bar{u}, \bar{v})dz \tag{4}$$

where $\bar{u}$ and $\bar{v}$ are x and y components of the Reynolds averaged velocity.

Using Eq. (1) we may express the Eqs. (2) & (3) in the flux form and thus, Eqs. (1)-(3) may be written as

$$\frac{\partial \zeta}{\partial t}+\frac{\partial u_0}{\partial x}+\frac{\partial v_0}{\partial y}=0 \tag{5}$$

$$\frac{\partial u_0}{\partial t}+\frac{\partial (uu_0)}{\partial x}+\frac{\partial (vu_0)}{\partial y}-fv_0=-g(\zeta+h)\frac{\partial \zeta}{\partial x}+\frac{T_x}{\rho}-\frac{C_f\ u_0\sqrt{u^2+v^2}}{(\zeta+h)} \tag{6}$$

$$\frac{\partial v_0}{\partial t}+\frac{\partial (uv_0)}{\partial x}+\frac{\partial (vv_0)}{\partial y}-fu_0=-g(\zeta+h)\frac{\partial \zeta}{\partial y}+\frac{T_y}{\rho}-\frac{C_f\ v_0\sqrt{u^2+v^2}}{(\zeta+h)} \tag{7}$$

where $(\tilde{u},\tilde{v})=(\zeta+h)(u,v)$

Here u and v in the bottom stress terms of Eqs. (2) and (3) have been replaced by $\tilde{u}$ and $\tilde{v}$ in Eqs. (6) and (7) in order to solve the equations numerically in a semi-implicit manner.

2.2 *Boundary Conditions*

At the closed boundaries of the mainland as well as of the islands the normal components of the mean current are taken as zero. The following radiation type of boundary conditions, due to Johns et al. [13], are taken for open sea boundaries:

$$\text{At the west boundary:} \qquad v+(\frac{g}{h})^{\frac{1}{2}}\zeta=0 \tag{8}$$

$$\text{At the east boundary:} \qquad v-(\frac{g}{h})^{\frac{1}{2}}\zeta=0 \tag{9}$$

$$\text{At the south boundary:} \qquad u-(\frac{g}{h})^{\frac{1}{2}}\zeta=0 \tag{10}$$

2.3 *Generation of wind stress*

The wind stress is, in general, parameterized in terms of the wind field associated with the storm. This is frequently done by the conventional quadratic law:

$$T_x=C_D\rho_a u_a(u_a^2+v_a^2)^{1/2} \qquad \text{and} \qquad T_y=C_D\rho_a v_a(u_a^2+v_a^2)^{1/2}$$

where C_D, ρ_a are the drag coefficient and air density respectively.

Generally, the storm information is available in terms of the maximum sustained wind speed (v_0) and the corresponding radius (R). The circulatory wind field is then generated by the empirical formula due to Jelesnianski [1], which is given by

$$v_a=\begin{cases} v_0\sqrt{(r_a/R)^3} & \text{for all} \quad r_a\leqslant R \\ v_0\sqrt{(R/r_a)^3} & \text{for all} \quad r_a>R \end{cases}$$

where r_a is the radial distance at which the wind field is desired. The x and y components (u_a,v_a) of the wind field are derived from v_a by the above empirical formula.

3. Numerical Method

3.1 *Set-up of the Nested Schemes*

In order to estimate the water level due to surge in the study area, it is necessary that the study area should be considerably big so that, a storm can move over the area at least for, say 3 days before crossing the coast. This is because; the surge response along the coast becomes significant well before a storm reaching the coast. On the other hand, in order to include the major islands in the estuary the mesh size (the distance between two consecutive grid points) should be considerably smaller whereas, this is unnecessary away from the estuary. Considering the above facts, a high-resolution numerical scheme (FMS) is nested into a coarse mesh scheme (CMS). Further, a very fine mesh scheme (VFMS) is nested in the FMS. The CMS covers the area between 15^0 N to 23^0 N latitude and 85^0 E to 95^0 E longitudes. The mesh size along north-south direction (along x-axis) is $\Delta x = 2.15$ km and along east-west direction (along y-axis) is $\Delta y = 3.29$ km. There are 92×95 grid points in the computational xy-plane. The VFMS covers the area between 21.77^0 N to 23^0 N latitude and 90.40^0 E to 92^0 E longitudes. The mesh size along north-south direction (along x-axis) is $\Delta x = 720.73$ m and along east-west direction (along y-axis) is $\Delta y = 1142.39$ m. There are 190×145 grid points in the computational xy-plane. As the computational method uses a rectangular grid, the coastline and the island boundaries have been approximated following the grid lines using stair step representation.

3.2 *Model Data Set-up*

All of the schemes have the same dynamical equations (5)-(7) with different boundary conditions. An important feature of this doubly nesting is that the CMS is completely independent. On the other hand, along the open boundaries of the FMS the parameter ζ is prescribed from those obtained in CMS in each time step of the solution process. Similarly for the VFMS, the parameter ζ is prescribed from those obtained in FMS in each time step of the solution process.

The governing equations given by (5)-(7) and the boundary conditions given by (8)-(10) are discretised by finite-difference (forward in time and central in space) and are solved by conditionally stable semi-implicit method using staggered grid system. For numerical stability, the x and y components of the momentum equations are modeled in a semi-implicit manner. For example, from Eq. (6) the term $\tilde{u}\sqrt{(u^2 + v^2)}$ is discredited as $u_0^{k+1}\sqrt{(u^{k^2} + v^{k^2})}$ where the subscript $k+1$ indicate that $\tilde{u}$ is to be evaluated in advanced time level. Moreover, the CFL criterion has been followed in order to ensure the stability of the numerical scheme. Along the closed boundary, the normal component of the velocity is considered as zero and this is easily achieved through appropriate stair step representation as mentioned earlier. The initial values of ζ, u and v are taken as zero. The time step is taken as 60 seconds that ensures stability of the numerical scheme. In the solution process, the values of the friction coefficient C_f and the drag coefficient C_D are taken as uniform throughout the physical domain, which are 0.0026 and 0.0028 respectively.

4. Analysis of Results and Model Verification

For the purpose of analysis, the computed results are presented at ten locations along the coast of Bangladesh for the severe cyclonic storm SIDR that hit south coast of Bangladesh on 15 November 2007. Records show that SIDR was one of the most severe of this century having maximum wind speeds 250 km/h (156m/s). History about SIDR is shown in Table 2, the data are received from the Bangladesh Meteorological Department (BMD).

Table 2. Track History of stom SIDR.

Synoptic Time	Latitude	Longitude
200711160000	25	91.9
200711151800	22.8	90.3
200711151200	20.9	89.5
200711150600	19.3	89.3
200711150000	17.8	89.2
200711141800	16.6	89.3
200711141200	15.7	89.3
200711140600	15	89.4
200711140000	14.3	89.6
200711131800	13.7	89.5
200711131200	13	89.6
200711130600	12.5	89.8
200711130000	12.1	89.9
200711121800	11.6	90
200711121200	11	90.3
200711120600	10.8	90.4
200711120000	10.4	90.8
200711111800	10.4	91.4
200711111200	10.2	91.9
200711110600	10	92.3

SIDR was located as a low pressure area over the southeast Bay of Bengal at 00 UTC of 11 November 2007. The low pressure system concentrated into a depression at 09 UTC of 11 November and lay centered at latitude 10.0 N, longitude 92.0 E about 200 km south-southwest of Port Blair and intensified into deep depression at 18 UTC of the same day. Moving in a northwesterly direction, the system intensified into cyclonic storm at 03 UTC of 12 November. Thereafter, the system rapidly intensified into severe cyclonic storm at 12 UTC and into a very severe cyclonic storm (90 knots) at 18 UTC on 12 November. The system continued to move in a northwesterly direction till 00 UTC of 13 November. Afterwards the system moved in a northerly direction up to 12 UTC on 15 November and then started to move north-northeastwards. It maintained the same intensity (90 knots) from 18 UTC on 12 November to 00 UTC on 15 November. The system further intensified to 115 knots at 03 UTC on the same day and crossed the west Bangladesh coast around 17 UTC near latitude 21.7 N, longitude 89.8 E with same intensity. After landfall the system weakened into cyclonic storm at 21 UTC of 15 November. The system further weakened into depression at 03 UTC and remained depression until 06 UTC on 16 November. The observed track of the system is presented in Figure 1. Figure 2 shows photo-like image of tropical cyclone SIDR which was captured by NASA's Terra satellite on November 12, 2007.

Figure 3 depicts the computed surge levels associated SIDR at Hiron Point, Chital Khali and Chittagong. It may be observed that, the maximum surge level is increasing with time as the storm approaches towards the coast and finally there is recession. At Hiron Point a strong recession is occurred after 14 hrs of 15th November, earlier than in any other locations and about 3 hrs before landfall of the storm (Fig.3). The recession takes place due to backwash of water from the shore towards the sea. In fact, Hiron Point is situated far left (west) of the storm path and so the direction of the anti-clock wise circulatory wind becomes northerly (i.e. towards the sea) at Hiron Point long before the storm reaches the coast and thus driving the water towards the sea. The recession reaches up to -1.6 m at 1730 hrs of 15th November. It may be noticed that the recession at Kuakata, Char Madras, Chital Khali, Cox's Bazar and Chittagong began approximately

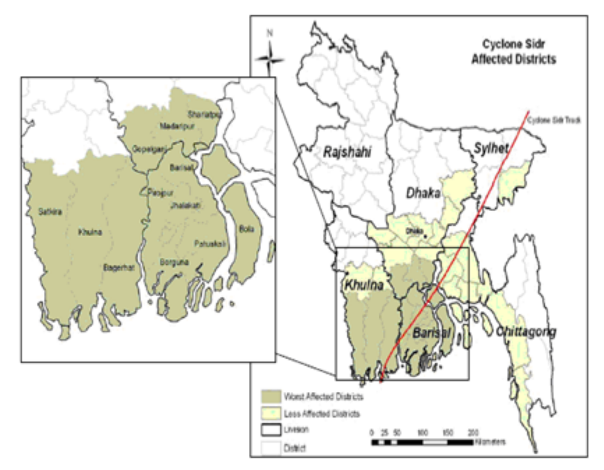

Figure 1. Observed Track of Cyclonic Storm "SIDR" with affected area.

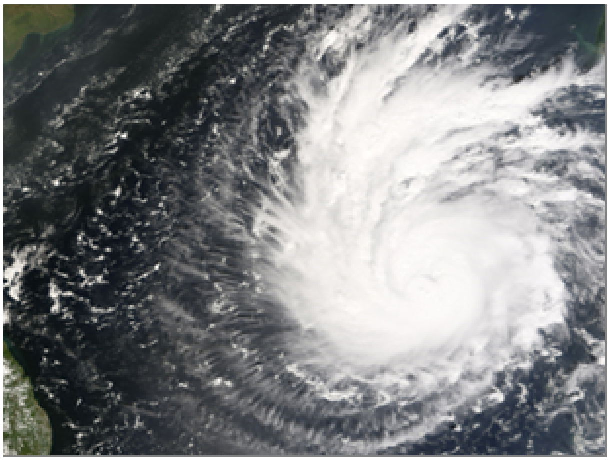

Figure 2. Tropical Cyclone Sidr. NASA's Terra satellite captured this photo-like image on November 12, 2007.

at 1500, 1630, 2000, 1700 and 2100 hrs (Figs. 3, 4 & 5) respectively. Thus the beginning of recession delays as we proceed towards east as is expected. At every location, the peak surge is attaining before the land falls time of the storm. This is expected, as the circulatory wind intensity is highest along the coast when the storm reaches near the coast. The maximum surge responses associated with SIDR at ten representative locations from west to east along the coastal belt of Bangladesh are shown in Figure 6. It is observed that the maximum surge increases from west to east up to Char Jabbar and then gradually decrease reaching minimum at Cox's Bazar. The maximum elevation varies between 1.90 m (at Cox's Bazar) to 6.68 m (at Char Jabbar). Finally, figure 7 shows the contours of the recommend water level associated with SIDR in the Meghna estuary region (in the VFMS region of our computed domain).

We could not compare our computed results of SIDR to the observed data due to non availability of observed time series data. But, according to storm surge analysis by the Institute of Water Modeling (IWM), there was 5.5 to 6 meters surge height at the outfall of Baleswar River and 3.5 meters at Hiron Point [21]. Thus, the computed surge heights are almost identical with the report that prepared by the Government of Bangladesh (2008).

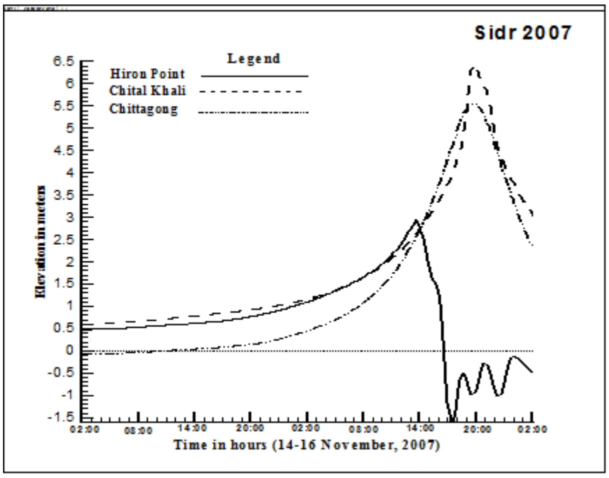

Figure 3. Computed water level (w.r.t. MSL) due to surge at different locations associated with Cyclonic storm Sidr 2007.

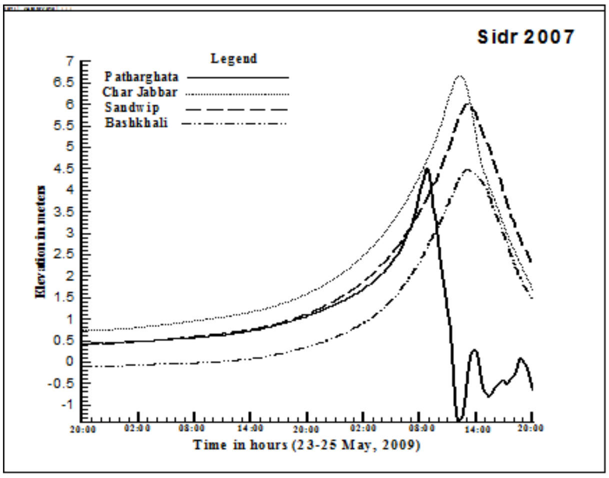

Figure 4. Computed water level (w.r.t. MSL) due to surge at different locations associated with Cyclonic storm Sidr 2007.

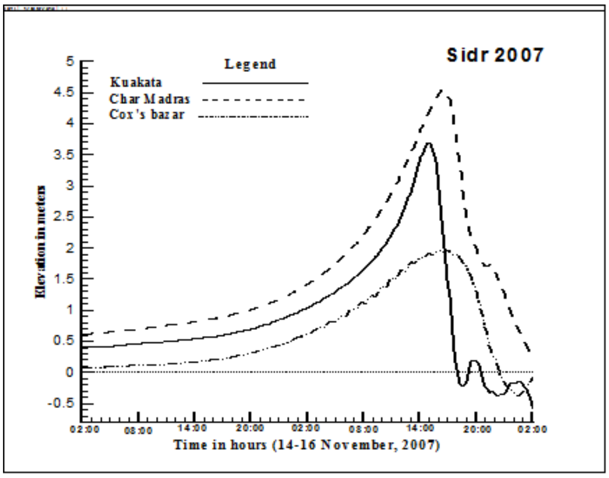

Figure 5. Computed water level (w.r.t. MSL) due to surge at different locations associated with Cyclonic Storm Sidr 2007.

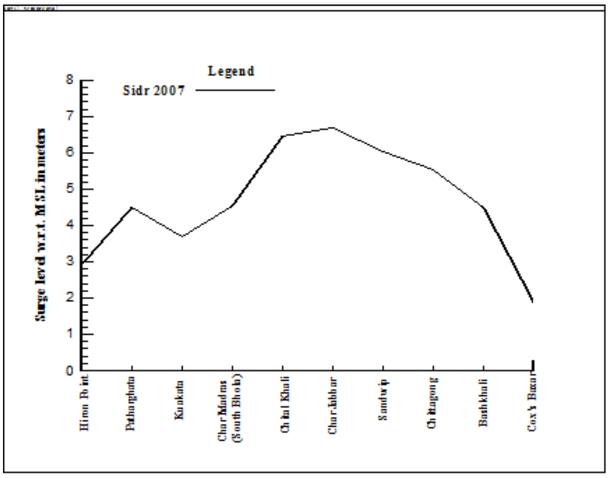

Figure 6. Computed maximum surge levels associated with Sidr 2007 along the coastal belt between Hiron Point and Cox's Bazar.

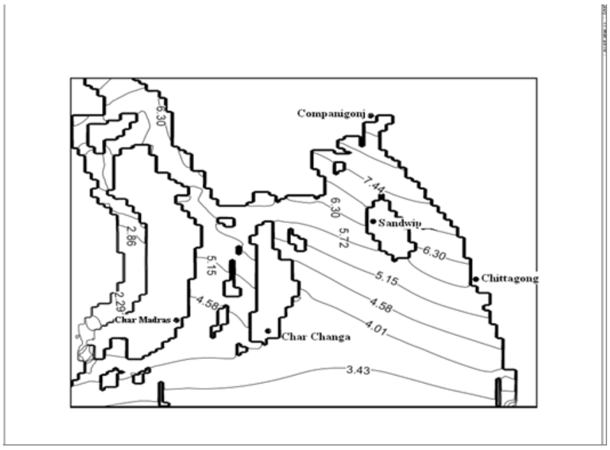

Figure 7. Contours of the recommend water level (in the VFMS region of the computed domain) associated with Sidr 2007 Cyclon.

5. Conclusion

In this study we have developed a nested numerical scheme for a Cartesian coordinate model that ensures fine resolution near the coast and coarse resolution in the deep sea. The scheme is found to be suitable for incorporating bending of the coastline and the island boundaries more accurately. The model is efficient to compute surge in the head Bay of Bengal, especially along the coast of Bangladesh. This model may be used to develop a operational forecasting model for the coast of Bangladesh and thus warning system may be improved during a storm period.

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